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INTEREST-RATE RISK AND THE TERM STRUCTURE OF INTEREST RATES: COMMENT*

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IN A recent issue of this *Journal*, James Van Horne (1965) argued that liquidity premiums are inversely related to the level of interest rates. Even if one were to accept the reasoning he presents in support of the a priori plausibility of such a relation,[1] the evidence he offers in support of it is far from convincing.

As a specific test in favor of his hypothesis, Van Horne (1965, p. 348, Fig. 2) presents a scatter diagram that shows a negative correlation between the residuals from a Meiselman error regression (Meiselman, 1962, pp. 19–21) and the beginning forward rate, ${}_{t+n}r_{1t-1}$. What he seems to have overlooked, however, is the possibility that this negative relation may be entirely a statistical artifact stemming from his use of the *beginning* forward rate rather than the ending forward rate as the measure of level. More precisely, the regression equation from which the residuals were obtained was of the form:

$$x - y = \hat{a} + \hat{b}z + u ,$$

where y is the beginning forward rate; x, the ending forward rate; z, the Meiselman "error"; and u, the residual. The simple correlation between y and u is given by

$$R_{yu} = \frac{(R_{xu}\sigma_x - \sigma_u)}{\sigma_y},$$

where the σ's are the sample standard deviations. Hence R_{yu} may well be negative even though R_{xu} is positive; and it may be substantially so if σ_u is large, that is, if the original regression fits the data relatively poorly.

To see whether such is indeed the case in the present instance, Van Horne's tests were rerun with the ending forward rate rather than the beginning rate. The results are presented in Table 1. Since this rerun was made without splitting the sample into two clusters as Van Horne did, the tests were also run with the beginning forward rate so that a consistent basis for comparison would be available.

The picture that emerges is striking. Wherever Van Horne found an inverse relation between the level of rates and the residuals using the beginning forward rates, I found a direct relation by using the ending forward rates. Furthermore, in eleven out of fifteen cases, the correlation coefficient between the residuals and the ending rate was larger in absolute value. Clearly this evidence supports the proposition that liquidity premiums vary directly with the level of rates as well, if not better, than it supports Van Horne's thesis.

Since he found a strong negative correlation between the beginning forward rates and the residuals from a simple regression, Van Horne went on to argue that the beginning-level forward rate should be employed as an additional explanatory variable in the following modified form of the error-regression model:

$${}_{t+n}r_{1t} - {}_{t+n}r_{1t-1} = \hat{a} + \hat{b}_1 E_t + \hat{b}_2\, {}_{t+n}r_{1t-1} + u_t .^2 \qquad (1)$$

But this too is a treacherous way of estimating a level effect, because ${}_{t+n}r_{1t-1}$ appears on both sides of the equation *and* contains

* I am grateful to Merton Miller for his assistance in the preparation of this note and to Reuben Kessel for many helpful comments.

[1] And not everyone will accept it. For a persuasive argument to the contrary, see Kessel (1965, p. 25).

[2] He actually added $({}_{t+n}r_{1t-1} - r_a)$, where r_a is the "accustomed level."

errors of measurement. Furthermore, it is likely that the errors are large since the data were read from yield curves fitted freehand by the Treasury Department. Consequently, $\hat{b}_2$ would probably be significantly negative even if the true value of b_2 were zero.

