U.S. TREASURY INFLATION-INDEXED BONDS: THE DESIGN OF A NEW SECURITY

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In May 1996, the U.S. Treasury announced its intention to issue "inflation protection" bonds with cash payments linked to a general price index. It solicited the opinions of interested parties about the form of such a security, and engaged in several months of collective security design.

This is surely one of the most interesting episodes in the history of U.S. fixed-income markets, for the Treasury has never before issued an indexed bond. If it is properly designed, it will be among the least risky of all assets, virtually immune to both default and inflation risks. Hence, it is destined to become the datum for all other fixed-income securities.

In its official request for suggestions on May 15, the Treasury asked about five specific design issues: 1) the inflation index; 2) the cash flow structure; 3) maturities; 4) the auction mechanism; and 5) amounts. This article discusses each question and describes the choices made for the first bond, scheduled to be auctioned in January.

Most attention is given to the structure. Four structures were considered, and each has its attractions and drawbacks. Although the structure of the first Treasury indexed bond has now been announced, future bonds may well have other structures.

The central conundrum here, as with any security's design, is the trade-off between broad market liquidity and specific structures that appeal to individual clientele groups. The Treasury could issue many different indexed bonds in an effort to obtain high prices from heterogeneous investors. If tastes change, however, each small issue might have limited secondary market liquidity.

To maximize proceeds and minimize Treasury borrowing costs, there has to be a compromise between the variety of individual bonds and the extent of the market. This implies a circumscribed variety of distinct bonds. The first one in U.S. history will be coming to market soon.

I. THE TREASURY'S REVOLUTIONARY PROPOSAL

Treasury bonds, notes, and bills have long been recognized as the safest of all nominal fixed-income instruments, yet they remain exposed to the considerable risk of an erosion in purchasing power. A properly designed inflation-protected Treasury will eliminate that risk. Provided that it enjoys adequate liquidity, it should become the benchmark against which all other fixed-income securities are compared.

Since an indexed bond will be highly valued by investors averse to inflation risk, the Treasury's funding costs should be reduced by risk premiums paid on nominal bonds. Actually, there could be two sources of such savings: direct savings from reduced real interest payments on the new indexed bonds themselves, and indirect savings from reduced real yields on the remaining nominal bonds. The most risk-averse investors will be attracted to indexed bonds, leaving nominal bonds to investors more tolerant of inflation risk and satisfied with lower risk premiums.

Security design involves compromise. A security should possess specific features that appeal to particular investor clientele groups. The more tailored the security to a clientele's requirements, the more the clientele is willing to buy.

The universal desire for liquidity, however, argues for a more standardized, less tailored product. The demand for liquidity itself derives from the prospect that clientele preferences and anticipations may change. Specific requirements are not immutable; consequently, an active secondary market has great appeal. Although a standardized product may not be perfect *initially* for every investor clientele, its greater liquidity will give it a higher market value.

The scholarly literature on security design is extensive but devoted mostly to privately issued securities where asymmetric information and agency costs are the main difficulties (see Boot and Thakor [1993]). This literature provides scant guidance to the Treasury. Even when the published results are somewhat more general, they have limited applicability.

For instance, Allen and Gale [1988] recommend that securities be designed so that payoffs are allocated in each possible future circumstance to whatever investors value them most in that circumstance. In the extreme, this prescribes a different Treasury bond for every investor. Liquidity and unknowable future changes in preferences are not part of this recipe.

Of somewhat more help is literature about the indexation experience of other countries. Governments of the United Kingdom and Canada currently issued indexed bonds, and a number of other countries have considerable experience, e.g., Brazil, Finland, and Israel. In general, empirical studies confirm the popularity of indexed bonds and document the cost savings available to the sovereign borrower.

