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The Journal of Finance, Volume 27, Issue 2, Papers and Proceedings of the Thirtieth Annual Meeting of the American Finance Association, New Orleans, Louisiana, December 27-29, 1971 (May, 1972), 251-277.

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The Journal of Finance

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INTEREST RATES ON MONETARY ASSETS AND COMMODITY PRICE INDEX CHANGES

RICHARD ROLL*

I. INTRODUCTION

A SIMPLE STORY is told about the response of interest rates to anticipated changes in commodity prices. A consumer expects the dollar cost of apples to be twenty per cent higher next period than now. He could be induced to give up 100 apples for the promise of 105 apples next period but he would not part with ten dollars for a promise of less than \$12.60 in repayment.¹ Moral: The nominal (money) rate of interest is equal to the real (apple) rate plus² the rate of inflation. This relation is named the Fisher effect after its originator and most distinguished proponent.³ It is one of the standard paradigms of monetary economics and is frequently used in explaining bond prices to spouses and physicists. Of the theoretical and empirical importance of the asserted relation there can be no doubt. The rate of interest has a central role in macrotheory, because it is a primary transmitter of both monetary and fiscal actions. For bond buyers and issuers, the course of inflation would seem to be at least as important as changes in default probability and in the real productivity of capital.

Despite its wide acceptance, however, a seemingly innocuous change in the story raises a question of whether the Fisher relation can be observed in data. Let us suppose consumers hold expectations about future commodity price *levels* and not about rates of change in prices. By adjusting freely, current prices will cause changes in anticipated rates of inflation; and consumers will be just as happy having adjusted current prices to bring about say a ten per cent inflation rate and a fifteen and one-half per cent interest rate as they were with a twenty per cent inflation and a twenty-six per cent interest rate. The system actually has indefinitely many equilibrium points. All except one, however, are eliminated by adding the existence of a currency which pays no interest and must be held by someone between periods. With this complication,

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1. Let p_t be price per apple in period t and q_t be the quantity consumed. The consumer is willing to trade apples intertemporally at the rate $q_2 = 1.05q_1$. He anticipates apple prices to be described by $p_2 = 1.20p_1$. Thus, he would trade dollars [$= p_t q_t$ (dollars/apples) \cdot (apples)] intertemporally at the rate $p_2 q_2 = (1.20)(1.05)p_1 q_1$ or at 26 per cent.

2. Strictly speaking, it is "plus" only when the rates are continuously compounded.

3. Irving Fisher [1930, chs. 2 and 19].

the *nominal* rate of interest is fixed at zero (on the asset money). Current commodity prices, along with anticipated rates of inflation, *must* adjust, therefore, to unique levels. The consumer will be indifferent to further transactions only after current prices have shifted to the point that he expects a zero rate of inflation or a rate just equal to the implicit non-pecuniary return from holding money balances.⁴

Ball [1964, p. 43] has emphasized this point as follows:

Full stock equilibrium requires now the further condition that the actual price level be equal to the expected price level, or in other words that a value of a unit of the monetary stock now be equal to its expected value in the next period. If this condition is not fulfilled, decision units will attempt to substitute goods now for goods in the future, i.e., they will attempt to reduce their real balances, hence driving the price level up to the expected price. When the two are equal this movement ceases and equilibrium is restored.

To Ball's statement about commodity prices should be added the following corollary about interest rates: Since the anticipated inflation rate will be just equal to the implicit non-pecuniary return from holding money (which was zero in Ball's case because he ignored it), the nominal interest rate will be just equal to the relative disutility of holding bonds rather than currency. If an asset provides non-pecuniary services of as high a quality as those provided by money, its nominal interest rate will show no relation to the expected rate of inflation.⁵

Ball finds that his above "... conclusion is justified only to the extent that ... the price level rises sufficiently rapidly over time to catch up with the

4. Of course, this will be the exact indifference position only if the consumer regards both returns as equally risky. If, for example, he is a risk averter and he regards the non-pecuniary return to money balances as more uncertain than the rate of commodity price change, he would be indifferent to further trading of money for current commodities only if the anticipated return from money included a risk premium in excess of the inflation rate's risk premium.

One must also recognize that the real interest rate is implicitly being held constant as we speak of commodity price and nominal interest rate changes here. As a first approximation, this is probably not a bad assumption since the real interest rate is determined by the aggregated marginal rates of time preference of consumers and the marginal products of physical capital goods.

5. Symbolically, the Fisher relation under certainty is

$$1 + R = (1 + r)(1 + I)$$

where R is the nominal and r is the real rate of interest and I is the anticipated rate of commodity price inflation. For money, $R \equiv 0$; and thus $(1 + r)(1 + I) = 1$ if money provides no non-pecuniary services. If, however, money and bonds provide non-pecuniary services, their nominal returns would be decreased by a premium and we would have

$$\begin{aligned} 1 + R &= (1 + r)(1 + I) - \lambda_B && \text{for bonds} \\ 1 &= (1 + r)(1 + I) - \lambda_m && \text{for currency} \end{aligned}$$

and thus

$$R = \lambda_m - \lambda_B$$

where the λ 's measure the implicit returns to non-pecuniary services. In general, if λ_B and λ_m differ, the difference will depend on the rate of inflation. When $\lambda_B = \lambda_m$, R will not depend on inflation. This conclusion is readily extended to the case of uncertainty in both asset returns and inflation rates.

expected price," (p. 43). In other words, the conclusion of no Fisher effect between nominal and real interest rates is valid only if commodity markets are efficient in the sense that has been applied to asset markets⁶ so that current prices adjust immediately to any relevant new information and thus continually stand at levels equal to expected prices.⁷ This is, I believe, an important fact concerning the likely occurrence of the Fisher effect. It forces us to suspect, at least based on the demand side for assets and commodities, that observation of a positive relation between nominal interest rates and commodity price changes would imply *inefficient* asset or commodity markets.

This suggests an empirical analysis in two parts: First, is there empirical evidence of inefficiencies in the asset or commodity markets? Second, has there been direct empirical confirmation of the Fisherian relation?

II. COMMODITY MARKET EFFICIENCY

If asset and commodity markets are efficient information processors, one should expect little comovement between nominal interest rates on assets that are good money substitutes and *any* measure of inflation. Thus, it is worth knowing whether these markets exhibit a reasonable degree of efficiency. For asset markets, several studies have found a very high, although not a perfect degree of efficiency [cf. Fama, 1970]. Thus the source of an easy-to-observe Fisher effect is not likely to be found in asset markets.

In commodity markets, the evidence is less complete. High serial dependence in commodity price changes would be tell-tale evidence of inefficient markets because it would imply (a) suppliers of goods with a production period shorter than the unit of observation could increase profits by producing more for periods when high prices are predicted on the basis of the observed serial dependence, and (b) consumers could time purchases to coincide with periods of lower predicted prices. The absence of serial dependence would mean that such profit-and utility-increasing opportunities are not available and that serial dependence which might have been observed has been eliminated by producers and consumers acting to take advantage of any that did exist.

In the first study of this question known to me, Laffer and Zecher [1970] analyzed the annual behavior of the U.S. Bureau of Labor Statistics Wholesale Price Index (WPI) from 1900 to 1963 and concluded that observed serial dependence in the index changes was small enough to imply efficient commodity markets.⁸ Table 1 presents a replication of their study for both the Wholesale and Consumer Price Indexes (CPI) over the period when monthly observations are available, January 1913 to the present. In the first panel, logarithmic annual rates of change in both indexes are related to past annual rates

6. Cf. Fama [1970].

7. Discounted at the appropriate risk-adjusted rates.

8. They actually calculated the serial dependence in $\log_e(p_t/p_{t-1}) - r_{1,t}$ where p_t is the Wholesale Price Index at time t and $r_{1,t}$ is a one-year bond rate at t . Sub-indexes of the Wholesale Price Index were also analyzed with similar results. Using their data, I have found that the subtraction of $r_{1,t}$ had no significant effect on the observed serial dependence. (A supplementary table, not printed here, will be provided to readers interested in this result).

TABLE 1
 AUTO-REGRESSIVE STRUCTURE OF CONSUMER AND WHOLESALE PRICE INDEXES

INDEX	β_0	β_1	β_2	β_2	R ²	D-W	N
Annual Observations between Successive Januaries, 1913-1971							
CPI	.0121 (1.73)	.827 (5.93)	-.404 (-2.35)	.0725 (.519)	.435	1.99	55
WPI	.0151 (1.07)	.356 (2.57)	-.171 (-1.18)	-.0109 (-.0787)	.123	2.00	55
Monthly Observations, January 1913-October, 1971							
CPI	.000387 (2.79)	.473 (12.6)	.0488 (1.18)	.145 (3.87)	.336	2.05	702
WPI	.000560 (1.16)	.363 (9.82)	.0939 (2.39)	.213 (5.77)	.298	2.03	702

1. t-ratios are in parentheses; R² = coefficient of determination; D-W = Durbin-Watson Statistic; N = Sample size.