TABLE 1

CORRELATION COEFFICIENTS BETWEEN REGRESSION RESIDUALS AND BEGINNING AND ENDING FORWARD RATES

Regression No.	$R_{u_t, {}_{t+n}r_{1t}}$	$R_{u_t, {}_{t+n}r_{1,t-1}}$
1954–63:		
DR 1	.209	−.086
DR 2	.274	−.128
DR 3	.415	−.0315
DR 4	.460	−.0244
1958–63:		
DR 1	.401	−.0199
DR 2	.489	−.139
DR 3	.428	−.292
DR 4	.336	−.477
DR 5	.357	−.545
DR 6	.519	−.446
DR 7	.558	−.389
DR 8	.493	−.411
DR 9	.359	−.441
DR 10	.628	−.458
DR 11	.464	−.530

To test the extent to which the negative $\hat{b}_2$'s Van Horne found were due to common errors in the variables, the beginning forward rate, ${}_{t+n}r_{1t-1}$, was replaced on the right-hand side by the ending forward rate, ${}_{t+n}r_{1t}$, to give the following model:

$$ {}_{t+n}r_{1t} - {}_{t+n}r_{1t-1} = \hat{a}' + \hat{b}_1' E_t + \hat{b}_2' \, {}_{t+n}r_{1t} + u_t \, . \tag{2} $$

The coefficients from (1) and (2) are shown in Table 2. Again, the simple act of changing the measure of the level of rates completely changes the results. Everywhere Van Horne found negative coefficients, I found a positive coefficient when I used the ending forward rate. However, model (2) is able to offer no additional information about the true value of the level-effect coefficient (if it exists), because the presence of common errors would tend to bias $\hat{b}_2'$ just as strongly in the positive direction as it biases $\hat{b}_2$ in the negative direction.

In an attempt to circumvent these biases, a third model was formulated in which the absolute level of rates was measured by the three-period yield at the beginning of each period, that is, by ${}_{t-1}R_{3t-1}$.[3] This model is written

$$ {}_{t+n}r_{1t} - {}_{t+n}r_{1t-1} = \hat{a}'' + \hat{b}_1'' E_t + \hat{b}_2'' \, {}_{t-1}R_{3t-1} + u_t \, . $$

The values for $\hat{b}_2''$ and its corresponding t-ratios are given in the last two columns of Table 2. The coefficients turn out to be much closer to zero than those obtained under the Van Horne Test (eq. [1]) and under its variant (eq. [2]), thus confirming the likelihood that his results are largely attributable to the presence of common errors. The pattern of coefficients might seem at first glance to provide some support for his hypothesized inverse relationship, since all the coefficients from DR 5 on have negative

TABLE 2

INTEREST-RATE-LEVEL REGRESSION COEFFICIENTS, 1958–63

Regression No.	$\hat{b}_2$	$\hat{b}_1'$	$\hat{b}_1''$	t-Ratio for $\hat{b}_1''$
DR 1	−.0146*	.341	.0979	1.27
DR 2	−.240*	.510	.0977	1.09
DR 3	−.513	.530	.0919	1.17
DR 4	−.715	.491	.0414	0.535
DR 5	−.837	.574	−.0969	−1.24
DR 6	−.735	.645	−.000472	−0.00716
DR 7	−.698	.647	−.128	−2.24
DR 8	−.659	.566	−.0299	−0.606
DR 9	−.640	.423	−.0819	−1.79
DR 10	−.898	.889	−.0349	−0.667
DR 11	−.829	.693	−.0168	−0.329

* t-ratios of −0.149 for DR 1 and −1.79 for DR 2. All other t-ratios for $\hat{b}_1$ and $\hat{b}_1'$ are greater than 2.

[3] Errors in ${}_{t+n}r_{1t-1}$ and ${}_{t-1}R_{3t-1}$ will, of course, still be correlated when $n = 2, 3$ because the forward rate is calculated from spot rates (see Van Horne, 1965, n. 4, p. 345). The correlation will be negative when $n = 3$ and positive when $n = 2$, *ceteris paribus*.

signs. However, in over half the cases the t-ratios are substantially below unity. Moreover, his hypothesis fails completely in the critical case of DR 3, which has a positive coefficient despite the fact that the coefficient is known to be biased in his favor (see n. 3 of this comment). Everything considered, the safest conclusion is that this particular set of data throws little light on the relation, if any, that exists between the magnitudes of liquidity premiums and the levels of interest rates.

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