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On Valuing American Call Options with the Black-Scholes European Formula

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ABSTRACT

Empirical papers on option pricing have uncovered systematic differences between market prices and values produced by the Black-Scholes European formula. Such "biases" have been found related to the exercise price, the time to maturity, and the variance. We argue here that the American option variant of the Black-Scholes formula has the potential to explain the first two biases and may partly explain the third. It can also be used to understand the empirical finding that the striking price bias reverses itself in different sample periods. The expected form of the striking price bias is explained in detail and is shown to be closely related to past empirical findings.

THE BLACK-SCHOLES European option pricing model tends to exhibit systematic empirical biases when used to value American call options. The reported biases have occurred with respect to the exercise price, the time until expiration, and the stock's volatility. Using Chicago Board Options Exchange (CBOE) prices, Black [1] reported that the model systematically underpriced deep out-of-the-money options and near-maturity options while it overpriced deep in-the-money options during the 1973–1975 period. MacBeth and Merville [8] confirmed the existence of a striking price bias for the 1975–1976 period, but found that it was the reverse of the bias reported by Black. Rubinstein [10] also found that the striking price bias had reversed in the 1976–1977 time period. Both Rubinstein [10] and Emanuel and MacBeth [7] reported that the original striking price bias observed by Black had reestablished itself in late 1977 and 1978.

Before listed option trading had commenced, Black and Scholes [3] tested their model on over-the-counter (OTC) call options. They found that the model underpriced options on low variance stocks and near expiration options while it overpriced options on high variance stocks. Geske et al. [5] argued that these "biases" detected using OTC data should be expected because the OTC dividend protection (i.e., reducing the exercise price by the full amount of the dividend on the ex-dividend date) is imperfect.

Whaley [14] also found striking price, time-to-expiration, and variance biases using CBOE data for the period 1975–1978. Whaley does not report on the reversal of the striking price bias. However, he does demonstrate empirically that the striking price and time-to-expiration biases are virtually eliminated, and that the variance bias is reduced when a corrected version of Roll's [9] American call option formula is employed. Whaley concludes that the dividend-induced prob-

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ability of early exercise is an important ingredient in American call option pricing.

Black [1] proposed an ad hoc adjustment of the European call option formula to account for the American feature of early exercise. Black's adjustment has been employed in studies by himself, MacBeth and Merville, Rubinstein, and numerous other researchers. The idea is to compute several different call values: values assuming early exercise at each of the scheduled ex-dividend dates during the call option's life and a value assuming that no early exercise will occur. The European call option formula is used for each computation. In the no early exercise computation, the relevant stock price used in the European formula is the current stock price reduced by the present value of all the scheduled dividend payments over the life of the option, discounted at the risk-free rate (since the dividend payments are assumed to be certain). The time-to-expiration is the same as the option's scheduled expiration date. In each of the certain early exercise computations, the time until expiration is shortened to the particular ex-dividend date, and the relevant stock price is only reduced by the present values of any dividends paid over the option's life before that ex-dividend date. Finally the maximum of these call prices, termed the "pseudo"-American call price, is selected as the value closest to the actual American value.

Assuming the option is exercised at an ex-dividend date shortens its life, thereby reducing its value. However, there is a compensating tradeoff because the stock price is not reduced by the present value of further dividends planned before the contractual expiration date. Whether the "certain to be prematurely exercised" option, with its shorter life but larger relevant stock price, is greater or less in value than the "certain to be held to expiration" option, with its longer life but reduced stock price, depends upon the size of the dividend, the interest rate, and the time period between the relevant ex-dividend date and the options expiration date.

The pseudo-American call value will be less than the actual American value because the pseudo method does not reflect the full opportunities of the American call holder. The American call value reflects the conditional probability (conditional on the stock price) of exercising prematurely at each ex-dividend date, rather than certain early exercise or certain no exercise; the American valuation formula (Roll [9]) incorporates this additional flexibility.[1] Because the Black-Scholes European formula, used in the pseudo-American way, does not reflect the full flexibility of the American call option contract, it *should* exhibit systematic biases, particularly with respect to the probability of early exercise. Thus, it is not surprising that Black, MacBeth and Merville, and Rubinstein report such biases.

