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A Theory of Noise Trading in Securities Markets

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ABSTRACT

In a recent article, Black [1] introduces a type of trading that he terms noise trading. He asserts that noise trading, which he defines as trading on noise as if it were information, must be a significant factor in securities markets. However, he does not provide an explanation of why any investors would rationally want to engage in noise trading. The goal of this paper is to provide such an explanation for one type of investor, managers of investment funds. As shown here, the incentive for a manager to engage in noise trading arises because of the positive signal that the level of the manager's trading provides about his or her ability to collect private information concerning current and potential investments. If the manager's compensation is directly related to investors' perceptions of his or her ability, the manager will then trade more frequently than is justified on the basis of his or her private information. In addition to providing this explanation for noise trading, the results of this analysis may also be useful for further empirical exploration of the relation between investment fund portfolio turnover and subsequent performance.

IN A RECENT ARTICLE, Black [1] introduces a type of trading that he terms noise trading. As he states, "Noise trading is trading on noise as if it were information. People who trade on noise are willing to trade even though from an objective point of view they would be better off not trading. Perhaps they think the noise they are trading on is information. Or perhaps they just like to trade." Although he does not give a reason why investors would rationally want to engage in noise trading, Black asserts that it must account for a significant proportion of total trading in securities markets. He states, "If there is no noise trading, there will be very little trading in individual assets." He gives two reasons for this. First, he says that people who desire to change their exposure to market risks or who want liquidity will prefer to trade in mutual or money market funds rather than in individual securities. Second, he says that people who have private information about these individual securities may also be reluctant to trade once they take into account that those with whom they are trading may also have private information.

The goal of this paper is to provide an explanation for why a certain class of investors, namely managers of investment funds, might find it in their interests to engage in noise trading even though it is not expected to result in positive returns. To understand the incentive for such trading, note that an investment manager's ability to generate returns on his or her fund's portfolio depends not

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only on the precision of any private information received but also on the frequency with which he or she obtains and trades on such information. Since realized performance is, in general, a noisy signal of the manager's ability, the level of informationally based trading then provides a supplementary signal of his or her ability. Investors will, therefore, find it valuable to use the level of informationally based trading in deciding how much to invest in the manager's fund; the greater the level of such trading, the more they will invest in the fund. However, investors can directly observe only the total amount of trading by the investment manager; they cannot distinguish between that which is informationally based and that which is noninformationally based. They must then make inferences about the level of informationally based trading given the total amount of trading. If investors believe that they are positively related, then the investment manager will have an incentive to trade more than is justified on the basis of private information alone. He or she will engage in noise trading. The manager will do so in an attempt to convey to investors a higher level of informationally based trading so that they will invest more in the manager's fund.

This study is similar in nature to that of Kanodia, Bushman, and Dickhaut [4] and Trueman [5], who show that agents may take actions that increase investors' perceptions of their ability to collect information. Kanodia, Bushman, and Dickhaut demonstrate that a manager may not give up an investment project found to be unprofitable since dropping it would reveal that the manager did not have accurate information at the time of investment. Trueman shows that a security analyst may be unwilling to fully revise his or her earnings forecast upon the receipt of new information because of the negative signal that such revision provides about the accuracy of the original forecast.

A further implication of the model developed here is that the amount of noise trading by any fund manager will increase with the riskiness of the fund's investments. This is because the prices of riskier assets are more likely to change over time. Consequently, it is more likely that a manager who receives private information about a riskier asset's future price will be trading in that asset. The absence of trading would more strongly indicate to investors that the manager did not receive any private information about the asset's future price. Uninformed managers, therefore, will have a greater incentive to trade in the riskier assets.

This model and its implications have a bearing on the recent empirical finding of Grinblatt and Titman [2], who show that portfolio turnover is positively related to subsequent mutual fund performance. Their result can be explained by the positive relation between the level of portfolio turnover and the extent of informationally based trading and between the latter and abnormal performance. However, as shown here, the relation between turnover and performance will be attenuated by the noise trading of uninformed managers; the more noise trading there is, the more tenuous will be this relation. Since noise trading is expected to be greater for riskier assets, it then follows that the observed relation between turnover and performance should be weaker for those funds with riskier investments.