For instance, Kandel, Ofer, and Sarig [1996] find that Israeli nominal bonds with maturities as short as one month still entail statistically significant inflation risk premiums. These premiums have ranged from about 5 basis points per month during the period of lowest Israeli inflation to more than 200 basis points per month during a period (1984-1985) when inflation was high.[1]

Liquidity, however, has sometimes been lacking. Canada's indexed bond market is in the liquidity doldrums, and Britain's is not much better. Although British indexed "gilts" currently constitute about 15% of all government debt outstanding, trading is thin.

Nonetheless, international experience offers insights about the design of indexed U.S. Treasuries. Other sovereign governments have tried alternative indexes, maturities, and structures, the very features that the Treasury now must decide for its own future issues. While the U.S. market is, of course, much larger and probably better-developed, a glaring failure or brilliant success of a particular feature in another country may still apply to the U.S. environment.

On September 25, 1996, President Clinton announced that the very first Treasury indexed bond, to be issued in January 1997, will be a ten-year final maturity, "Canadian" style "note" with payments linked to the Consumer Price Index. Time will tell whether this proves to be a durable and popular design.

II. BASIC DESIGN FEATURES

Choice of Index

Four price indexes were considered by the Treasury: 1) the Consumer Price Index (CPI-U) of all items; 2) the "core" CPI, which excludes energy and food prices; 3) the GDP deflator; and 4) the employment cost index (ECI) of average wages.

The index choice runs squarely up against the trade-off between specific investor clienteles and liquidity. There is no doubt about investor heterogeneity in terms of preferences across possible indexes. For example, defined-benefit pension plans might favor the employment cost index, because their liabilities are often linked to wage rates prevailing shortly before an individual's retirement. Funding a pension currently necessitates an actuarial prediction of future wages. A bond indexed to wages would hedge these future pension liabilities and make the funding provision more transparent.

Corporate treasurers, on the other hand, might prefer the GDP deflator, because it would offer better protection against wholesale costs of production. Individual investors might advocate the full CPI-U, as they are presumably concerned about the overall cost of living. The core CPI might be preferred by those who want less short-term volatility in their indexed bond payments.

From the liquidity perspective, many have argued that the Consumer Price Index of all items would be best, at least until indexed bonds become more familiar, and in fact the Treasury selected the CPI-U for its first issue.[2] The CPI-U has the greatest name recognition and is the most widely followed and understood by the public.

It has been the choice of other countries. British indexed "gilts" are linked to the retail price index, an analogue to the U.S. CPI. Israel uses a "CPI" with ten sectors whose weights change but are announced in advance. Canadian indexed bonds use the Canadian CPI.

In future years, after Treasury indexed bonds become commonplace, some consideration might well be given to issuing bonds linked to indexes other than the CPI. The ECI might be a good second choice. Ignoring liquidity, linkages could be to even more narrowly defined indexes such as medical expenses or college tuition. Those bonds would appeal strongly to particular niche buy-and-hold investors, and would also be politically attractive, given widespread concerns of various segments of the public.

The literature about indexation in other countries sometimes warns about a "moral hazard." That is, *any* price index constructed by the government could someday be subject to unwise manipulation, particularly when a large volume of index-linked bonds has been issued and inflation has not been restrained. Of course, a similar risk is inherent in nominal bonds; a future government could inflate unexpectedly to reduce the real value of its nominal payments. The index-linked bond is unambiguously preferable to the nominal bond in this regard.

Nonetheless, the potential for real value erosion via index manipulation suggests that the index might be constructed by an independent, non-government entity. Such a system is actually in effect in some countries (Israel is one). This mechanism has not, however, been suggested by the Treasury.

Structure of the Security

The choice of structure, like that of the index, involves a compromise among clientele groups to achieve the greatest liquidity. Unlike the index, however, structure can be reengineered after issuance. *Whatever* the initial structure, flexibility is greatest when a security is easy to reengineer. If it's a multiple payment structure (either coupon or annuity), the bond should be strippable down to each individual payment.