The purpose of this paper is to explain why these biases occur and how they

[1] Technically, Roll's [9] formula is also an approximation to the problem of valuing an American call option on a stock paying constant dividends. By assuming the dividends are certain, and thus escrowing them from the current stock price, Roll avoids the intractable discontinuity at the ex-dividend date. However since the diffusion process and its parameters (i.e., the instantaneous variance) may be affected by the level of the stock price, this is an approximation to the true problem. Roll's formula based on a duplicating portfolio is unnecessarily complex. Geske [4] derives an equivalent but simpler formula, and Whaley [13] corrects errors in both Roll and Geske.

may reverse themselves. Since the probability of early exercise depends on whether the option is in- or out-of-the-money, a striking price-related bias should be of major importance, but we also report on the time-to-expiration and variance biases.

Finally if the observed biases are alleviated by the American call formula, it should provide more accurate values than the Black-Scholes (pseudo) formula. The accuracy of the American formula should be most noticeable between "certain exercise" and "certain no exercise" where the flexibility of the American call holder is most valuable and the error of the pseudo-American technique is greatest. Due to problems discussed in this paper, earlier research could not detect the expected dominance of the American formula. In three recent papers, Whaley [14] and Sterk [11, 12] show that the American formula yields statistically and economically superior prices to the pseudo-American formula.

I. Biases in Pricing American Call Options

This section demonstrates how improper treatment of dividends and thus of the probability of early exercise can induce the striking price, time-to-expiration, and variance biases currently observed and reported in the literature on American call option pricing.

A. Striking Price Biases

The "original" striking price bias (Black [1]) was that the Black-Scholes model underpriced out-of-the-money options (i.e., stock price less than the exercise price or $S < X$) and overpriced in-the-money options. This bias acts as if the volatility were related to the exercise price; i.e., if the Black-Scholes European formula were employed with different volatilities at each exercise price, the observed pattern of market prices could be produced.

Table I provides some information about this problem. Implied volatilities from the original Black-Scholes formula were computed for a range of exercise prices assuming both (i) no early exercise and (ii) certain early exercise. The stock price is \$40.00, it pays a \$0.50 dividend in 2.5 months, and the call option expires in 3 months. American call values are computed by the corrected Roll formula, assuming the true volatility is $\sigma = 0.3$. The implied volatilities are computed with the Black-Scholes formula using the computed American call prices as if they were market prices. For the no exercise case, the time until the option's expiration is $T = 3$ months, and for the certain early exercise case, $T =$ 2.5 months, since this option would be exercised at the ex-dividend date.

For the no early exercise case, Table I shows that the implied volatility is inversely related to the striking price. This inverse bias was reported by MacBeth-Merville and Rubinstein. Conversely, for the certain early exercise case, the implied volatility is directly related to the striking price. This direct bias was originally reported by Black. Thus, it is possible to obtain either bias using the Black-Scholes model depending upon whether no exercise or certain exercise is employed in the pseudo-American valuation technique.

The probability of exercising early is inversely related to the interest rate and

Table I

Implied Volatilities versus Exercise Prices
(S = \$40.00, r = 5%, D = \$0.50, t_D = 2.5 months, T = 3 months)

	Implied Volatility	
Exercise Price X(\$)	No Early Exercise (Inverse Bias) $\hat{\sigma}(T = 3 \text{ months})$	Early Exercise (Direct Bias) $\hat{\sigma}(T = 2.5 \text{ months})$
10	1.831	0.300
20	0.970	0.300
30	0.482	0.313
36	0.344	0.315
38	0.328	0.316
40	0.318	0.317
42	0.313	0.318
44	0.310	0.319
50	0.306	0.322
60	0.306	0.325

directly related to the dividend payment. Whether the pseudo-American technique selects early exercise or no early exercise will depend primarily on these two parameters.[2] Because their effects on the probability of early exercise are opposite, the dominant effect must be determined. The determination of the dominant effect is complicated empirically by the degree to which dividend and interest rate movements are correlated. If dividends and interest rates move together, and if the dividend effect dominates, then in times of relatively low dividends and/or relatively low interest rates, the no early exercise alternative is more likely to be selected. In this case, we would expect to see the inverse bias wherein the implied volatility is inversely related to the exercise price. In times of relatively high dividends and/or high interest rates, the early exercise option may be selected. This would induce Black's direct bias. Thus, a possible explanation of the bias reversals reported by MacBeth and Merville, MacBeth, and Rubinstein is the effect of dividend and interest rate movements on the probability of early exercise. Table II provides evidence on this hypothesis.