2. The regression equation is

$$\log_e(p_t/p_{t-1}) = \beta_0 + \beta_1 \log_e(p_{t-1}/p_{t-2}) + \beta_2 \log_e(p_{t-2}/p_{t-3}) + \beta_3 \log_e(p_{t-3}/p_{t-4})$$

p_t is the price index for period t.

3. The two indexes used were

- 1) U.S. Bureau of Labor Statistics "Consumer Price Index for Urban Wage Earners and Clerical Workers, U.S. City Average: All Items—Series A."
- 2) U.S. Bureau of Labor Statistics Wholesale Price Index.

of change.⁹ These results, taken alone, do not support Laffer's and Zecher's conclusion that commodity markets are efficient because the first-lagged annual rate of change in each commodity price index is a significant explanatory variable. The same is true of the monthly rates of change reported in Table 1 (Panel 2). In fact, lagged monthly changes are even more significant (in terms of the coefficients exceeding their standard errors).

The prices sampled for these indexes are spot rather than future so it is perhaps surprising that annual changes show such strong serial dependence. One might have expected a seasonal in the monthly observations and this would, of course, have increased the observed serial dependence. The Bureau of Labor Statistics [1967] reports, however, that "Patterns of seasonal price movements are apparent for many components of the Consumer Price Index, but they largely counterbalance one another and cause little or no seasonal variation in the total index", (p. 14). I mention this because seasonal variation in spot prices does not contradict the efficient market hypothesis for commodities if it occurs in non-storable items such as some agricultural products. No obvious market mechanism (storage in inventory by consumers or production smoothing by producers) can offset the predictable price changes in such items. In fact, the strongest seasonals do occur in items that cannot be stored easily and must be produced when the weather is favorable; fresh fruits and vegetables have the greatest seasonal variation while meats, poultry and

9. The annual change was taken between successive Januaries for a rather arbitrary reason: It was the first month reported in the series. Annual changes between other months would display a similar autoregressive structure. Note, however, that averages of monthly observations must not be used to obtain yearly estimates because this would induce spurious positive serial correlation, cf. Working [1960].

TABLE 2
AUTOREGRESSIVE STRUCTURE IN CONSUMER AND WHOLESALE PRICE
INDEXES USING PUBLICLY AVAILABLE EXPLANANDS

INDEX	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	R ²	D-W	N
March-to-January Changes Lagged on January-to-January Changes, 1913-1971							
CPI	.0147 (2.16)	.626 (4.62)	-2.73 (1.63)	.0431 (.317)	.326	2.02	55
WPI	.0165 (1.25)	.182 (1.40)	-.0732 (-.537)	-.0239 (-.184)	.0412	2.02	55
Observations every Third Month, January, 1913-October, 1971							
CPI, t = 7, 10, 13,0000850 (.181)	.184 (2.30)	.142 (1.95)	.236 (3.09)	.211	2.01	233
CPI, t = 8, 11, 14,00134 (3.22)	.254 (3.70)	.206 (2.72)	.0721 (1.19)	.239	2.12	233
CPI, t = 9, 12, 15,00124 (2.24)	.140 (1.98)	.357 (3.60)	.0577 (.690)	.158	2.11	233
WPI, t = 7, 10, 13,000381 (.455)	.335 (4.26)	.141 (2.22)	.0166 (.249)	.220	1.95	233
WPI, t = 8, 11, 14,00197 (1.38)	.153 (2.16)	.261 (3.03)	.00727 (.112)	.138	2.01	233
WPI, t = 9, 12, 15,00101 (.996)	.387 (5.81)	.197 (2.34)	-.0393 (-.458)	.205	1.89	233

1. t ratios are in parentheses; R² = coefficient of determination; D-W = Durbin-Watson Statistic; N = sample size.

2. For the March-to-January models, the regression equation is

$$\log_e(p_t/p_{t-10}) = \hat{\alpha}_0 + \hat{\alpha}_1 \log_e(p_{t-12}/p_{t-24}) + \hat{\alpha}_2 \log_e(p_{t-24}/p_{t-36}) + \hat{\alpha}_3 \log_e(p_{t-36}/p_{t-48})$$

t = 49, 61, 73, 85, ... months
(t = 1 is January, 1913)

3. The monthly regression equations are

$$\log_e(p_t/p_{t-1}) = \hat{\alpha}_0 + \hat{\alpha}_1 \log_e(p_{t-3}/p_{t-4}) + \hat{\alpha}_2 \log_e(p_{t-4}/p_{t-5}) + \hat{\alpha}_3 \log_e(p_{t-5}/p_{t-6})$$

t = 7, 10, 13, 15 ... for first model
t = 8, 11, 14, ... for second model
t = 9, 12, ... for third model.

4. Full titles of price indexes are given in notes to Table 1.

fish are next. The natural production seasons in these last items are no doubt reduced by their being partly storable in frozen or canned form. The only other items which display seasonality are apparel and automobile transportation, both of which have high fall prices when new models are introduced. However, this must be regarded as a defect in the index caused by its inability to measure seasonal quality variation.

Before concluding that commodity markets are grossly inefficient based on the results of Table 1, we should ask whether there are other explanations of the observed serial dependence that are consistent with efficiency.

One potentially important fact for interpreting the auto-regressive structure in price index changes is the information lag between gathering individual prices and publishing the Index. For example, The Consumer Price Index for a given month is not announced until around the 25th of the following month¹⁰ and hence, not until after many prices for that month have already been

10. Between January, 1960 and December, 1970, the announcement date of the Consumer

sampled for use in calculating its Index value. An investor could not use the monthly equations in Table 1 for predictive purposes unless he personally were sampling commodity prices more quickly. He could not rely on BLS announcements since they would be received too late to act upon.

For example, the first lag in annual changes between successive Januaries is not predictively useful because January Index values are not announced publicly until near the end of February. However, the auto-regressive structure in March-to-January changes, (see Table 2, first panel), still indicates a strong effect from the first lagged CPI change. In this calculation, the logarithmic Index change from March of year t to January of year $t + 1$ was regressed on January-to-January changes in years $t - 1$, $t - 2$, and $t - 3$. This was done to insure that the BLS announcement in February of year t of the January Index, took place prior to the decision period of March in year t . The total explanatory significance of this regression has declined slightly from the annual regressions of Table 1, indicating that some of the serial dependence was indeed caused by an information lag. For the Wholesale Price Index, lagged annual rates of change which are known in time to act upon "explain" only four percent of the variation in the next 10-month change. For the Consumer Price Index, however, the predictive usefulness of the equation is considerably higher; (33 per cent of the variation is explained). Moreover, the monthly models show that about 20 per cent of the monthly variation in index changes can be accounted for by lagged changes.

For the same reason of timing the public availability of index announcements, the first monthly explanatory variable had to be lagged two full months in the monthly regression. This causes an econometric difficulty which is explained in the footnote.¹¹ It is necessary to use every third observation only in

Price Index varied between the 18th and the 31st of the following month and in one case, for January, 1964, was delayed until March 3. I am indebted to Bureau of Labor Statistics Commissioner Geoffrey H. Moore for providing these data.

11. The difficulty is caused by spurious autocorrelation introduced into the residuals by omitting the first two lagged monthly index changes (which should be omitted because they are not known in time to make decisions.) Defining $I_t = \log_e(p_t/p_{t-1})$, the results of Table 1, indicate that β_1 and β_2 in the model

$$I_t = \beta_0 + \beta_1 I_{t-1} + \beta_2 I_{t-2} + \beta_3 I_{t-3} + \varepsilon_t$$

are significant explanatory variables. However, I_{t-1} and I_{t-2} are not known in time to make commodity purchases which would bring gains of I_t . Thus, a predictive equation would contain only I_{t-3} and be of the form

$$I_t = \beta_0 + \gamma_3 I_{t-3} + u_t$$

Note, however, that the disturbance of this model is

$$\begin{aligned} u_t &= \beta_1 I_{t-1} + \beta_2 I_{t-2} + \varepsilon_t \\ &= \varepsilon_t + \beta_1 (\beta_0 + \gamma_3 I_{t-4} + u_{t-1}) \\ &\quad + \beta_2 (\beta_0 + \gamma_3 I_{t-5} + u_{t-2}) \end{aligned}$$

So that the appropriate econometric model would be

$$I_t = \alpha_0 + \alpha_1 I_{t-3} + \alpha_2 I_{t-4} + \alpha_3 I_{t-5} + \xi_t$$

where

$$\xi_t = \varepsilon_t + \beta_1 u_{t-1} + \beta_2 u_{t-2}$$

the regression calculation in order to avoid serial dependence in the residuals. This is why Table 2 contains three monthly models for each index. Each model has been fitted with every third observation. Fortunately, the very large number of available observations makes it possible to do this and still retain a large sample size in each model.