Different clientele groups probably prefer particular structures. For example, a tax-exempt defined-benefit pension fund might favor relatively long-term zero-coupon bonds. This would let the fund immunize future liabilities with minimal reinvestment risk. Conversely, a taxpaying entity would consider a zero-coupon bond inconvenient because, in the absence of other sources of cash inflow, an annual partial sale would be necessary to fund the tax liability.

A taxpaying retiree might prefer a constant real-valued annuity, a level payment (in real terms) including amortization of principal. If taxes are insignificant, payments from such a structure would permit the retiree to maintain a constant level of real consumption consisting of a basket equivalent to the index. This structure might have a broader appeal to more than retirees. One can envisage, for example, insurance companies offering real-valued annuities to clients, and then hedging their liabilities through the purchase of an index-linked Treasury annuity.

Four different structures were considered by the Treasury:

1. A zero-coupon bond whose final (and only) payment is linked to the inflation index.
2. A mortgage-type bond, whose periodic interest-

EXHIBIT 1 ■ Cash Flows of Indexed and Nominal Bonds

Bond	CF_t, $(t < N)$	CF_N
Zero-Coupon	0	$\Pi_{\tau\leq N}(1 + I_\tau)$
Mortgage	$c_{Mor}(1 + I_t)B_{t-1}\,[1 - (1 + c_{Mor})^{t-N-1}]^{-1}$	$(1 + c_{Mor})(1 + I_N)B_{N-1}$
Canadian	$c_{Cdn}[\Pi_{\tau\leq t}(1 + I_\tau)]$	$(1 + c_{Cdn})[\Pi_{\tau\leq N}(1 + I_\tau)]$
Floater	$(1 + c_{Flot})(1 + I_t) - 1$	$(1 + c_{Flot})(1 + I_t)$
Nominal Bond	c_{Nom}	$1 + c_{Nom}$

Symbol definitions:

CF_t cash payment (nominal) in period t;
N number of periods until maturity;
I_τ actual inflation in period τ;
c original issue coupon (for issue price of par = 1), (% per period/100);
B_t remaining nominal principal mortgage balance at the end of year t;
Π product operator;

plus-principal payment is linked to the index and thus remains level in real terms.

3. The "Canadian" model, a fixed-coupon bond whose outstanding principal is linked to the index. As the principal grows with inflation, coupon payments also increase because the fixed coupon rate is multiplied by the accreted principal amount on each payment date. At maturity, the accreted principal is paid in full.
4. A "floater" structure, similar to the Canadian model, except that principal accreted is paid on each coupon date, not postponed until final maturity.

The floater was not among those initially under consideration in May, but was suggested by various interlocutors. The Treasury held subsequent open meetings to discuss its merits and defects, but the Canadian structure was adopted for the first bond to be issued in January 1997.

The nominal cash flow patterns of these indexed structures plus the pattern for current nominal bonds are given in Exhibit 1. Note that coupons have identifying subscripts. Since all but the zero-coupon bond sell originally at par, their stated coupons can and probably will be different. We'll drop the subscripts when identification is obvious.

The nominal bond has the familiar semiannual cash flow pattern. The zero-coupon indexed bond is also straightforward; it differs from the standard case only in the terminal payment's link to cumulative inflation. The other structures require some explanation.

For the mortgage structure, the nominal principal balance, B, is most easily computed with a recursion formula that includes indexation:

$$B_t = B_{t-1}(1 + c_{Mor})(1 + I_t) - CF_t \qquad (1)$$

Using the mortgage annuity formula, the annual *real* payment determined on the issuance date is

$$A \equiv c_{Mor}/[1 - (1 + c_{Mor})^{-N}] \qquad (2)$$

It is straightforward to verify that the mortgage payment has a constant real value equal to this amount; i.e., $CF_t/[(1 + I_1)(1 + I_2)...(1 + I_{t-1})] = A$. Hence, if inflation is a constant, say, I^c, the mortgage's nominal cash flow can be written in a simplified form as

$$CF_t = A(1 + I^c)^t \qquad (3)$$

We can think of payments to the Canadian bond as consisting of three distinct elements:

1. The nominal coupon paid on each scheduled date.
2. The nominal par principal paid at maturity.
3. A payment on each scheduled coupon and principal date equal to the inflation accrual since origination on that scheduled payment.