In Table II dividend yields for the S&P 500 composite index and yields to maturity for short-term governments are given from January 1976 to December 1978. The bars indicate the months for which Rubinstein and MacBeth-Merville report Black's direct bias and the inverse bias. The inverse bias is detected when the average dividend yield and average yield-to-maturity (reported next to each bar graph) are both relatively low compared with those averages during the period when the direct bias was detected.

Since the mispricing by the Black-Scholes model is most noticeable for in- and out-of-the-money options, a common assumption is that their model correctly prices at-the-money options. This assumption is incorrect. The Black-Scholes model employed in its pseudo-American form will underprice *all* dividend-paying

[2] We assume that other effects, such as the average proximity of the last ex-dividend date to the expiration date, do not change much over different sample years.

Table II

Dividend Yields and Yields to Maturity For Time Period of the Striking Price Bias Reversal

		Dividend Yield S&P 500 Composite	Yield-to-Maturity Short-Term Governments	Rubinstein	MacBeth-Merville MacBeth
1976	J	3.80	6.76		Inverse Bias (1976 J–D)
	F	3.67	6.84		
	M	3.65	6.91		
	A	3.66	6.51		
	M	3.76	7.02		
	J	3.75	7.07		$\overline{D/S} = 3.77$
	J	3.64	6.82		$\bar{r}_F = 6.60$
	A	3.74	6.60		
	S	3.71	6.32	Inverse Bias (X & σ Inversely Related) (1976 S – 1977 O)	
	O	3.85	5.84		
	N	4.04	5.84		
	D	3.93	5.31		
1977	J	3.99	5.78	$\overline{D/S} = 4.39$	
	F	4.21	5.98	$\bar{r}_F = 6.07$	
	M	4.37	5.93		
	A	4.47	5.78		
	M	4.57	6.03		
	J	4.60	5.89		
	J	4.59	5.99		
	A	4.79	6.35		
	S	4.82	6.44		
	O	4.97	6.83	Direct Bias (X & σ Directly Related) (1977 O – 1978 S)	
	N	5.02	6.78		
	D	5.42	6.70		
1978	J	5.32	7.23	$\overline{D/S} = 5.29$	Direct Bias (1978 J–D)
	F	5.49	7.38	$\bar{r}_F = 7.54$	
	M	5.62	7.36		
	A	5.42	7.54		
	M	5.20	7.83		
	J	5.19	8.08		$\overline{D/S} = 5.28$
	J	5.25	8.33		$\bar{r}_F = 8.24$
	A	4.93	8.21		
	S	4.97	8.45		
	O	5.11	8.82		
	N	5.45	9.65		
	D	5.42	9.95		

Source: Standard and Poor's *Security Price Index Record,* 1980. The direct bias was originally reported by Black in a paper published in mid-1975. He did not report dates, but the observations must have been for a period between mid-April, 1973 and the end of 1974, since the CBOE began trading listed options on the former date and since some lag had to occur between Black's observation and his publication. The 4/73–12/74 period was characterized by dramatically increasing dividend yields and interest rates. For example, in the last 6 months of 1974, the average dividend yield and average interest rate were $D/S = 5.12$ and $r_F = 8.00$, respectively.

American options, including at-the-moneys. MacBeth and Merville, in their well-known paper [8], forced the Black-Scholes model to "correctly" price at-the-moneys on the grounds that this would enable the model to better fit the data. Their procedure creates a more complex bias.

In order to examine the difference between Black-Scholes model prices and observed market prices, the MacBeth-Merville procedure must obtain an implied variance rate for an at-the-money option. Since at-the-money options are not usually available, they estimate this implied volatility using linear regression. Define M as percentage in- or out-of-the-money, i.e.,

$$M = \frac{S - Xe^{-rt}}{Xe^{-rt}}.$$

Then run the regression

$$\sigma_{ijt} = \theta_{i0t} + \theta_{i1t} M_{ijt} + \epsilon_{ijt}$$

where σ_{ijt} is the implied standard deviation of option j on security i on day t.[3] The estimated at-the-money implied volatility, $\hat{\sigma} = \theta_{i0t}$ is used as the true volatility of the security.