Another very important possible cause of spurious serial dependence is non-stationary in the process, and in particular, a non-stationary mean inflation rate. This is a very reasonable possibility considering the long time period being used and the changes that have occurred in economic institutions and in modes of behavior since 1913. In addition, a model involving lagged dependent variables as regressors is particularly susceptible to incorrect inferences caused by changes in the underlying process. To see why, consider what would happen if the inflation rate conformed to the process

$$I_t = a(t) + \epsilon_t$$

where the function $a(t)$ is a constant at period t which does not depend on past rates of inflation but is presumed to wander over time. In efficient market theory, $a(t)$ would correspond to the "normal" rate of return for time t , the maximum rate of return that a trader could earn by using some prediction model such as lagged variable regression. Since $a(t)$ fluctuates slowly from month to month, inflation rates of adjacent months will have means that differ only slightly while inflation rates for months that are separated by several years could have very different means.¹² This implies that serial regressions fitted with lagged dependent variables will display spurious serial dependence. Actually, the lagged variables are only providing an estimate of the current normal rate, $a(t)$.

There are at least two ways to check whether non-stationarity in the process has introduced spurious serial dependence. The first is to divide the time series into segments and inquire whether the serial dependence in each segment is smaller than the serial dependence calculated from the overall record. Some evidence on this question is provided by Table 3 which reports on monthly serial regression equations for sub-periods. A composite hypothesis that all three lagged monthly¹³ index changes have no effect on the current index change is tested by a Gaussian-based F statistic. Asterisks indicate those regressions where serial dependence is detected at a five percent level of significance. The multiple R^2 's reported are not significantly different from zero in

Thus, disturbance term autocorrelation is present in the model because $\text{Cov}(\xi_t, \xi_{t-1})$ and $\text{Cov}(\xi_t, \xi_{t-2})$ are not zero. However, since $\text{Cov}(\xi_t, \xi_{t-3}) = 0$, an appropriate procedure is to use every third observation.

12. Suppose $a(t)$ followed a random walk, $a(t) = a(t-1) + \xi_t$. Then $I_t = I_{t-1} + \xi_t + \epsilon_t - \epsilon_{t-1}$ for adjacent months, while $I_t = I_{t-n} - \epsilon_{t-n} + \sum_{j=0}^{n-1} \xi_{t-j}$. Thus the disturbance of the second equation has a variance which is on the order of n times as large as the variance of the first equation's disturbance. This implies that a lagged variable regression would detect a spurious autoregressive structure which decays with the length of lag.

13. As mentioned before, because of the lag in public availability, the explanatory variables are lagged 3, 4, and 5 months.

TABLE 3
TESTS OF AUTOREGRESSIVE SIGNIFICANCE IN THE CONSUMER AND WHOLESALE
PRICE INDEXES' MONTHLY CHANGES FOR SUBPERIODS, 1913-1971

INDEX	Sample Period	Beginning Month of Dependent Variable		
		July	August	September
Coefficient of Determination Using Every Third Monthly Change in the Calculation				
CPI	1913-22	.193	.426*	.240
	1923-32	.191	.120	.188
	1933-42	.197	.0111	.118
	1943-52	.255*	.111	.137
	1953-62	.0864	.117	.187
	1963-71	.360*	.233*	.312*
	All years	.211*	.239*	.158*
WPI	1913-22	.299*	.186	.255*
	1923-32	.119	.0738	.0357
	1933-42	.125	.143	.221*
	1943-52	.300*	.119	.382*
	1953-62	.0216	.0607	.167
	1963-71	.131	.0511	.143
	All years	.220*	.138*	.205*

1. The regression equation is the same as the monthly equation given in the notes to Table 2. Sample size is 38 for regressions beginning in the years 1913-1953. Sample sizes are 33 for regressions beginning in 1963 and 233 for regressions spanning all years.

2. *Indicates that the F statistic associated with a null hypothesis of no serial dependence, i.e., ($H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0$), exceeds the .95 fractile of the sampling distribution, thus rejecting the null hypothesis at a five percent level of significance. This test is based on an assumption on independent Gaussian disturbances.

3. Exact titles of price indexes are given in Table 1's footnote.

most cases but they *are* significantly non-zero in more cases than one would expect by chance.¹⁴ Naturally, one must consider the possibility that the process is non-stationary *within* a sub-period, but the number of observations becomes rather small when the time series is divided into smaller segments.

Fortunately, the adaptive regression model recently developed by Cooley and Prescott [1971] uses the full sample and still allows the constant term to vary.¹⁵ Table 4 reports the adaptive regression fit of annual rates of index change for 1913-71. This table's contents should be compared with the annual ordinary least squares regressions reported in the top panel of Table 2. The very significant coefficient of the first lagged consumer price index change was

14. Note well, however, that the six reported coefficients within a given sub-period cannot be presumed independent. Thus, a *single* binomial test of the number of significant coefficients is not available.

15. In the notation used here, the adaptive regression model is

$$y_t = \alpha_0(t) + \alpha_1 y_{t-3} + \alpha_2 y_{t-4} + \alpha_3 y_{t-5} + \varepsilon_t$$

where

$$\alpha_0(t) = \alpha_0(t-1) + v_t; \quad \varepsilon_t \text{ and } v_t$$

are independent disturbances with zero means and with variances $(1 - \gamma)\sigma^2$ and $\gamma\sigma^2$ respectively. Output of the solution includes estimates of γ , σ^2 , and $\alpha_0(N + 1)$ in addition to the slope coefficients.

TABLE 4
ADAPTIVE REGRESSIONS OF ANNUAL CHANGES IN CONSUMER
AND WHOLESALE PRICE INDEXES, 1913-1971

INDEX	$\hat{\alpha}_0(N+1)$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\gamma}$	$\hat{\sigma}^2$	N
March-to-January Changes Lagged on January-to-January Changes, 1913-1971							
CPI	.0342 (1.27)	.321 (2.45)	-.230 (1.67)	-.147 (1.15)	.162 (2.70)	.150	55
WPI	.0167 (.535)	.149 (1.18)	-.086 (.662)	-.045 (.360)	.012 (.160)	.800	55

1. The regression equation is

$$\log_e(p_t/p_{t-10}) = \hat{\alpha}_0(t) + \hat{\alpha}_1 \log_e(p_{t-12}/p_{t-24}) + \hat{\alpha}_2 \log_e(p_{t-24}/p_{t-36}) + \hat{\alpha}_3 \log_e(p_{t-36}/p_{t-48})$$

$t = 49, 61, \dots, \dots$ months ($t = 1$ in January, 1913).

2. The constant term is assumed to be described by a stochastic process with parameters γ and σ^2 . See footnote 15.

3. Exact titles of price indexes are given in footnotes to Table 1.

4. t-ratios are in parentheses; N = sample size.

dropped to about one-half its former size. Together with a significant $\hat{\gamma}$, which is a measure of the importance of nonstationarity, this does indicate that a substantial part of the serial dependence formerly measured in Consumer Price Index changes had been caused by a shifting mean. For the Wholesale Price Index, no significant autoregressive structure is evident in this table (4); the estimated extent of nonstationarity ($\hat{\gamma}$) is not significant either and, of course, the other parameter estimates have changed little from their ordinary least squares counterparts of Table 2.

The difference between these two indexes over exactly the same time period is rather mystifying. Several colleagues have suggested the difference is due to the absence of service prices in the WPI, (but not in the CPI), the implication being that services are not storable commodities and are therefore not subject to the speculative activity which reduces intertemporal price change dependence for storable goods. Others have emphasized differences in sampling techniques between the indexes. For example, since some consumer prices are obtained only quarterly, the monthly Index values necessarily contain some extrapolated components. Such arguments may ultimately explain more of the remaining measured serial dependence in price indexes but in order to get back to our primary subject, the Fisher effect, it is necessary to leave them unexamined. Furthermore, there does seem to be enough dependence in monthly changes of both indexes and in annual changes of one index to make the Fisher relation a still-viable hypothesis; and when one considers the possibility of constructing a price index from privately-conducted surveys, this conclusion is strengthened.