The floater structure is similar except for the timing of inflation accrual payments. The nominal payments are the same as with the Canadian structure, but the third element of the floater is instead: a payment on each scheduled coupon date equal to the inflation accrual on a single coupon *and* upon the original nominal principal *since the previous coupon date*.

This current-pay bond is called a "floater" because its cash flows are similar in many respects to an ordinary floating-rate note. There is, however, a subtle but significant difference. The coupon on a floating-rate note is set at the *beginning* of the period (e.g., to the six-month LIBOR rate), while the coupon on the Treasury floater structure is set at the end of the period when inflation is known.

The floating-rate note's coupon must depend only on inflation *expected* at the beginning of each coupon period, while the Treasury floater's coupon depends on actual inflation over that same period.[3] Consequently, the Treasury floater structure has less inflation risk than an ordinary floating-rate note, an advantage that is offset by greater risk of fluctuations in real interest rates. The Treasury floater's coupon is fixed at origination to its real yield (because its issue price is par). In contrast, the floating-rate note's coupon depends on the ex ante real interest rate at the beginning of each coupon period.

Evidently, the Treasury has not yet considered an ordinary floating-rate note. This would be a structure very familiar to the market that would have wide appeal because of its high credit quality.

If the issue price is par, the nominal coupon at origination is the bond's real yield in the mortgage, Canadian, and floater cases. These bonds and the zero-coupon indexed bond are well-protected against inflation risk, so their real yields should include only a minimal inflation risk premium, if any at all. They are not at all similar in their sensitivity to fluctuations in real interest rates, however. To the extent that the market requires a premium for this source of risk, these bonds will have disparate real yields.

Maturities

From a trader's perspective, the question of maturity centers on short-term price volatility. From an investor's perspective, the optimal maturity of an inflation-protection instrument is dictated by inflation-sensitive liabilities or future consumption expenditures. Traders will be nervous about this new security if its duration is substantial; they would prefer short maturities.[4] Investors, by contrast, are likely to have just the opposite attitude. Many will want *long-term* inflation protection.

Investors are probably more concerned with long-term inflation because near-term inflation volatility is not large. Nominal intermediate-term bonds, rollover investments in Treasury bills or similar instruments, and floating-rate notes provide protection against inflation over short and intermediate horizons. The larger inflation fear is psychologically associated with some longer-term structural change in the economy or in monetary policy, perhaps induced by a shift in government or in the attitude of the public.

Maturities of at least ten years, and probably significantly longer, seem likely to provide the greatest reduction in Treasury borrowing costs. True, there will be initially some trepidation and perhaps illiquidity until the trading community becomes more accustomed to the new instrument, but this should prove to be a short-term problem. It may mean that the very first issue, or even the first few, will not provide the full reduction in Treasury borrowing costs that will ultimately be achieved. Funding costs should be lower after the market becomes accustomed to the vagaries of the new instrument.

Perhaps in an effort to satisfy several constituencies, the Treasury announced in August that it would eventually issue indexed bonds at several maturities, although it did not specify which ones or in what amounts. The inaugural bond will have a final maturity of ten years, a very understandable compromise first choice.

The Auction Mechanism

The Treasury and various constituencies shared a broad sentiment in favor of a single-price auction at origination. In this instance, the total issue amount is announced and the bond's price is established at par. Competitive bids are solicited in units of real yield. The highest accepted (i.e., market-clearing) bid becomes the coupon on every auctioned bond, even for bidders who offered to buy the bond at a lower yield. This will be the procedure in the January auction.