To facilitate comparison between options on different securities, MacBeth and Merville define the percentage difference between C, the market (or American) price of the option, and the Black-Scholes model price of the same option, $c(\hat{\sigma})$,

$$V = \frac{C - c(\hat{\sigma})}{c(\hat{\sigma})}.$$

For a given stock, they plot the extent (V) of Black-Scholes mispricing for various options on various days in the sample, against the extent (M) that the option is in- or out-of-the-money.

We replicated their technique using computed American call prices as market prices. Figure 1 graphs four possible representations of call values versus exercise prices. The upper curve is the Black-Scholes European call value, $c(S, X, \sigma, T)$, assuming no dividend payments. Just below this upper bound is the curve of computed American call values, $C(S, X, \sigma, \langle t \rangle, T, \langle D \rangle)$, where σ is assumed to be the true volatility of the security, $\langle t \rangle$ represents the sequence of ex-dividend dates, T is the option contractual expiration date, and $\langle D \rangle$ is the sequence of dividend amounts. The next two curves (which intersect near-the-money) represent pseudo-American values in two cases: (1) early exercise, $c(S, X, \sigma, t)$, where the option expiration date is taken as an ex-dividend date, and (2) no early exercise, $c(S - De^{-rt}, X, \sigma, T)$, where the stock price is reduced by the present value of the dividends to be paid and the option's original expiration date remains valid. The box on the American call curve above the at-the-money point, $X = Se^{rT}$ (or $S = Xe^{-rT}$), represents the market call price which will be substituted

[3] Because the implied volatility is nonlinear with respect to the amount the option is in- or out-of-the-money, this linear regression technique is somewhat inaccurate in itself. It will tend to overestimate the true σ. However this problem is minor relative to forcing the Black-Scholes model to "correctly" price at-the-money options.

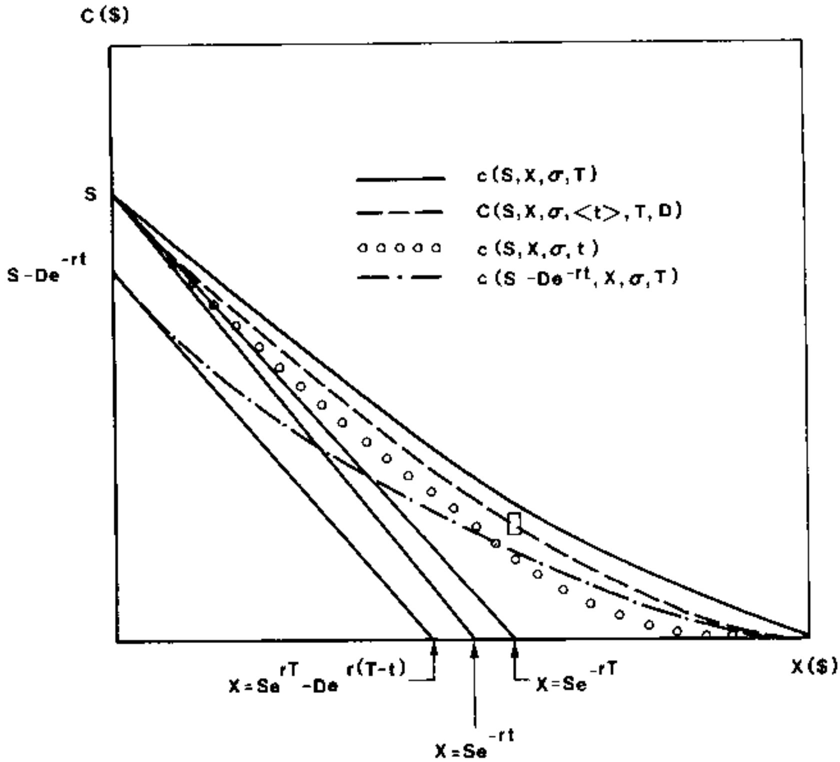

Figure 1. Call Values versus Exercise Prices. In- and out-of-the-money call options. American (C) and Black-Scholes European calls (c). True volatility is σ. The order is European no dividends (——), American (- - -), European early exercise (ooo), and European no early exercise (-·-).

into each of the pseudo-American methods in order to compute an at-the-money implied volatility. Figure 1 illustrates that the pseudo-American values always lie below the correct American values.