III. DIRECT TESTS OF THE INTEREST RATE-INFLATION RATE RELATION

Studies that Relate Interest Rates to Functions of Commodity Prices Only

Using a simple two variable regression between annual observations of yields-to-maturity from Standard and Poor's Composite Corporate Bond Av-

erage and percentage rates of change in a combination price index,¹⁶ Weil [1970] found no significant relation for 1900 to 1968. From 1961-1968, however, the relation was positive and highly significant. The same results were obtained using Durand's [1942] one-year rate. These results could have been due to a basic structural change in the 60's or to some other variable that affected interest rates having been omitted from the fitted equation.

Meiselman [1963] used an annual interest rate series composed of Macauley's [1938] high-grade railroad bond yields from 1873 to 1936 and a combination of Moody's Aaa and Durand's [1947] 30-year basic yield from 1937 to 1963. He regressed the yield on its lagged value and on current and lagged values of the Wholesale Price Index. The lagged yield was a uniformly powerful explanatory variable as was the current *level* of wholesale prices. However, the current *change* in prices was not significant, thus contradicting the Fisher effect with a rather simple measure. After this inauspicious (for Fisher's Theory) beginning, Meiselman went on to use several more sophisticated functions of past prices including distributed lags. How many different measures one is unable to determine, however, because to ". . . keep the paper within bounds . . ." Meiselman only reported ". . . what appear[ed] to be the most promising results", [p. 129]. The most successful equations related the bond yield to a difference between the current price level and an expected price level taken as a distributed lag of past prices *one period earlier*. If the distributed lag was taken in the same period, the Fisher effect was not found.

A slightly different procedure was employed by Gibson [1970] whose interest rate data consisted of six different yields-to-maturity;¹⁷ two with quarterly observations, 1948-63, and four with annual observations, 1869-1963. Each yield was regressed on a current price index change and up to eleven past changes. For annual observations, the price index used was the "implicit NNP deflator . . . obtained from Milton Friedman and Anna J. Schwartz," (p. 34). We are not told how many other indexes were tried but this one ". . . proved best at measuring expectations. It tended to produce more statistically significant coefficients and coefficients closer to those expected theoretically than any other indexes considered," (pp. 33-34). With annual data, even this index produced significant coefficients for only the effect of *current* inflation rates on two long-term yields. Neither the current inflation rates for short-term yields nor any of the lagged inflation rates for any yield was significant. However, when *changes* in yields were regressed on current and lagged changes in inflation rates,¹⁸ none were significant for the long-term yields while the

16. The price index was based on the BLS Wholesale Price Index from 1900-1929 and on the GNP deflator from 1930-1968. Unfortunately, both were annual averages of monthly observations and, in addition, a weighted average of adjacent yearly observations was used to obtain a final index which was supposed to have coincided in time with the interest rate observations. This is not intended as a criticism. Weil was one of the few writers who even took the trouble to ascertain that his data were temporally aligned.

17. Short-term rates were the Treasury bill rate, the call money rate and the commercial paper rate. Long-term rates were the Treasury bond rate, a combination of Macauley's and Durand's long-term corporate rates, and Standard and Poor's index of high-grade industrial bond yields.

18. Evidently, changes were used in an attempt to reduce auto-correlation in the residuals. In the first equations mentioned, for annual observations using interest and inflation rate levels, the *highest* Durbin-Watson statistic was 0.685.

current and 3-year lagged inflation rate changes were significant for the short-term yields.¹⁹ For quarterly data,²⁰ not a single coefficient of the 46 reported was significant in any of the four regressions, (two with interest and inflation rate levels and two with changes).²¹ Gibson states that coefficients of the two quarterly regressions with interest and inflation rate changes "are positive as hypothesized, frequently significantly so," (p. 23). However, although 17 of the 20 coefficients are indeed positive, only three exceed their standard errors and the *largest* t-ratio is 1.75; (the R^2 's are 0.054 and 0.161).

Studies That Relate Interest Rates to Functions of Commodity Prices and Other Variables

Table 5 lists five recent articles that attempted to measure *several* determinants of nominal interest rates including the Fisherian factor of anticipated inflation. In addition to inflation, the explanatory variable common to all five studies was a measure of aggregate liquidity, although not all the authors described it as such.²² This and the other variables are intrinsically interesting. Their inclusion is well-founded on theoretical grounds in each case, and they are undoubtedly important in isolating the pure effect of inflation on interest rates but since this paper is exclusively concerned with inflation, the reader is referred to the cited articles for a detailed analysis of the other variables.²³

Only Ball purported to find no support for the Fisher effect.

What has not been established is that the expected rate of price inflation has been a significant factor affecting the long term trend in interest rates, independently of the effects of inflation on income. In a sense the rate of inflation has been partly responsible for the rise in the level of interest rates, for it has helped to reduce the money/income ratio. But this reflects the relationship of the *level* of interest rates to the level of prices, [*italics his*]. What is at issue is whether the long term rate of interest is affected by the *rate* at which prices rise. *Since no apparent relation between them has been established, contrary to much distinguished opinion, it is worth considering whether this is because there are plausible reasons why such an effect might not be found,*" (p. 85, *italics added*).

Although Ball apologizes for his ". . . too Keynesian liquidity preference results," I have quoted his conclusion at length because it clearly enunciates the impact of the Gibson paradox²⁴ on attempts to find empirical support for the Fisher effect and is therefore very relevant for the discussion of the Gibson paradox that follows.

19. For the commercial paper rate, the one-year lagged inflation rate was also significant.

20. The Treasury bill and bond rates and the GNP deflator.

21. See his Table 3, p. 26.

22. In fact, Feldstein and Eckstein (p. 367) criticised Sargent for excluding a liquidity measure even though he included the rate of change of the real money supply, explaining its inclusion through a Wicksellian theory of the spread between nominal and real interest rates; Sargent, (pp. 130-132). Feldstein and Eckstein included a similar variable, the change in the real per capita monetary base.

23. You should know in advance, however, that many different functions of the other listed variables were used. For example, Ball reported a total of 53 regressions with various functions of five original variables (and 36 observations).

24. The positive relation between interest rates and the *level* of prices.

TABLE 5
FIVE RECENT EMPIRICAL STUDIES RELATING INTEREST RATES TO
COMMODITY PRICE CHANGES AND OTHER ECONOMIC MEASURES

Author(s) (date)	Interest Rate	Price Index	Other Variables	Time Period and Frequency of Observations	Functions of the Price Index Used as Proxies for Anticipated Inflation
Ball (1965)	British 2 1/2 per cent Consol Yield	British Wholesale Price Index	Bank Rate, (a short- term interest rate), Real money supply; (notes and coin plus "total deposits of the London Clearing Banks" divided by National Income).	1929-1963 Annual	$p_t/p_{t-1} - 1 = I_t + \rho \Delta I_t$; i.e., an adaptive expecta- tions model. Also an at- tempt at a Koyck function but speed of adjustment was pre-set rather than searched for.
Sargent (1969)	Durand's one and ten year basic yields.	"Commodity price in- dex" (source given as G. C. Chow, "On the Long-Run and Short- Run Demand for Money" IBM Re- search Paper, [1964]).	Real Money Supply; Gross National Pro- duct. (Same source as price index).	1902-54 Annual	$\sum_{i=1}^{\infty} \beta^{i-1} \Delta p(t-i)/p(t-i)$ - 1 (lag of zero was omitted, geometric declining weights thereafter). Also tried second order distri- buted lags which tended to show a regressive expecta- tional element on short-term rates but not long-term rates.

TABLE 5 (Continued)

Author (s) (date)	Interest Rate	Price Index	Other Variables	Time Period and Frequency of Observations	Functions of the Price Index Used as Proxies for Anticipated Inflation
Yohé and Karnosky (1969)	Four-to Six-month commercial paper rates and Aaa Cor- porate Bonds (Source not given), both seasonally ad- justed and non- seasonally adjusted.	Consumer Price Index for all items (pre- sumably by Bureau of Labor Statistics). (In one set of tests, the mortgage rate component of the CPI was purged. The results were "quite close" to those using the all-items CPI.) In multiple regres- sions, the "GNP de- flator (or a monthly proxy)" was used.	Level of and rate of change in real GNP; Change in the real money stock.	Quarterly and monthly Jan- uary, 1952 to Septem- ber, 1969. Regression with all variables only done for 1961-69 (quarterly.)	p_t undefined but stated to be the "rate of change," with lags up to 48 months entered separately. Also, Sixth-degree Almon [1965] polynomial was used to estimate lag structure of p_t .
Feldstein and Eckstein (1970)	Yields-to-maturity on seasoned and newly issued Moody's Aaa Corporate bonds.	Price deflator for the consumer expendi- ture component of GNP.	Real Monetary base (as calculated by Federal Reserve Bank of St. Louis); private gross national product; pri- vately-owned Federal government debt. All were divided by the GNP price deflator and by resident popu- lation to obtain real per capita measures. Other variables too in various equations.	1954-1969 Quarterly	Distributed lags of $I_t = (p_t -$ $p_{t-1})/p_{t-1}$ using current I_t and $\sum_{j=1}^{23} a_j I_{t-j}$ where Almon (1965) Polynomial of 3rd degree was employed to determine the coefficients, a. The 23 quarter lag was chosen after trying both shorter and longer lags.