In a concurrent innovation, an auction reopening procedure is on the drawing board. Reopening is used in other countries and is closely related to whether the Treasury should set limits on the amount purchased by a single bidder in a given auction.

Reopening is the perfect antidote against short squeezes in the when-issued market. The possibility of reopening removes the incentive to attempt a corner in the first place; consequently, it eliminates any harmful effect of an abnormally large bid and thus removes the necessity for a limit. Should the price around an auction begin to drift out of line, immediate reopening is curative.

More generally, reopening at later dates is an effective means of reducing Treasury borrowing costs. Once the index-linked market is established and continuous, the term structure of real interest rates should acquire a smooth appearance. A blip downward in the real yield at any term implies an opportunity for the Treasury to reopen a nearby issue and thereby borrow at relative low cost. Conversely, an upward blip suggests that the Treasury could repurchase bonds at this maturity and finance the repurchase by issuing greater quantities at other maturities. This has proved effective in both the nominal and indexed markets of other countries, where "tap" and repurchase facilities are common.

The exact mechanism for reopening should be carefully planned, because it has implications for market quality. There will be an established market price for each outstanding bond, and the Treasury will be selling additional amounts close to that same price. This could be accomplished in units of original par or in inflation-adjusted units on a coupon date.

An important design element is that new bonds be identical in every respect to existing bonds; their coupons, payment schedule, and accumulated inflation adjustments should be perfectly matched. The new and old bonds should even have the same CUSIP number. They should be legally indistinguishable, so that delivery can be achieved with either an existing or a new bond.

Amounts in Initial Auctions

In the interest of establishing liquidity and fostering confidence in market continuity, relatively large amounts are likely in the initial auctions. Although the amount in the first auction remains to be specified, Deputy Secretary of the Treasury Lawrence Summers stated on September 25 that "we're going to issue enough that it's going to be a liquid market."[5]

Since a rollover strategy in bills represents the best inflation protection currently available from Treasury issues, one would expect at least some investors to substitute the new indexed bonds for current positions in bills. As an example, on September 30, the Treasury auctioned approximately $29 billion in new bills, equally divided into 91- and 182-day maturities. If the new indexed bonds had come to auction for the first time on that date, perhaps 20% of the bills total could have been sold; hence, at least $5 billion and perhaps more might be expected in the first indexed bond auction in January.

Assuming that indexed bonds will be issued quarterly (the initially announced calendar), $20 to $30 billion would not be out of question during the first year. Although this is a minuscule portion of total Treasury debt, it is large enough to support a reasonable degree of liquidity, provided that additional auctions are expected.

The ultimate aggregate outstanding amount of Treasury indexed debt is anyone's guess, but it could easily match Britain's 15% of total debt.

III. TAXATION

Federal taxes will be imposed on *all* cash flows except the final return of *nominal* principal. Taxable income will include every coupon payment and every inflation accrual, and taxes will be due currently.

To see how this works, consider the Canadian structure with an original coupon of 3%. If inflation is 5% during a given year, taxes payable that same year will be (3% + 5%)τ, where τ is the Federal tax rate on ordinary income. Although the inflation accrual does not represent real income, it is nonetheless taxed at the full ordinary rate.[6]

Outrage is the first reaction of many when they learn that taxes will be imposed on the inflation accrual of Treasury indexed bonds. "It's not fair," they say. And it's not. The Treasury has little choice, however, if it wants to foster a liquid market in its indexed securities.