Figure 2 depicts American call values and pseudo-American values for the case of no early exercise where an implied volatility $\hat{\sigma}$ is computed so as to force equality between the at-the-money Black-Scholes value and the at-the-money market price of the option (see box, Figure 2). This figure illustrates that the Black-Scholes model will overprice out-of-the-money calls and underprice in-the-money calls, i.e., the reverse striking price bias reported by MacBeth-Merville and Rubinstein. Using the same data, Figure 3 graphs V, the extent of mispricing, versus M, the extent the option is in- or out-of-the-money. This figure is identical in form to Figure 2, p. 1180 in MacBeth and Merville [8]. When V and M are both positive, the Black-Scholes model underprices in-the-money options, and V and M both negative implies an overpricing of the out-of-the-money options. For example, a -10% error in value when the option is out-the-money would imply an error of 50 cents on a \$5.00 call. As Figure 3 illustrates, the graph is quite nonlinear, as was the empirical scatter shown in MacBeth and Merville. When MacBeth and Merville's [8, p. 1180] figure from actual data is overlaid on our Figure 3 from simulation, the two match almost perfectly.

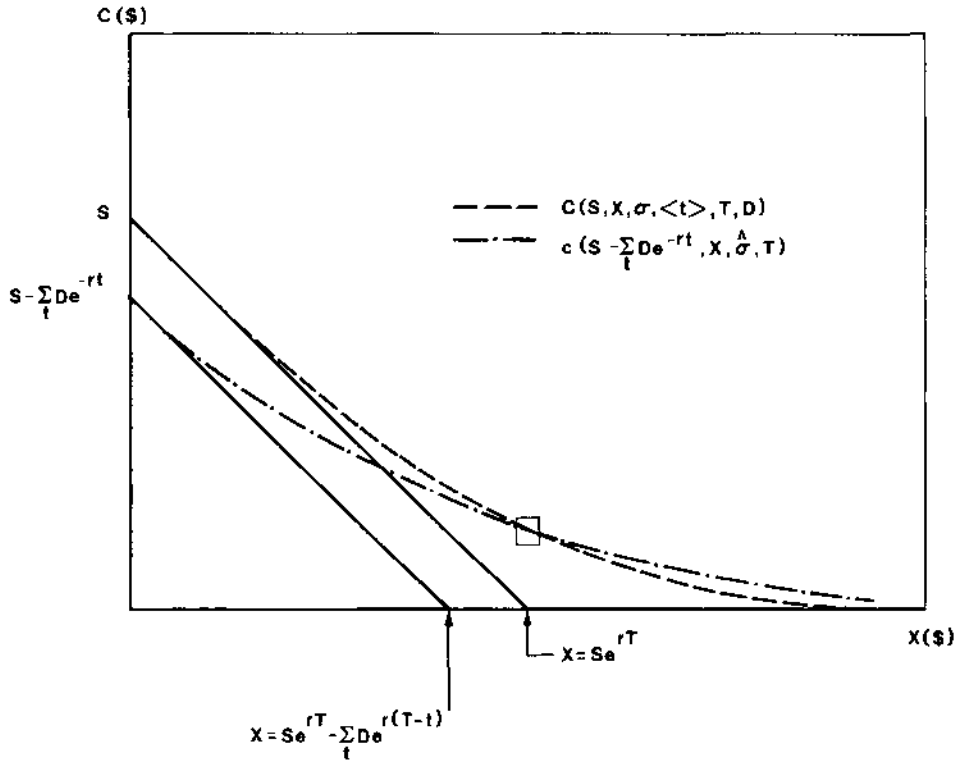

Figure 2. No Early Exercise. In- and out-of-the-money call options. Call values versus exercise prices for American calls (C) and Black-Scholes European calls (c) for no early exercise. True volatility is σ. Estimated volatility, $\hat{\sigma}$, forces $c = C$ for at-the-money call options.

When MacBeth and Merville were able to detect a significant probability of early exercise, they eliminated that option from the sample. Thus, they were more likely to have found the reverse bias, which they indeed did find. They report finding no evidence of an early exercise effect for options with between 90 and 100 days to expiration. However, they did find evidence of an early exercise effect for options with less than 90 days to expiration. If they had left these options in the sample, there would have been more data points in the second and fourth quadrants of their V versus M graph, as we now illustrate.