Author(s) (date)	Interest Rate	Price Index	Other Variables	Time Period and Frequency of Observations	Functions of the Price Index Used as Proxies for Anticipated Inflation
Tuttle and Wilbur (1971)	Annual holding period yields computed from Durand's and Winn's basic yields assuming a 3 per- cent coupon. (Var- ious maturities from 1 to 25 years).	Bureau of Labor Statis- tics Wholesale Price Index, (average of monthly values). Consumer Price In- dex also tried but not reported.	Federal demand debt (member bank re- serves, other Federal deposits, and cur- rency outside Trea- sury and Federal Re- serve Banks) divided by Nat'l Income; Change in the money supply; English Bank Rate.	1900-1957 Annual	p_t/p_{t-1} . The value was sup- posedly lagged one period because the holding period yield for year t was defined as $H_t = (p_{t+1} + c_{t+1} -$ $p_t)/p_t$ where p is the bond's price and c is its annual coupon. However, the actual lag is uncertain. Cf. foot- note 26.

Actually, Ball's econometric efforts were not as sophisticated as those of Sargent, Feldstein and Eckstein and Yohe and Karnosky. Ball tried only very simple functions of past inflation rates as proxies for anticipated inflation while Sargent used non-linear estimation of both first- and second-order geometrically declining distributed lags, Feldstein and Eckstein used third-degree Almon [1965] polynomials and Yohe and Karnosky tried both direct linear estimation with past inflation rates and estimation with a sixth-degree Almon polynomial. All of these authors claim to have confirmed the Fisherian theory. Nevertheless, I believe Ball's economic insights are still worth considering; for it is easy to show that a combination of the Gibson paradox with just about any type of stochastic process in the *level* of commodity prices will produce a set of observations that contains a positive relation between the current interest rate and a distributed lag of past commodity price *changes*.

For example, consider the idealized case of an efficient market so that the price level index, P , follows a random walk of the form

$$P_t = P_{t-1} \varepsilon_t$$

where ε_t is a temporally independent positive random variable with unit mean. The Gibson paradox is a positive relation between interest rates, r , and price levels. As an illustrative example, it can be described by the simple process,

$$r_t = k \log_e(P_t) + u_t; \quad k > 0$$

where u_t is another random variable independent of ε_t and of time. Combining this process with the random walk in commodity prices gives

$$\begin{aligned} r_t &= k \log_e(P_{t-1} \varepsilon_t) + u_t \quad \text{or} \\ &= k \sum_{j=0}^N \log_e(\varepsilon_{t-j}) + u_t; \quad (P_{t-N} \equiv 1.) \end{aligned} \quad (1)$$

Now suppose an empirical test of nominal interest rates and inflation is conducted by defining anticipated inflation as a geometrically-distributed lag of past inflation rates thusly:

$$\begin{aligned} r_t &= \alpha \sum_{j=0}^N \beta^j \log_e(P_{t-j}/P_{t-j-1}); \quad 0 < \beta < 1 \\ &= \alpha \sum_{j=0}^N \beta^j \log_e(\varepsilon_{t-j}) \end{aligned}$$

Except for the presence of β , this is almost identical to the equation (1) derivable from an efficient commodity market and the Gibson paradox. Although when $\beta \neq 1$, $E(\hat{\alpha})$ will not equal k , the estimated α will definitely be positive and thus supposedly support the Fisherian theory.²⁵

25. In a later working paper, Sargent (1971) pointed out essentially the same fact. He added in insight that if β is estimated from the data using a non-linear search technique and if the estimate turns out to be close to unity. The distributed lag brings nothing but confirmation of the Gibson paradox. Several authors, including Fisher himself, in fact estimated β to be very

Tuttle and Wilbur were the only authors who used holding-period yields rather than yields-to-maturity. One should note that the holding-period yield is closely negatively related to the change in yield-to-maturity between adjacent periods. The negative and significant relation Tuttle and Wilbur found between holding-period yields and changes in price index is therefore indicative of a positive association between the yield-to-maturity level and the price index level.²⁶

Sargent attempted to resolve the ambiguity caused by the Gibson paradox by including the current price index level in one set of equations. Its coefficient was negative but not significant and he concluded that the sign opposite to that implied by the Gibson paradox settled the issue in favor of Fisher's hypothesis. However, a closer examination shows that this price level was not measured concurrently but was predated by his interest rate observations.²⁷ This fact, together with (a) a high value of β (.98) in his distributed lag and (b) the absence of a "current" inflation rate in his distributed lag; shows that the most relevant price level, that for the nearest time period, had not been expunged.

At this point, the importance of timing different variables should be emphasized. For the Gibson paradox relation, there is no theoretical reason for not using interest rates and price levels of the same calendar dates even though they become publicly available at different times. No predictive relation in either direction is necessarily implied and as long as the fitted equation is not regarded as predictively useful but only as a description of simultaneous economic events, no harm is done by disregarding the information lag. This is also true of included variables such as those related to national income and to the money stock. More care is warranted in the case of the anticipated inflation rate, which is always measured as a function of past rates. If anticipations are presumed to be formed on the basis of those past rates, one must ensure they are observable at least by the date when interest rates are observed.

Yohe and Karnosky's results should be interpreted with this in mind. In all of their regressions, the *current* rate of commodity price change was the strongest explanatory variable. This was true for both the Consumer Price Index and the GNP Deflator, for simple regressions and for regressions with other explanands. Naturally, the fact of neither index being available to investors concurrently with interest rate observations reduces the significance of

close to unity. This was taken for many years to imply a very long lag in the formation of expectations.

26. Although the holding-period yield was reported regressed against the change in wholesale prices lagged one year, the observations were in fact as nearly simultaneous as lagged. This is because the Durand data for year t were computed from monthly high and low observations for the first three months of t . These data were available to investors as soon as the transaction was made. The wholesale price index for year $t-1$ was an average of 12 monthly observations, none of which are published in the month of sampling.

Thus the wholesale price index for year $t-1$ might actually not have been known to investors completely until after all the bond price observations for year t were available.

27. This was because he used Durand's data which were obtained from first quarter observations in each year.

Yohe and Karnosky's results vis-a-vis the current inquiry *if* investors form expectations on the basis of past inflation rates.

Since Yohe and Karnosky did not include a price index level in any test, they offer no evidence on the Gibson paradox effect.²⁸

The weights in Feldstein's and Eckstein's distributed lag seem to support the Fisherian theory strongly because "The sum of the coefficients . . . implies that a permanent one per cent increase in the annual rate of inflation . . . would increase the interest rate by 0.92 per cent," (p. 366).²⁹ However, they do not report and presumably did not calculate an equation that included a price level. They too used a price index whose latest observations post-dated the latest interest rate observations but their careful documentation of all variables permits an approximate reconstruction of their data and replication where the price *level* is included.

A series of Feldstein-Eckstein type models is reported in Table 6. Since the interest rate and price index were intentionally selected differently from those used by Feldstein and Eckstein in order to determine how specifically their results depended on those measures, and since some of the other variables could not be exactly the same,³⁰ it was necessary to fit a model identical in form to theirs in order to ascertain the impact of measurement differences. This is the role of regression number one in Table 6. It is identical in form to Feldstein's and Eckstein's eq. 19 which also repeated here. All the estimated coefficients are comparable in significant level³¹ so the conclusions of their basic model are confirmed by a different set of interest rate and price index measures.

In regression 2, the current *level* of the Wholesale Price Index is included along with a distributed lag of current and past inflation rates, (price index rates of change). The Gibson paradox is strongly in evidence here as the price level's coefficient is positive and more than six times its standard error. Anticipated inflation, as measured by the Almon lag, has no significant effect in this equation. Equation 3 is a mirror image of 2 in the sense that the current

28. However, Yohe and Karnosky gave a convincing argument that extremely long lags (values of β close to one), found with the Koyck method of estimating geometrically-declining distributed lags are specific to the method, (see their pp. 28-29.) They actually estimated $\beta > 1$ using the Koyck method but found a very rapid decay, that is, an equivalent result to β much less than one, using other methods.