The plan of this work is as follows. In Section I, the model is described. This is followed in Section II by an analysis of an equilibrium in which noise trading occurs and in Section III by a discussion and comparative statics analysis of the equilibrium. Possible extensions to the analysis are then described in Section IV. A summary in Section V concludes the paper.

I. The Model

Consider a two-period setting in which there are many risky assets and one riskless asset available for investment each period. Without loss of generality, the return on the riskless asset is set equal to zero. There are many (weakly) risk-averse investors in this economy. They have the choice of either investing directly in the available securities or placing their money in the one available investment fund. The fund is offered and managed by a risk-neutral investment manager who chooses the composition of the fund portfolio.

The investment manager's success at managing the investors' money is a function of his or her ability to collect private information each period about the future cash flows of the risky assets in the fund's portfolio. In order to simplify the analysis, it is assumed that the investment manager invests only in the riskless security and in the shares of a single risky firm.[1] This risky asset will generate one of three possible cash flows each period, either v_o, v_h, or v_l, where $v_h > v_l$ and $v_o = (v_h + v_l)/2$. Further, the cash flow of period 1 is assumed to be independent of that in period 2. At the beginning of each period, date $t = 0$, all individual investors have identical beliefs about the probability of occurrence of these cash flows. They believe that the probability of the period's cash flow equalling v_h (and also of equalling v_l) is given by r while the probability of it equalling v_o is given by $1 - 2r$. v_o can then be thought of as their expectation of the per-period cash flow of the risky asset. With probability p, the investment manager will be able to collect private information, supplementary to that of the other investors, at date 0 of each period about the magnitude of the risky asset's cash flow that period. With complementary probability $1 - p$, the manager will not be able to collect any private information during either period. It is assumed that investors do not know whether the manager has the ability to collect this private information.

The private signal that an informed manager receives has the following properties, which are common knowledge to the manager and to all investors:

$$pr(\text{signal } v_i \text{ received} \mid \text{period's cash flow equals } v_i) = t; \qquad i = h, l, o, \quad (1)$$
$$pr(\text{signal } v_j \text{ received} \mid \text{period's cash flow equals } v_i) = (1 - t)/2; \qquad j \neq i, \quad (2)$$

where $t > 1/3$.

The parameter t can be thought of as the precision of the private signal, with a larger value of t corresponding to a more precise signal. The condition that $t > 1/3$ ensures that the signal is valuable. Given this condition, if signal v_i is received, there is an upward revaluation of the probability of occurrence of cash flow v_i and a downward revision of the probability of occurrence of the other two possible cash flows.

All trading each period takes place at date 1 of the period. Before the first period's trading, the investment fund's portfolio consists of a positive number of

[1] Including many risky securities in the fund's portfolio would not affect the results of the analysis.

shares of both the risky and riskless assets. The investment manager may change the composition of the portfolio by buying or selling shares of the two assets. Short selling of either asset is precluded.

It is assumed that the manager's trading decision has a negligible impact on market prices. This is a reasonable assumption as long as the manager's investment demand is small relative to the total investment demand by all investors and as long as there is sufficient noise in the market. Given these conditions, investors will be unable to infer the manager's private information by observation of the amount of trading in the market.[2] The price of the risky asset will then be determined by the trading of the individual investors. To simplify the mathematics, it is assumed that the price set equals their expectation of the risky asset's future cash flows.

At the end of each period, date 2, the risky asset's cash flow that period is revealed and is paid out as a dividend to shareholders.

The following time line depicts the sequence of events for the two periods.

Date 0	Date 1	Date 2
Individual investors have same information, but manager may receive private information about period-1 cash flow of risky asset.	Trading takes place. Period-1 price of risky asset determined.	Period-1 cash flow of risky asset revealed and paid out as a dividend.