Figure 4 depicts American call values and pseudo-American values for the certain early exercise case. Again, an implied volatility, $\hat{\sigma}$, is computed so as to force equality between the at-the-money Black-Scholes value and the at-the-money market price (see box, Figure 4). In this case, the Black-Scholes model tends to underprice out-of-the-money calls and overprice in-the-money calls; Figure 5 illustrates this using the same data in a graph of V versus M. Note that some observations lie in the second and fourth quadrants. MacBeth and Merville's Figure 3, which included some options showing evidence of early exercise effects, also supports this conclusion. Even after screening out options "likely" to be exercised early, it contained a few points in the second and fourth quadrants.

In summary, both the original and the reverse striking price biases detected

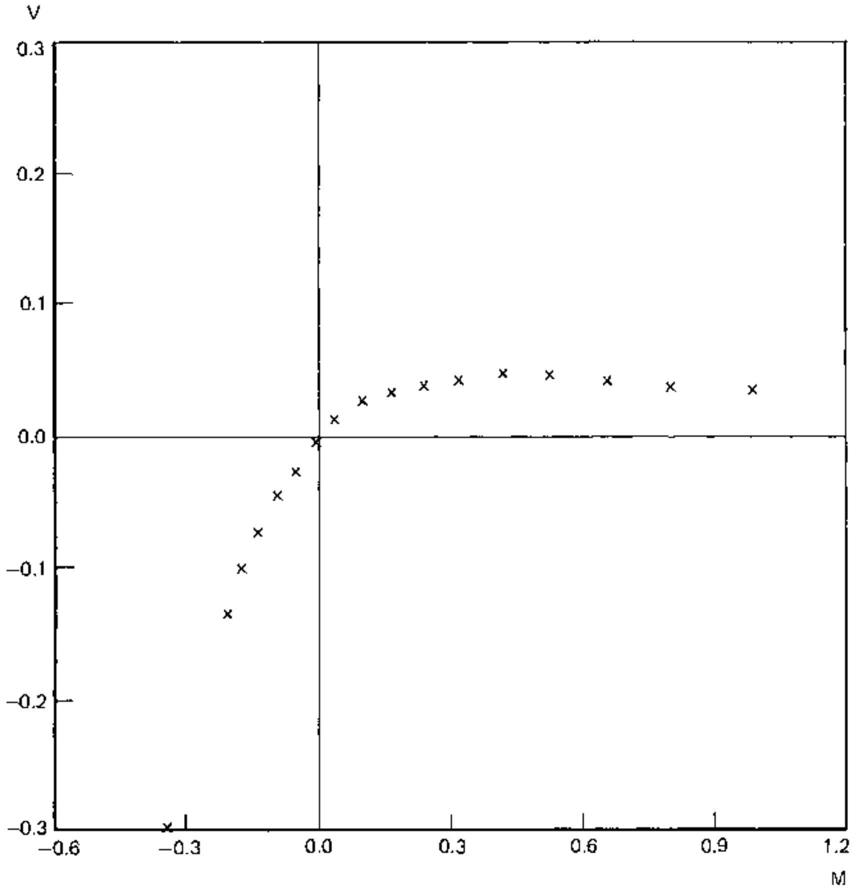

Figure 3. No Early Exercise. "V", percent difference between American call value and Black-Scholes model value assuming no early exercise, versus "M", percent in- or out-of-the-money. Parameters are: $S = \$40.00$, $\sigma = 0.3$, $r = 0.05$, $X = \$20.00, \cdots, 60.00$, $D = \$0.50$ (T, t_2, $t_1 = 4$, 3.5, 0.5 months).

when using the pseudo-American method to price options can be explained by the early exercise phenomenon. The approximation implicit in the method can be expected to and actually did cause systematic discrepancies between market prices and predicted values.