Although it may be less obvious, nominal bonds are also taxed on inflation. The nominal bond's coupon includes not only a real yield, but also a component for expected inflation. This is not a risk premium, but simply the extra yield required, even by a risk-neutral investor, for the *expected* erosion in purchasing power. Thus, if real yields are, say, 3% while expected inflation is 5%, the taxpayer who owns $100 worth of nominal Treasuries pays ($3 + $5)$\tau$ in taxes each year even though the purchasing power of the bond is expected to decline from $100 to $95 and the investor's real pre-tax income is expected to be only $3.[7]

Since nominal bonds are taxed on inflation, if indexed bonds had tax-exempt inflation accruals, they would sell at relatively high prices, and low yields. Tax-exempt institutions would not buy them because nominal bonds would have dominant pre-tax returns. Consequently, indexed bonds would be held mainly by tax-paying entities, who constitute only a fraction of all investors, well less than half.

Liquidity would be poor, as it is in Britain where indexed bonds' inflation accruals are not taxed. Treasury indexed bonds would be similar to municipal securities, which are eschewed by tax-exempt institutions because of low pre-tax yields. Inescapably, so long as Treasury nominal bonds are unfairly taxed, indexed bonds must be treated equally, and taxed unfairly too! Otherwise, they will be illiquid.

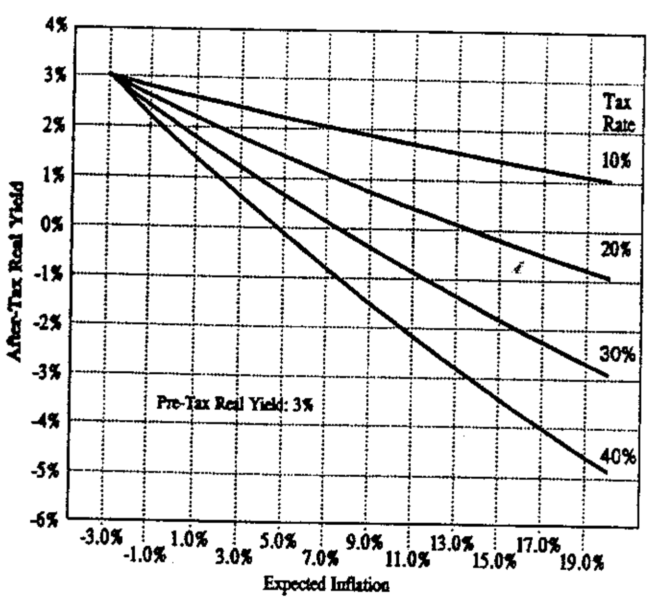
The Incidence of Taxation

For simplicity of illustration in this section, a flat (marginal) tax rate is assumed. Complicated sections of the tax code will be ignored.

For the taxation of bonds selling at discounts or premiums, we'll use that part of the tax code applicable to financial institutions and dealers, which does not apply to certain other entities such as insurance companies; that is, the ordinary tax rate is imposed on the nominal yield as computed on the purchase date. Each period, taxable income is this original yield multiplied by the bond's accreted principal value.

Hence, if a nominal bond is purchased at par, the tax falls only on the coupon. For a bond purchased at a discount, cash received from the coupon is less than taxable income, thereby resulting in "phantom" taxable income. A bond purchased at a premium enjoys a symmetric excess of coupon receipts over taxable income.[8]

EXHIBIT 2 ■ After-Tax Real Yields from Treasury Indexed or Nominal Bonds



Although the exact tax treatment of the new Treasury inflation-protection bonds has yet to be determined for discount and premium cases, it will likely conform to the current treatment of nominal Treasuries. If so, taxable income per period for all structures is given by the generic formula:

$$TI_t = [(1 + r)(1 + I_t) - 1]P_{t-1} \tag{4}$$

where r is the real yield on the purchase date of the bond and P_{t-1} is the accrued principal value at the beginning of each tax period. For tax purposes, the real yield (r) does not change over time until there is a sale and a new basis is established.