29. Their rather short "mean lag of 8.1 quarters" does not imply anything because it is confounded with the coefficient k if the Gibson paradox is valid at all. (They did not employ a non-linear search technique to isolate the lag structure from the overall effect as did Sargent).

30. Several variables are exactly the same. These include the real per capita money base and, (I believe, but am not certain because Feldstein and Eckstein did not mention the source), real per capita privately-owned Federal Government Debt. In addition to price index and interest rate differences, I used real per capita GNP rather than real per capita *private* GNP as a measure of income. The price index used here was the BLS Wholesale Price Index for all items. They used the consumer expenditure component of the GNP deflator. The interest rate series used here was "current coupon" yields on Aa utilities. In equation 19, they used Moody's newly-issued Aaa corporate bonds yields.

31. Only one coefficient, that for the change in the real per capita money base, has changed sign. This particular coefficient is not significantly different from zero in either equation. The major difference between the two equations is the great degree of positive serial correlation estimated in the disturbances of regression model no. 1.

TABLE 6
 REPLICATION OF FELDSTEIN-ECKSTEIN MODEL OF INTEREST RATE DETERMINATION WITH EXPLICIT CONSIDERATION OF THE
 GIBSON PARADOX, U.S. DATA, 1947-1971, QUARTERLY

Model	Constant	Log (R.P.C. Money Base, M_t)	Log (R.P.C. GNP)	Log (R.P.C. Privately-Held Fed. Govt. Debt)	Change in R.P.C. Money Base, $M_t - M_{t-1}$	Current Inflation Rate	Almon Lag of Inflation Rates ^a	Lagged Change in Dep. Vbl, $R_{t-1} - R_{t-2}$	Current Price Index Level 1957-59 = 1.0	Almon Lag of Price Index Level ^c	R ²	D-W
Equations Containing Distributed Lags												
Feldstein & Eckstein, eq. 19, p. 372	-17.6	-9.83* (1.32)	+* (1.14)	2.89 (1.45)	-39.6 (21.1)	.227* (.110)	4.58* (.040)	0.64* (0.18)	n.i.	n.i.	.976	2.23
1	-55.4* (14.9)	-6.47* (1.07)	5.66* (1.03)	4.70 (2.52)	.147 (.127)	.112 (.0722)	4.03* (.0881)	.366* (.163)	n.i.	n.i.	.948	1.01
2	4.02 (15.0)	1.69 (1.60)	1.26 (1.14)	-5.26* (2.56)	.0334 (.109)	n.i.	.216 (.179)	.687* (.147)	24.8* (3.97)	n.i.	.963	1.56
3	-43.3 (13.1)	4.51* (.964)	-1.31 (1.05)	2.34 (2.10)	-.0733 (.126)	.0338 (.0598)	n.i.	.690* (.175)	n.i.	25.2* (5.30)	.943	1.09
4	8.99 (16.5)	1.14 (1.73)	1.30 (1.19)	-5.76* (2.71)	.0406 (.110)	n.i.	.143 (.137)	.662* (.148)	n.i.	25.3 (16.9)	.965	1.53
Equations Without Distributed Lags												
5	-85.8* (11.6)	4.08* (.798)	-1.52 (1.08)	9.13* (1.83)	-.0368 (.127)	-.0965* (.0367)	n.i.	.556* (.194)	19.0* (1.68)	n.i.	.926	.822
6	p Constant	0.9875* (.302)	2.28 (2.37)	-2.55 (1.75)	-.0819 (.01714)	-.0247 (.0237)	n.i.	.0495 (.0843)	3.61 (2.88)	n.i.	.103	1.85

Standard errors are given in parentheses under coefficients.

ρ Autoregressive Parameter.

* Indicates significant difference from zero at 2.5% level based on an assumption of Gaussian disturbances.

n.i., variable not included in regression.

+ This coefficient was printed as .82 but other similar regressions reported by Feldstein and Eckstein make it seem likely to have been a typographical error. The correct value is probably in the neighborhood of 8.2.

^a Third degree Almon polynomial of order 23. This distributed lag structure was chosen to match that used by Feldstein and Eckstein. If P_t is the price index in quarter t let $I_t \equiv [(P_t - P_{t-1})/P_t] - 1$. The distributed lag is a series of coefficients, denoted a_j , weighting past values of I as $\sum_{j=1}^{23} a_j I_{t-j}$ in

model one and $\sum_{j=0}^{23} a_j I_{t-j}$ in other models. Numbers reported are $\Sigma \hat{a}$. The estimated lag structure was very similar to that reported by Feldstein and Eckstein.

^b In Feldstein and Eckstein's eq. (19), this variable was the lagged first differences in Moody's *seasoned* Aaa corporate bond yield. (Their dependent variable was Moody's newly-issued Aaa corporate bond yield).

^c Third degree Almon polynomial of order 10, $\sum_{j=0}^{10} a_j P_{t-j}$. The numbers reported are $\sum_{j=0}^{10} \hat{a}_j$. The order 10 was chosen after longer lags were discovered

to have no significance.

Sample Size: The basic period of observation was 1947: I to 1971: II, a total of 98 quarters. However lagged variables reduced the number of effective observations so that, for example, model one had only 74 observations remaining after the order 23 Almon lag (plus the single lag used to compute the inflation rate).

Sources and Definitions:

1. Quarterly GNP, GNP Deflator (all items), real per capita GNP are from U.S. Dept. of Commerce, *Business Conditions Digest*; October, 1967; October, 1970; August, 1971.
2. Population—defined as GNP/[$(R.P.C. \text{ GNP})$ (GNP deflator)].
3. Total nominal money base for March, June, September, and December; January, 1947, through June, 1948, from Anderson and Jordan [1968]; After June, 1948, from Federal Reserve Bank of St. Louis *Monetary Trends*, releases of October 16, 1969 and April 14, 1971. (Used after division by population and the GNP deflator).
4. Privately-Held Federal Government Debt for March, June, September and December defined as "Interest Bearing Public Debt Securities Held by Private Investors—Total End-of-month Figures," from "Survey of Ownership" in various *Treasury Bulletin*. (Used after division by population and the GNP deflator).
5. Price Index is BLS Wholesale Price Index for all items, March, June, September, and December.
6. Yields (dependent variable) are "current coupon" on seasoned Aa, long-term utilities from Salomon Brothers and Hutzler, (see reference list). Yields are from "offering side of the dealer market on first [trading] day of month," for April, July, October, and January.

inflation rate is included with a distributed lag on current and past price index levels. The sum of distributed lag coefficients is close in absolute value and in significance to the single coefficient of regression 2 for the current price index level³² but again the current inflation rate has no significance. In regression 4, both the price index level and the rate of inflation have been estimated with Almon polynomials. Both are positive in effect but neither are significant.

Other explanatory variables change capriciously from one regression to another. The real per capita money supply is actually significant with a negative sign in equation 1 and significant with a positive sign in 3. Real per capita GNP and Privately-held Federal debt behave similarly. In fact, only one variable, the lagged change in yields, is consistently significant with the same sign in every equation. This would argue for hesitation on econometric grounds of uncritical acceptance of any particular interpretation of any of the five regression models. Whether the basic difficulty arises from measurement error³³ or, (what is more probable), from a bald fitting of a single equation to a multiple equation system, one should beware of making a judgment about the relative effects of anticipated inflation, the price level, and other variables on interest rates.

In the lower part of Table 6, some additional computations are presented. These were done for two reasons: one, to avoid some interpretation difficulties attendant to the Almon procedure and two, to indicate further how difficult a task it is to find useful predictors of speculative prices (i.e., interest rates).

One should be suspicious of the Almon procedure in a single equation where the Almon explanatory variable is going to be compared with other explanatory variables introduced linearly. The suspicion arises from a methodological bias against those other variables. They are permitted to enter the regression linearly and in no other way while the variable entering as an Almon polynomial has a chance to benefit from an essentially infinite number of transformations; the procedure is to search over an entire class of polynomials of given degree to find the member of best fit. Imagine what the effect would be of first running an explanatory variable linearly, then logarithmically, trigonometrically, and then with various polynomials. With a given set of observations, the chance of finding a significant transformation must increase with the number of transformations attempted, even if the two variables are statistically independent. Therefore, one should not compare the significance of an Almon lagged variable with that of another explanand unless the latter has been transformed over at least as large a class—e.g., polynomials of the same degree, and unless every *combination* of transformation of the two variables has been calculated and sorted by some criterion such as R^2 .

32. However, the estimated lag structure does not show all the weight on the current index level. Coefficients for price index levels lagged zero, one, five, six, seven, and eight quarters are positive and have corresponding *t*-ratios in excess of 2.0.