Sequence of events for period 1

Date 0	Date 1	Date 2
Individual investors have same information, but manager may receive private information about period-2 cash flow of risky asset.	Trading takes place. Period 2 price of risky asset determined.	Period-2 cash flow of risky asset revealed and paid out as a dividend.

Sequence of events for period 2

The investment manager's compensation in period i, C_i, $i = 1, 2$, is assumed to be a linearly increasing function of the fund portfolio's end-of-period market value, MV_i, given by

$$C_i = a + bMV_i, \tag{3}$$

[2] Even if this assumption were relaxed to allow for investors to make some inference about the manager's information from observation of the risky asset's price, the results of this analysis would still go through.

where a and b are positive constants. Denoting by R_i the return earned during period i and by I_i the total amount invested in the fund by investors at the beginning of period i, (3) can be rewritten as follows (assuming no withdrawal of funds by investors during the period):

$$C_i = a + bI_i(1 + R_i). \tag{4}$$

The amount I_i that investors place in the fund at the beginning of period i is assumed to be a function of their perception of the manager's ability to generate positive returns on their investment. Their perception of the manager's ability, in turn, is increasing in their assessment of the probability that the manager receives private information each period. (It is also a function of the precision of the private information. However, since the level of precision is a known constant over the two periods, it need not be explicitly considered.) For tractability, it is assumed that I_i is a linear function of this probability assessment. In period 1, it is given by $j + kp$, where j and k are positive constants.[3] In period 2, it is given by $j + kp'$, where p' denotes investors' posterior probability (as of the beginning of the second period) that the manager receives private information. p' is a function of investors' observation of the manager's first-period trading decision and that period's realized cash flow for the risky asset. Given this setting, the manager's compensation can be rewritten as

$$C_1 = a + b(j + kp)(1 + R_1) \tag{5}$$

for the first period and

$$C_2 = a + b(j + kp')(1 + R_2) \tag{6}$$

for the second period.

II. The Investment Manager's Trading Decision

In this section, the investment manager's trading decision for each of the two periods will be analyzed. In making his or her decisions, the risk-neutral manager's objective is to maximize the expected value of his or her compensation over the two periods. As an introduction to the analysis, consider what the manager's optimal trading decisions would be in the case where investors could directly observe whether the manager receives private information during each period. In such a setting, the manager's decisions would be straightforward. An informed manager who observes a signal v_h (v_l) at date 0 of period i would purchase (sell) shares of the risky asset and sell (purchase) shares of the riskless asset at date 1 in order to maximize the expected return on the portfolio and, therefore, his or her expected compensation. An informed manager observing signal v_o would weakly prefer not to trade (and would strongly prefer it if transactions costs were introduced). Similarly, a manager who does not receive private information would also weakly prefer not to trade.

[3] The magnitudes of j and k are expected to be functions of the composition and riskiness of the fund's portfolio as well as of the size of any fees charged by the fund.

When investors cannot observe whether the manager receives private information, the optimal trading decision of an uninformed manager changes. It will now be optimal to trade in the first period with positive probability. That is, the manager will engage in noise trading. To see why, note first that the uninformed manager's first-period trading decision does not affect the expected return on the fund's portfolio; it is zero whether or not the manager trades. Therefore, his or her first-period expected compensation is also invariant to the trading decision. However, this trading decision indirectly impacts the second-period compensation. This is because it affects investors' perception of the probability that the manager receives private information each period, p', which, in turn, affects the amount that investors place in the manager's fund in the second period.

The uninformed manager's objective, in deciding whether to trade, then will be to maximize p'. The incentive for the manager to trade with positive probability results from the observation that, if he or she did not trade at all, investors would set p' at a value less than one. Further, in that case investors would know for certain that all trading was by informed managers and so would set p' to one for a manager who trades. Because of this, it is strictly profitable for an uninformed manager to trade in the first period with some positive probability. In fact, the uninformed manager will increase the probability with which he or she trades until the point is reached where the expected value of p' conditional on trading in the first period is just equal to the expected value conditional on not trading. This argument will be formalized below. In the second and last period of the model, however, the uninformed manager will again weakly prefer not to trade at all on the fund's portfolio. Since there are no future periods, there is no value in trying to raise investors' assessment of the probability that he or she receives private information.