B. Time-to-Expiration Bias

Many papers have noted that the Black-Scholes model underprices near-maturity options. Reconsider Figure 1 when there are fewer than 90 days until the option's expiration but when one dividend payment still remains before expiration. The shorter the remaining term, the closer all the curves will be to their lower bounds. Still, since there has been no screening or forcing equality at-the-money, in Figure 1, *all* the European options are underpriced relative to the American values. Assuming no early exercise, the in-the-moneys are the most underpriced because of the dividend. Since an option is a wasting asset, the

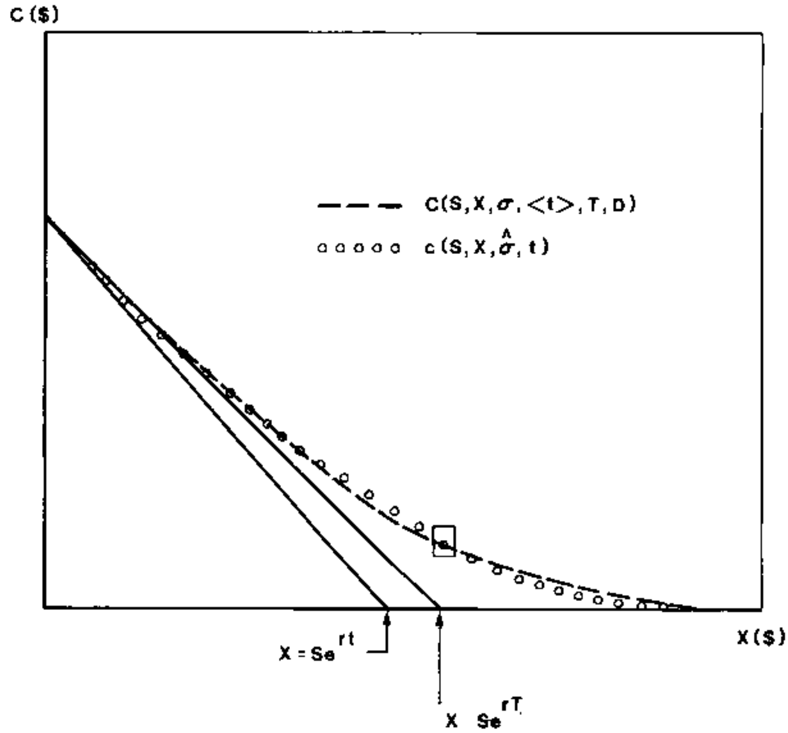

Figure 4. Early Exercise. In- and out-of-the-money call options. Call values versus exercise prices for American calls (C) and Black-Scholes European calls (c) for early exercise. True volatility is σ. Estimated volatility, $\hat{\sigma}$, forces $c = C$ for at-the-money call options.

premium above parity is a maximum at-the-money. The premium is actually negative for in-the-money, no early exercise, European options, since they lie below $S - X$.

If screening takes place, some of the certain early exercise options are removed from the sample, leaving a predominance of no early exercise options; so the underpricing bias is larger on average for options remaining in the sample after screening. The in-the-money options, which are the most underpriced, are the calls which trade most frequently. The out-of-the-money calls do not trade frequently and both their European and market values are close to zero.

Thus, it appears that the European model will exhibit an underpricing of near-maturity American call options and that the underpricing will be more severe when options likely to be exercised early are screened from the sample. One offsetting factor is that the nearer the option's contractual expiration, the less likely the stock is to have a dividend remaining; with no dividends remaining, of course, the European and American values should be equal. However, the nearer the ex-dividend date to the expiration, the more likely the option (in-, at-, or even out-of-the-money) is to be screened because of probable early exercise. This should leave the sample with only two types of options: first, those with no dividends remaining and thus with no bias in valuations, and second, those with a dividend closer to the current date than to the expiration date (and thus with substantial downward bias).