33. As emphasized before, the particular price index chosen has an important effect on the results. I recomputed equation 1-4 using the GNP deflator for all items in place of the Wholesale Price Index and in each equation, the inflation rate variable had a much stronger effect relative to the price index level variable. Thus, the reported results in the first four equations of Table 6 are less favorable to the Feldstein-Eckstein model than the results using the GNP deflator. A table with those latter results is available on request.

In models 5 and 6 of Table 6, all the regressors are entered linearly (or logarithmically in the same cases that Feldstein and Eckstein used logarithmic transformations). Model 5, using the *levels* of all variables, finds many significant explanatory variables. Both the price index level and the inflation rate are significant (but the inflation rate has a theoretically-incorrect sign). However, an extreme degree of positive autocorrelation is indicated by the Durbin-Watson statistic of .822. Model 6 represents an econometric remedy for autocorrelation, constrained estimation with transformed variables.³⁴ In model 6, not a single explanatory variable remains significant at a significance level in common use³⁵ and the multiple coefficient of determination is only 0.103. Since the autoregressive parameter estimate is very close to one, the results are more consistent with the hypothesis that nominal interest rates follow a random walk than they are with any other hypothesis that has been presented in the literature.

Indirect Measures of Anticipated Inflation Inferred From Differences Between Real and Paper Assets

Although direct correlations between nominal interest rates and functions of past inflation rates are indicative tests, they can never be decisive; for they measure *anticipated* inflation imperfectly, they do not account adequately for changes in the "real" rate of interest, and they are beclouded by the Gibson paradox. Fortunately, there are other tests that do not suffer from the same problems (although they may have unique difficulties of their own.) Fisher himself, [1930, pp. 401-407] introduced one of these. He attempted to relate the difference in returns between nominal (or paper) assets and real assets to some measure of inflation. Naturally, the difficult task was to find a real asset; for as Fisher noted, "There are, theoretically, just as many rates of interest expressed in terms of goods as there are kinds of goods diverging from one another in value," (p. 42).

However, he did discover two examples that were at least suggestive: Both examples used bonds issued payable in gold while paper currency bonds of the same issuer were outstanding. According to his theory, differences in yield-to-maturity between the two bonds should have measured the market's anticipation of gold's future price change in currency units. The first example consisted of gold and paper bonds of the U.S. government from 1870 to 1896. Before 1878, the yield difference was distorted by a constant anxiety about the re-

34. This procedure is as follows. Let the original model be $y_t = \pi'_t \beta + \varepsilon_t$ where ε is assumed to display a first-order autoregressive structure, $\varepsilon_t = \rho\varepsilon_{t-1} + u_t$, such that u_t is distributed identically and independently over time. Combining the autoregressive structure and the original model gives a model in transformed variables,

$$y_t - \rho y_{t-1} = (x_t - \rho x_{t-1})' \beta + u_t.$$

A search is conducted over possible values of ρ to find the ρ which minimizes the estimated variance of u . At each trial, including the final trial, least squares estimates of β are obtained also. The final estimates ρ and β are the maximum likelihood estimates of ρ and β .

35. However, both the money base and privately-held government debt are significant at the ten per cent level.

sumption of specie payments,³⁶ (Cf. Unger [1964], or Friedman and Schwartz [1963], pp. 79-85), while from 1879 to 1896, the evidence directly contradicts the theory because gold yields were lower than or equal to paper yields in every period while commodity prices expressed in paper *declined* about 25 per cent.³⁷ The second example was Indian gold and rupee bonds traded in London from 1865 to 1906. Differences in these yields were compared to the London Rupee-Pound exchange rate and this example indeed tends to support the Fisherian hypothesis.³⁸

Among the more recent studies of this type, Friedman and Schwartz [1963, pp. 583-584] mention the difference between a Baa-rated bond yield³⁹ and the dividend and earnings yields on Moody's 125 Industrial Common Stocks as indications of changes in expectations of price inflation during three post-World-War II periods. The periods examined were: 1946 quarter I to 1948: IV, which was marked by a decline in anticipated inflation; 1950: III to 1951: III, a rise in anticipated inflation; and 1955: I to 1957: IV, a rise in anticipated inflation. Although these anticipations were stated by Friedman and Schwartz to have been "widely-held" during the periods given, no evidence is offered to support that contention. Presumably, such evidence could have been in the "Surveys of Consumer Finances" compiled by the Michigan Survey Research Center and reported in various issues of the *Federal Reserve Bulletin*. This might be inferred from Friedman's and Schwartz' later use of these surveys (see their p. 599) to justify similar statements about consumers' price expectations.⁴⁰

36. i.e., resumption of a Treasury guarantee to convert Greenback dollars to gold at par.

37. See Bureau of the Census, *Historical Statistics of the United States* [1960], pp. 115, 117. Prices increased from 1879 to 1882 and declined steadily from 1882 to 1896.

38. Fisher did not make these calculations but the Rupee-Gold yield differences is positively correlated with changes in the pound price in rupees over leads and lags. Between the Rupee-Gold yield difference and the next annual Rupee price change in pounds, the simple correlation is $-.206$. Between the difference and the most recent past change, the correlation is $-.250$.

39. The (quarterly) yield they used is an average of three monthly observations. Primary source was Moody's Investors Service.

40. These surveys of consumer expectations are very interesting in their own right for the information they provide on the relation between inflation attitudes and current commodity-buying attitudes. In Katona and Mueller [1953, pp. 70-75], this relation is discussed in detail. Respondents who believe prices have recently risen see the present as a "bad time to buy" as do respondents who predict prices will decline over the next five years. Those with opposite opinions, that prices have recently fallen or will go up over the next five years, view the present as a good time to buy. The surveys included a very large number of different questions and it is interesting to note that Katona and Mueller judge that ". . . the relationship of past price movements and attitudes toward spending is among the most pronounced and most consistent that has been detected in this series of surveys," (p. 71). For a later period, this relation in attitudes does not show up. See, e.g., Katona, *et. al.*, [1967, pp. 236-237]. The surveyors attribute its absence to the recent presence of "creeping" rather galloping inflation. However, one may wonder whether it is not just a deficiency in the question asked, which has been changed from "Do you think this is a good time or a bad time to buy such large household items (furniture, house furnishings, rugs, refrigerators, stoves, radios, and things like that)?" to "Now speaking for a moment about price increases and inflation, would you say that someone like you can do something when prices are going up so as to safeguard himself to some extent against price increases?" The most frequent answer to this last question was that one can't do anything, and a sizeable minority said one can buy selectively, postpone buying, or buy less. This indicates a confusion among respondents' about whether the price increase in the question has already or is just about to occur and about whether they are being asked a *theoretical* question about the likely effect of many consumers ceasing to buy.

TABLE 7
MEASURES OF ANTICIPATED INFLATION FROM SURVEY
RESPONSES AND BOND AND STOCK YIELDS

Period of Survey	Yield Quotation Dates	Percentage of Survey Respondents Anticipating Inflation ^a	Yields ^b		Bond Yield Minus Earnings Yield	Stock Price Index ^b
			Baa Bond	Earnings		
Early 46	Feb. 46	53	2.95	5.03	- 2.08	49.3
Early 47	Feb. 47	13	3.12	9.10	- 5.98	46.2
July 47	July 47	32	3.18	10.3	- 7.12	47.9
Jan. 48	Jan. 48	50	3.52	12.0	- 8.48	45.4
Feb. 14-	Feb. 48	15	3.53	13.0	- 9.47	43.2
Mar. 5 48						
July 48	July 48	42	3.37	13.0	- 9.63	48.6
Early 49	Feb. 49	8	3.45	15.9	-12.5	44.5
July 49	July 49	7	3.46	14.9	-11.4	46.0
Early 50	Feb. 50	12	3.24	12.5	- 9.26	52.9
Early 51	Feb. 51	67	3.16	12.5	- 9.34	68.6
Early 52	Feb. 52	49	3.53	10.0	- 6.47	72.0
Early 53	Feb. 53	15	3.53	9.17	- 5.64	79.2
Jan.-Feb. 63	Jan. 63	71	4.91	5.46	- 0.55	204.
Jan.-Feb. 64	Jan. 64	68	4.83	5.24	- 0.41	241.
Feb. 65	Feb. 65	72	4.78	5.29	- 0.51	278.
Aug. 65	Aug. 65	73	4.88	5.62	- 0.74	279.
Feb. 66	Feb. 66	86	5.12	5.69	- 0.57	293.
Aug. 66	Aug. 66	87	5.83	6.90	- 1.07	244.
Feb. 67	Feb. 67	78	5.82	6.14	- 0.32	267.
Aug. 67	Aug. 67	82	6.33	5.29	+ 1.04	295.
May 68	May 68	81	7.03	5.18	+ 1.85	318.
Aug. 68	Aug. 68	80	6.82	5.38	+ 1.44	318.