In order to show that an equilibrium exists in which there is noise trading, it must be demonstrated that there are a set of investor conjectures over the investment manager's first-period trading behavior (conjectures that include a positive probability of noise trading for the uninformed manager) that are consistent with the manager's optimal trading decision that period. The remainder of this section is devoted to showing that an equilibrium exists in which certain investor conjectures are confirmed—namely, conjectures that the investment manager will take the following actions in period 1:

1. If the manager receives a signal at date 0 that indicates that cash flow v_h (v_l) will occur in period 1, he or she will purchase (sell) shares of the risky asset that period;
2. If the manager receives a signal at date 0 that indicates that cash flow v_o will occur in period 1, he or she will not trade during that period;
3. If the manager does not receive a signal at date 0, he or she will purchase (sell) shares of the risky asset that period with probability x (x), where $0 < x < 1/2$.

Before showing that an equilibrium exists in which these conjectures are confirmed, it is necessary to calculate the investors' posterior probability p', as of the end of the first period, that the manager received private information during the period. This posterior probability is conditioned on whether the manager traded during the period and on the realized cash flow for the risky asset that period. Denote by $p'(b, v_i)$ ($p'(s, v_i)$) this posterior probability given that the manager bought (sold) shares of the risky asset at date 1 and given that the realized cash flow for the period was v_i. Similarly, let $p'(n, v_i)$ represent this posterior probability given that the manager did not trade at date 1. Note that, given investors' conjectures, the only signal that the manager could have received if he or she had purchased (sold) shares of the risky asset during the period is v_h (v_l). Similarly, v_o is the only signal that the manager could have received if he or she did not trade during the period. Consequently, $p'(b, v_i)$ ($p'(s, v_i)$) is equal to the posterior probability that the manager received signal v_h (v_l) during the period, while $p'(n, v_i)$ is equal to the posterior probability that the manager received signal v_o. It is straightforward to calculate these probabilities using Bayes' rule, and they are given in Subsection *A* of the Appendix. The following properties of these probabilities are easy to verify and will be useful for the subsequent analysis.

Observation 1: Investors' posterior probabilities have the following properties:

$$p'(b, v_h) > p'(b, v_l) = p'(b, v_o); \tag{7}$$

$$p'(s, v_l) > p'(s, v_h) = p'(s, v_o); \tag{8}$$

$$p'(n, v_o) > p'(n, v_h) = p'(n, v_l). \tag{9}$$

Conditions (7) through (9) state that investors' posterior probability that the manager observed information at date 0 is highest if the realized cash flow of the risky asset is consistent with the manager's trading decision, that is, if v_h (v_l) is realized when the manager purchases (sells) shares of the risky asset and v_o is realized when the manager does not trade.

In deciding at date 1 whether to trade, the manager must calculate his or her expectation at that time for the value of p' conditional on each possible action. (The reason that the manager can calculate only the expected value, rather than the realized value, of p' is that at date 1 the realized value of the risky asset's cash flow is not yet known.) This expectation is a function both of the manager's private information (if he or she has received any) and of his or her trading decision. Denote by $\mathrm{E}_i[p'(b, \cdot)]$ ($\mathrm{E}_i[p'(s, \cdot)]$) the expected value of p', over all possible realized cash flows, given that the manager received a signal of v_i and purchased (sold) shares of the risky asset, and denote by $\mathrm{E}_n[p'(b, \cdot)]$ ($\mathrm{E}_n[p'(s, \cdot)]$) the expected value of p' given that the manager did not receive a signal at date 0 but still purchased (sold) shares at date 1. Similarly, denote by $\mathrm{E}_i[p'(n, \cdot)]$ the expected value of p' given that the manager received a signal of v_i and did not trade at date 1, and denote by $\mathrm{E}_n[p'(n, \cdot)]$ this expected value given that the manager did not receive any signal at date 0 and did not trade. Given properties (1) and (2) of the private signal and using Observation 1, it is easy to show the following.