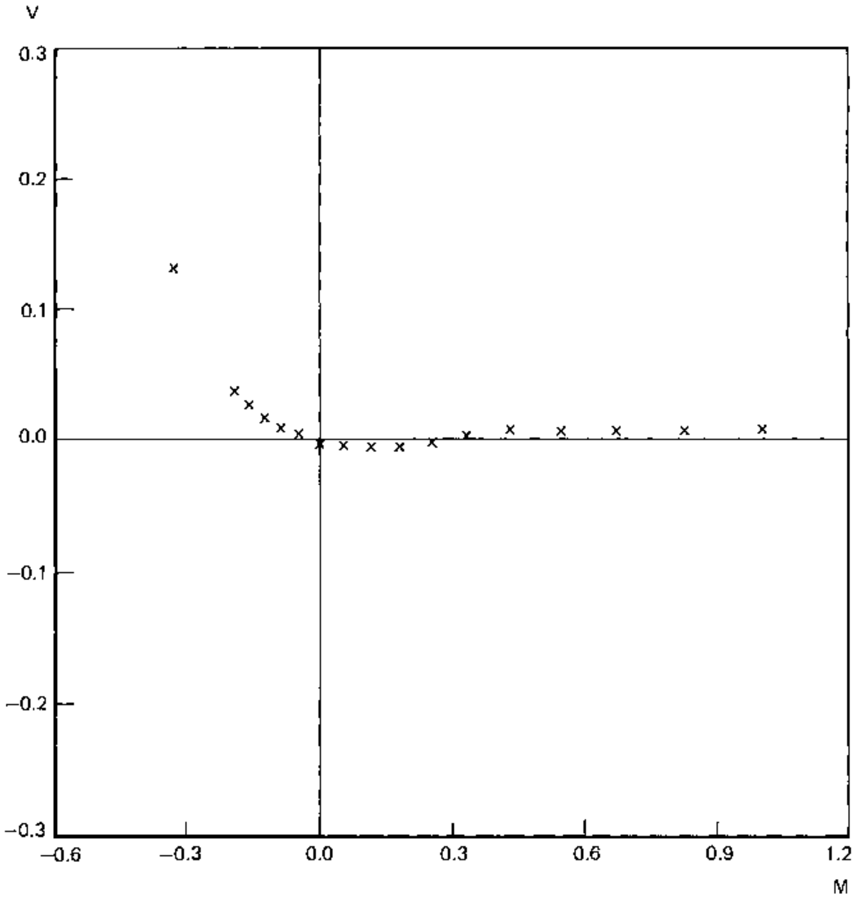

Figure 5. Early Exercise. "V", percent difference between American call value and Black-Scholes model value assuming early exercise at t_2 versus "M", percent in- or out-of-the-money. Parameters are: $S = \$40.00$, $\sigma = 0.3$, $r = 0.05$, $X = \$20, \cdots, 60$, $D = \$0.50$ (T, t_2, $t_1 = 4, 3.5, 0.5$ months).

C. Variance Biases

Black and Scholes [3], using over-the-counter (OTC) option data, found that their model underpriced call options on low variance stocks and overpriced call options on high variance stocks. Recently Geske et al. [5] demonstrated that these systematic "biases" should be expected when using the Black-Scholes model to value OTC dividend protected options. Whaley [14] detected these same biases using CBOE data, and found that the American model did not entirely remove the bias. It is well known that most stock volatilities are nonstationary. This could be a possible source of the variance bias, and a more complicated option pricing model with a changing and possibly stochastic variance would be necessary to alleviate the bias. One source of a nonstationary variance could be dividend uncertainty.

Geske and Shastri [6] demonstrated that uncertain dividend payments can significantly alter Merton's [8a] established properties for the rational pricing of American options, including the variance effect. Merton proved that an American or European option value is an increasing function of the underlying stock's volatility. Geske and Shastri showed that if the stock's dividend payments are uncertain, where the uncertainty takes the form of the dividend being suspendible, then the option value can be a decreasing function of the stock's volatility. Thus, in the case of suspendible dividend payments, the Black-Scholes model and the American model would systematically undervalue options on low variance stocks and overvalue options on high variance stocks, in conformance with the empirical results reported by Whaley [14].

II. Conclusion

The empirical literature on option pricing is in a state of conflict. Reports of biases in the Black-Scholes model from both traders and from academic studies are conflicting. A reading of papers by Black and Scholes [3], Black [1], MacBeth and Merville [8], Emanuel and MacBeth [7], and Rubinstein [10] confirms the conflicting findings. The original exercise price biases have been found to reverse themselves during certain time periods, so that both in- and out-of-the-money call options can be over- or underpriced by the Black-Scholes model depending on the sample of data being analyzed. More consistently, the model has been found to underprice near-maturity options. We offer an explanation of these biases based on the dividend-induced early exercise feature of American call options. We conclude that the American option model can explain the reported exercise price and time-to-expiration biases.

The variance bias could be attributed to nonstationary stock volatility which could be induced by dividend uncertainty. Based on the work of Geske and Shastri [6], we argue that when dividends are suspendible, both the Black-Scholes model and the American model will exhibit the variance biases reported by Whaley [14].

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