^a Source through 1953: *Federal Reserve Bulletin*, (1949, p. 641; 1951, p. 632; 1953, p. 592); From 1963-1966, Katona, *et al.*, [1967, p. 248, 254]; after 1966, Katona, *et al.* [1969, p. 214, 222]. There is a discrepancy between the latter two references in those cases where both contain results for the same dates.

^b Source: *Moody's Industrial Manual*, (1970, pp. a18, a27-a29). Earnings yield is reciprocal of price earnings ratio.

With a larger sample of similar data, 22 survey periods from 1946 to 1968, Table 7 supports the qualitative conclusion of a valid Fisher effect because it demonstrates a strong positive relation between the percentage of respondents who expected commodity price increases and the difference between bond and stock earnings yields. Before accepting this evidence as conclusive, however, one must consider the complications caused by three data pollutants: First, bond yields during most of the early period 1946-1951, were undoubtedly depressed by the pegging operations in the short-term government bond market during the tenure of the Federal Reserve-Treasury accord. Second: Dividend and Earnings yields on common stocks are poor estimates of the real rate of interest because common stock prices in real terms are not independent of inflation. Industrial corporations are generally net nominal debtors⁴¹

41. Of course, "financial corporations" are generally net creditors or are completely hedged, but the stock index used by Friedman and Schwartz (and here) did not include those groups.

and they have also benefited from a lag of contractual wage increases behind commodity price increases. On the other hand, they can be harmed as inflation alters tax liabilities calculated on the basis of historical costs.⁴²

Third and most important, the current earnings price ratio does not measure the real rate of interest alone even if common stocks are perfect inflation hedges. Instead it measures the real interest rate minus the real growth rate in earnings.⁴³ This can be discovered by examining simple discounting formulae. Let E_j be expected nominal earnings per share of the corporation in period j , P_j be the commodity price level, V_0 be the stock's current market price, and r be the real interest rate. The current market price must be equal to the discounted present value of future real earnings per share $V_0 = \sum_{j=1}^{\infty} (E_j/P_j)/(1+r)^j$. Now suppose commodity prices are inflating at the rate α and earnings are growing in nominal terms at the rate α (for inflation hedging) plus β (for a real earnings increase); i.e., $P_j = (1+\alpha)^j P_0$ and $E_k = [(1+\alpha)(1+\beta)]^k E_1$. For simplicity let the current commodity price level, P_0 , equal unity. Then

$$V_0 = [E_1/(1+\alpha)] \sum_{j=1}^{\infty} [(1+\beta)^{j-1}/(1+r)^j]$$

and thus after summing the series, assuming $r > \beta$,

$$E_1/V_0 = (r - \beta)(1 + \alpha)$$

The difference between the bond yield and the earnings yield is therefore

$$\begin{array}{c} \text{Bond yield} \\ [(1+r)(1+\alpha) - 1] \end{array} - \begin{array}{c} \text{Earnings yield} \\ [E_1/V_0 = (r - \beta)(1 + \alpha)] \end{array} = \alpha + \beta + \alpha\beta,$$

the sum of the inflation rate and the real growth rate of earnings plus a compounding term of small order. This explains why one could observe increases in both the bond yield and the bond-stock yield difference when there is no inflation anticipated at all; for β , the real growth rate of earnings, is highly positively related to the real interest rate.⁴⁴ Increases in the real rate of interest would cause an increase in nominal yields on bonds and also cause an increase in real earnings growth and hence in the bond-stock yield difference.

Although the problem caused by non-zero real rates of earnings growth was still present, an attempt to diminish some other measurement problems was made by Gupta [1970] in a truly unique and ingenious test of the Fisher hypothesis. Gupta noted that a valid Fisherian relationship of the form

$$R = r + I$$

where R is a nominal anticipated return, r is the real rate of interest and I is the anticipated rate of commodity price change, would imply that either R or

42. See Nichols [1968] and Motley [1969] for an analysis of inflation's effect on the tax deductions from different methods of accounting for depreciation.

43. For the illustration following, risk will be disregarded.

44. In fact, they are perfectly functionally related in the case of certainty.

$r + I$ could be used in any *other* economic relation that needed a nominal interest rate as an explanatory variable. Gupta chose to work with a simple demand for money function of the form $M = f(Y, R)$ where M is the money stock and Y is (permanent) net national product.⁴⁵ Gupta's procedure was to run one regression of the form

$$\log M = \hat{a}_0 + \hat{a}_1 \log Y + \hat{a}_2 R \quad (2)$$

and another of the form

$$\log M = \hat{b}_0 + \hat{b}_1 \log Y + \hat{b}_2 r + \hat{b}_3 I \quad (3)$$

and to test the hypothesis that $a_2 = \hat{b}_2 = \hat{b}_3$. R was measured by the same yields as those used by Meiselman [1963], while r was the ratio of an "expected" earnings per share number to an average common share price.⁴⁶ Expected earnings were computed by an exponentially weighted distributed lag of an actual earnings index; but unfortunately, the speed of adjustment coefficient was chosen "... to give the 'best' results in the tests ...", (p. 190). The anticipated inflation rate was measured by the difference, $I \equiv R - r$, between the bond yield and the expected return to equities; so the sum of the last two explanatory variables in equation (3) was identically equal to the last variable in (2) for every observation.

Most of the tests Gupta ran for different sub-periods and for the total period supported the hypothesis that $a_2 = b_2 = b_3$ and thus supported Fisher's theory. Again, however, one must hesitate to accept this as conclusive evidence because the variables were constructed so as to maximize the chance of obtaining the desired results. This is easy to see in the construction of the real interest rate, r , because a search was conducted over twenty possible definitions of r to find the "best" one. Recall that the speed of adjustment coefficient in the distributed lag of past earnings, (the numerator of r 's formula), was chosen this way. When one considers the substantial body of empirical evidence indicating corporate earnings changes cannot be predicted by using past changes,⁴⁷ the effect of Gupta's search procedure is readily seen to bias his empirical results in favor of the maintained hypothesis. It would be unfair to dismiss the results on these grounds alone, however, since it seems unlikely that other speed of adjustment coefficients would have altered the picture completely.

In summary, if one believes that asset markets are efficient processors of information, predicted rates of inflation measured by the difference between paper and "real" yields on assets should be among the best measures available. There have been relatively few studies of inflation with such measures, however, and this no doubt can be ascribed to a scarcity of "real" securities and hence to a scarcity of relevant price data. This scarcity is itself a cause of amazement. Why *are* there so few existing liquid securities with explicit contractual claims tied to commodity prices?⁴⁸ When one regards the variety of

45. Both M and Y are in "real" terms.

46. Both earnings and stock price averages were Cowles Commission indexes for years 1871-1925 and Standard and Poor's indexes thereafter. See Gupta (pp. 200-201) for more details.

47. See, e.g., Brealey [1969], Chapter 8.

48. Arvidsson's [1962] is an essay on just this point. He presents several reasons why one

other assets ranging from Savings Accounts, through Treasury Bills, Mortgages, Convertible Preferreds, Municipal Revenue Bonds, Warrants, Puts and Calls, etc., many obviously designed for the special needs of the issuer or to attract buyers by catering to their special needs and many redundant in the sense that a suitable combination of others would bring the same distribution of returns, how is one to explain the absence of bonds tied to the general price level? To be sure, a portfolio of commodity futures contracts provides some insulation against movements in wholesale prices but it seems doubtful that even a perfectly diversified commodity futures portfolio would possess qualities attractive to consumers worried about the future prices of refrigerators and automobiles rather than the prices of flax, soybeans, and pork bellies.

Since new varieties of assets are produced in a relatively competitive industry, the absence of this particular variety, bonds with claims tied to a price index, must be attributed to a lack of demand, such lack being due to no fear of inflation or to the availability of a cheaper adjustment mechanism though current commodity purchases should the fear of inflation arise.⁴⁹

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should or should not expect "Index loans" to arise in a market economy and he also investigates the benefits and costs a government might expect from issuing such loans.

49. Also, of course, the large proportions most consumers have invested in the human capital and real estate sections of their asset portfolios makes them less susceptible to uncertainties about inflation. Transaction costs are relatively higher for these two categories of assets, both in direct charges and in the time necessary to consummate a trade. Thus, their existence does not answer the question of why *liquid* price-index-linked bonds are hard to find.

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