Observation 2: The expected value of investors' posterior, p', has the following properties:

$$\mathrm{E}_h[p'(b, \cdot)] > \mathrm{E}_n[p'(b, \cdot)] > \mathrm{E}_o[p'(b, \cdot)] = \mathrm{E}_l[p'(b, \cdot)]; \tag{10}$$

$$\mathrm{E}_l[p'(s,\cdot)] > \mathrm{E}_n[p'(s,\cdot)] > \mathrm{E}_o[p'(s,\cdot)] = \mathrm{E}_h[p'(s,\cdot)]; \tag{11}$$

$$\mathrm{E}_o[p'(n,\cdot)] > \mathrm{E}_n[p'(n,\cdot)] > \mathrm{E}_h[p'(n,\cdot)] = \mathrm{E}_l[p'(n,\cdot)]. \tag{12}$$

Condition (10) states that a manager observing signal v_h at date 0 has the greatest expectation of investors' posterior conditional on purchasing shares of the risky asset. This is because the manager attaches the greatest probability to the realized cash flow equalling v_h and confirming the trade. Conversely, as conditions (11) and (12) show, the manager has the lowest expectation of investors' posterior conditional on either selling shares of the risky asset or not trading at all. This is because the manager attaches the lowest probability to the realized cash flow equalling either v_l or v_o and confirming these trading decisions. For similar reasons, a manager observing signal v_l at date 0 has the greatest expectation of investors' posterior conditional on selling shares of the risky asset (as seen in condition (11)) and the lowest expectation conditional on either buying shares or not trading (conditions (10) and (12)). Also, a manager observing signal v_o has the greatest expectation conditional on not trading (condition (12)) and the lowest expectation conditional on either buying or selling shares (conditions (10) and (11)).

From relations (10) through (12), the following proposition can be proven:

PROPOSITION: *In the economic setting described here, an equilibrium exists in which investors' conjectures about the investment manager's first-period behavior are fulfilled.*

Proof: Consider first the actions of the uninformed manager. As discussed above, the manager's first-period trading decision will not affect his or her first-period expected compensation; regardless of the action taken, the expected return on the portfolio is zero. Therefore, this trading decision will be made with the objective of maximizing the manager's expected second-period compensation, which, from (6), is equivalent to maximizing the expected value of p'. Note that, for the manager, $\mathrm{E}_n[p'(b,\cdot)] = \mathrm{E}_n[p'(s,\cdot)]$ for all x. Therefore, an uniformed manager will buy shares of the risky asset at date 1 with the same probability that he or she will sell shares of the risky asset. Further, as shown in Subsection *B* of the Appendix, $\mathrm{E}_n[p'(b,\cdot)]$ and $\mathrm{E}_n[p'(s,\cdot)]$ are decreasing and $\mathrm{E}_n[p'(n,\cdot)]$ is increasing in x. This is intuitive since the more likely it is that the uninformed manager trades at date 1, the less (more) can investors use their observation of trading (no trading) as a sign that the manager is informed. The quantity $\mathrm{E}_n[p'(b,\cdot)] - \mathrm{E}_n[p'(n,\cdot)]$ is then continuously decreasing in x. Further, it is positive at $x = 0$ and negative at $x = 1/2$. It is positive at $x = 0$ because, at that point, $\mathrm{E}_n[p'(b,\cdot)]$ is equal to one. (Since the uninformed manager trades with probability zero when $x = 0$, if investors observe trading, they know with certainty that the manager is informed.) It is negative at $x = 1/2$ because, at that point, $\mathrm{E}_n[p'(n,\cdot)]$ is equal to one. (Since the uninformed manager trades with probability one when $x = 1/2$, if investors observe that the manager does not trade, they know with certainty that the manager is informed.) Given these properties, $\mathrm{E}_n[p'(b,\cdot)]$ will equal $\mathrm{E}_n[p'(n,\cdot)]$ for exactly one value of x, x^*, where $0 < x^* < 1/2$. The unique conjecture by investors that is confirmed in equilibrium is that $x = x^*$, that is, that the uninformed manager will buy (and sell) shares of the risky asset with probability x^*. At this value of x, the manager is indifferent between trading and not trading at date 1; buying (and selling) with probability x^* is then consistent with maximizing the manager's objective function and is consistent with investors' conjectures. If investors conjectured a value of x less than (greater than) x^*, $E_n[p'(b, \cdot)]$ would exceed (be less than) $E_n[p'(n, \cdot)]$ and the manager would trade with probability one (zero). Those conjectured values of x would not be consistent with the manager's actions and, therefore, would not lead to an equilibrium.

Finally, it needs to be shown that the remaining investor conjectures are also consistent with the manager's actions. These conjectures are, first, that an informed manager receiving a signal of v_h (v_l) at the beginning of the first period will buy (sell) shares of the risky asset at date 1 and, second, that an informed manager receiving a signal of v_o will not trade. To see that the manager's optimal actions fulfill these conjectures, note from Observation 2 and from the condition that $E_n[p'(b, \cdot)]$ and $E_n[p'(s, \cdot)]$ must equal $E_n[p'(n, \cdot)]$ in equilibrium that the following relations hold:

$E_h[p'(b, \cdot)]$ is greater than $E_h[p'(n, \cdot)]$ and $E_h[p'(s, \cdot)]$;

$E_l[p'(s, \cdot)]$ is greater than $E_l[p'(n, \cdot)]$ and $E_l[p'(b, \cdot)]$;

$E_o[p'(n, \cdot)]$ is greater than $E_o[p'(b, \cdot)]$ and $E_o[p'(s, \cdot)]$.

Then, by following the pure strategy of buying (selling) at date 1, the manager who receives a signal of v_h (v_l) will maximize his or her two-period expected compensation. The manager will simultaneously maximize the expected first-period return on the fund's portfolio (which, in turn, will maximize first-period compensation) and maximize his or her expectation of investors' assessment of p' (which will maximize expected second-period compensation). Similarly, the manager who receives a signal of v_o will maximize his or her two-period expected compensation by not trading at date 1. The informed manager's optimal actions are, therefore, consistent with investors' conjectures.

This proves that an equilibrium exists in which all of the investors' conjectures for the manager's first-period behavior are confirmed by the manager's actions. Q.E.D.

III. Discussion

The preceding analysis has shown that an equilibrium exists in which an uninformed investment manager will trade in the first period with positive probability even though he or she does not possess any private information. The manager will engage in noise trading. He or she will do so in order to raise investors' assessment of the probability that he or she does have private information, p', so that they, in turn, will increase the amount that they invest in his or her fund. The uninformed manager will set the probability with which he or she trades so that the expected value of p' conditional on trading equals the expected value conditional on not trading. In other words, the manager will choose a mixed trading strategy so that, at the margin, the manager will be indifferent between trading and not trading. With the uninformed manager indifferent, the informed manager will strictly prefer to trade when his or her information indicates that it is appropriate to do so (if either signal v_h or v_l is received) and to refrain from trading otherwise (if signal v_o is received).

These actions imply that fund portfolio turnover will be positively related to subsequent portfolio performance, a result documented empirically by Grinblatt and Titman [2]. However, the strength of this relation will be affected by the extent to which uninformed investment managers engage in noise trading. The greater the magnitude of this trading, the more tenuous will be the observed relation between turnover and performance.

One factor that will affect the amount of noise trading is the riskiness of the assets in the fund's portfolio. In terms of the model presented here, an asset's riskiness is measured both by the difference between v_h and v_l and by the parameter r, the probability that the asset's cash flow will deviate upward (or downward) from its expected value during any period. While the difference between v_h and v_l does not affect the uninformed manager's decision as to the optimal amount of noise trading, the magnitude of r does have an effect. To see this, note that, as r increases, $E_n[p'(b, \cdot)]$ and $E_n[p'(s, \cdot)]$ increase while $E_n[p'(n, \cdot)]$ decreases. The reason for this is that the greater the value of r, the greater is the likelihood of cash flows v_h and v_l being realized while the smaller is the likelihood of cash flow v_o occurring. It is then more likely that, if the uninformed manager trades, the realized cash flow will be consistent with his or her trading decision. Conversely, it is less likely that the realized cash flow will be consistent with a decision not to trade. It is, therefore, more profitable for the uninformed manager to trade when r is higher. Consequently, the probability that the manager will engage in noise trading increases with r. An empirical implication of this result is that noise trading by investment managers is expected to be more common for riskier assets. Because of this, the observed association between fund turnover and performance should be weaker for funds that invest in such assets.

In contrast to this clear comparative static result, there is no clear relation between the ex ante probability, p, that the manager receives private information during the period and the extent of noise trading. This is because, as p increases, $E_n[p'(b, \cdot)]$, $E_n[p'(s, \cdot)]$, and $E_n[p'(n, \cdot)]$ all increase. How the probability of noise trading changes with p then depends on the values of the other parameters in the model. Viewing p as a proxy for the prior performance record of the fund, this implies that a fund that has performed better in the past will not necessarily engage in less noise trading.

The existence of noise trading provides an alternative explanation, not previously discussed in the literature, for why prices in securities markets may not fully reveal informed traders' information. As Grossman and Stiglitz [3] and others point out, less than fully revealing prices are a necessary condition in order for traders to have an incentive to collect costly information. To achieve such an equilibrium, it is usually assumed either that the supply of risky assets in the market is uncertain or that there is a random market demand for those assets by liquidity traders. As the analysis here suggests, even if the supply of the risky assets is known and there are no liquidity traders, noise trading will still prevent prices from fully revealing all private information.

The existence of noise trading also sheds light on the extent to which trading volume just prior to corporate news announcements reflects the magnitude of news leaks or insider trading in advance of the announcements. While some trading based on private information will occur, it is expected that there will also be some noise trading generated by those who want to mimic the actions of informed investors. The total level of trading volume will then be an overestimate of the extent of informed trading in advance of the corporate announcements.

IV. Extensions of the Analysis

There are several directions in which the preceding analysis can be extended so as to give additional insights into the expected level of noise trading in the marketplace. One such extension is the introduction of a positive transactions cost. It should be clear that the existence of such a cost will not affect the magnitude of trading by an informed manager as long as the manager finds it profitable to trade at all. If the manager trades, he or she will still invest the fund's assets fully in the risky security or fully in the riskless security, depending on the nature of the private information. Because of this, the amount of noise trading that an uninformed manager must engage in so as to mimic the informed manager's trading behavior is also unaffected by the existence of a transactions cost. However, since it is now more costly to trade, the probability, x, that an uninformed manager will engage in any noise trading at all decreases. Consequently, the expected amount of noise trading is reduced by the introduction of a positive transactions cost.

Another possible extension to the analysis is to allow the fund manager to be risk averse. This change, too, decreases the expected level of noise trading. To understand why, note first that a risk-averse manager who receives a signal that the risky asset is undervalued will not, in general, invest all of the fund's assets in that security (in contrast to the action of a risk-neutral manager). Because of this, an uninformed manager who decides to purchase the risky asset need not engage in as much noise trading in order to mimic the action of the informed manager. Further, since it is now more costly to trade, the probability that the uninformed manager will trade at all decreases. As a result, the more risk averse the fund manager, the less will be the expected level of noise trading.

V. Summary

While Black has asserted that noise trading must be an important factor in securities markets, he did not explain why anyone would rationally want to trade on noise. In this paper it has been shown that, for one type of investor, managers of investment funds, there is a motivation for such trading. An investment manager may trade on his or her fund's portfolio without having private information in an attempt to convey to investors that he or she is informed, so that the investors, in turn, increase the amount of money that they invest in the fund.

This analysis provides two related empirical implications. First, noise trading should be more commonly observed in riskier assets. Second, as a result of this, the positive relation between fund turnover and performance, documented empirically by Grinblatt and Titman [2], should be weaker for those funds that specialize in riskier assets. This result may be useful for more detailed empirical work measuring the association between turnover and performance.

Appendix

A. Derivation of Investors' Posterior Probabilities

The investors' posterior probabilities are derived using Bayes' rule, given their conjectures. For the case where the manager buys shares of the risky asset at date 1 and v_i is realized at the end of the period,

$$p'(b, v_i) = \frac{pr(\text{buy}, v_i \text{ realized} \mid \text{signal } v_h \text{ received}) \times pr(\text{signal } v_h \text{ received})}{pr(\text{buy}, v_i \text{ realized})}, \tag{A1}$$

where the denominator of (A1) is equal to

$$\begin{aligned} &pr(\text{buy}, v_i \text{ realized} \mid \text{signal } v_h \text{ received}) \times pr(\text{signal } v_h \text{ received}) \\ &\quad + pr(\text{buy}, v_i \text{ realized} \mid \text{signal } v_l \text{ received}) \times pr(\text{signal } v_l \text{ received}) \\ &\quad + pr(\text{buy}, v_i \text{ realized} \mid \text{signal } v_o \text{ received}) \times pr(\text{signal } v_o \text{ received}) \\ &\quad + pr(\text{buy}, v_i \text{ realized} \mid \text{no signal received}) \times pr(\text{no signal received}). \end{aligned}$$

Substituting into (A1) the specific values of these probabilities gives

$$p'(b, v_h) = \frac{tp}{tp + x(1-p)}, \tag{A2}$$

$$p'(b, v_l) = \frac{(1-t)p/2}{(1-t)p/2 + x(1-p)}, \tag{A3}$$

$$p'(b, v_o) = \frac{(1-t)p/2}{(1-t)p/2 + x(1-p)}. \tag{A4}$$

Given the symmetry of the signals v_h and v_l, $p'(s, v_l) = p'(b, v_h)$, $p'(s, v_h) = p'(b, v_l)$, and $p'(s, v_o) = p'(b, v_o)$. Following along similar lines gives

$$p'(n, v_h) = \frac{(1-t)p/2}{(1-t)p/2 + (1-2x)(1-p)}, \tag{A5}$$

$$p'(n, v_l) = \frac{(1-t)p/2}{(1-t)p/2 + (1-2x)(1-p)}, \tag{A6}$$

$$p'(n, v_o) = \frac{tp}{tp + (1-2x)(1-p)}. \tag{A7}$$

B. Proof That $\mathrm{E}_n[p'(b, \cdot)]$ *and* $\mathrm{E}_n[p'(s, \cdot)]$ *Are Decreasing in x and* $\mathrm{E}_n[p'(n, \cdot)]$ *Is Increasing in x*

The expression for $E_n[p'(b, \cdot)]$ is given by

$$E_n[p'(b, \cdot)] = rp'(b, v_h) + rp'(b, v_l) + (1 - 2r)p'(b, v_o), \tag{A8}$$

which is also equal to $E_n[p'(s, \cdot)]$, given the symmetry of the signals. The expression for $E_n[p'(n, \cdot)]$ is given by

$$E_n[p'(n, \cdot)] = rp'(n, v_h) + rp'(n, v_l) + (1 - 2r)p'(n, v_o). \tag{A9}$$

Since all terms in (A8) are decreasing in x (as can be seen from examination of (A2) through (A4), $E_n[p'(b, \cdot)]$ and $E_n[p'(s, \cdot)]$ are decreasing functions of x. Similarly, since all terms in (A9) are increasing in x (as can be seen from examination of (A5) through (A7)), $E_n[p'(n, \cdot)]$ is an increasing function of x.

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