Although taxable income has a similar *form* across all proposed bond structures, there are variations in details. The nominal yield on the nominal bond is a fixed constant; it is determined by the real yield and the *fixed* inflation, I^e, expected on the purchase date. Taxable income is yP_{t-1} where $y = (1 + r)(1 + I^e) - 1$, and I^e is a prospective and unchanging inflation prediction on the purchase date. Unlike indexed bonds, whose taxable incomes vary with *actual* inflation, the nominal bond's taxable income is locked in from the beginning (until it is sold and a new basis is established.)

The mortgage, zero-coupon, and Canadian indexed bonds all base taxable income calculations on updated face amounts. The floater bond has the same taxable income formula, but the face amount remains constant at par = 1. The principal value basis of the floater bond changes over time only when the purchase price differs from par. The principal value basis of the zero-coupon or Canadian bond changes for two reasons: 1) a discount or premium at purchase, and 2) the indexed face amount. If inflation is positive, taxable income for either the zero-coupon or Canadian bond will increase over the bond's lifetime.

For a *fixed* inflation rate, I^e, and equal risk premiums, the after-tax real yield will be the same for all bond structures:

$$\rho = \{1 + (1 - \tau)[(1 + r)(1 + I^e) - 1]\}/(1 + I^e) - 1$$

$$= r(1 - \tau) - \tau I^e/(1 + I^e) \tag{5}$$

Exhibit 2 reveals how inflation combines with taxation to erode real returns. Even at moderate tax rates, after-tax real yields become negative for levels of inflation within U.S. experience over the past few decades. Indexation offers a major improvement in risk control, but no return relief whatsoever to the taxpaying investor. (In this illustration, the pre-tax real yield is 3%.)

Phantom Income

Phantom income is taxable "income" not currently received in cash, so a taxpayer with no other revenue would have to sell part of the original investment in order to pay taxes. Obviously, an illiquid bond producing substantial phantom income would pose quite a hardship on such investors and make them unlikely to buy it in the first place. Cash emissions at least as large as current tax liabilities would be most efficient from the perspectives of trading costs and convenience.

The various indexed bond structures differ significantly in their production of phantom income. The zero-coupon structure has the greatest problem; 100% of its taxable income is phantom prior to maturity. The mortgage and Canadian structures can also be subject to a smaller measure of the same difficulty.

Cash receipts from a Canadian bond amount to $c(1 + I_t)P_{t-1}$ in period t. As a fraction of cash receipts, *after-tax disposable income* is $(1 - \tau) - \tau I_t/[c(1 + I_t)]$, a decreasing concave function of actual inflation. Cash receipts will not even cover tax liabilities on a Canadian structure bond when inflation exceeds a particular level. Cash will be deficient whenever

$$I_t > c(1 - \tau)/[\tau - c(1 - \tau)] \qquad (6)$$

ignoring intra-year interest on coupons and assuming $\tau - c(1 - \tau) > 0$.

For example, with the marginal federal tax rate of $\tau = 0.38$, if the annual bond coupon were $c = 3\%$, an inflation rate exceeding approximately 5.15% would preclude bondholders from paying taxes using only current coupon receipts.

Since its payments include principal amortization in addition to interest, one might at first think that the mortgage structure would always generate sufficient cash to cover taxes. Yet this is not the case, and the possibility of insufficient tax coverage also increases with inflation.

Taxable income for the mortgage structure includes the coupon payment on the remaining principal plus the increase in the nominal value of *remaining* principal induced by the linkage to inflation:

$$TI_t = [(1 + c)(1 + I_t) - 1]B_{t-1} \qquad (7)$$

True economic earnings do not include the return of capital; they are simply $c(1 + I_t)B_{t-1}$. As a fraction of these true earnings, after-tax disposable income from the mortgage structure is $(1 - \tau) - \tau I_t/[c(1 + I_t)]$, which is exactly the same as under the Canadian structure. Thus, phantom income also has the same form.

Total *cash* received each period from the mortgage structure is somewhat larger than earnings, because a portion of the remaining principal is repaid each period; this makes things a bit easier for the hapless taxpayer (although not in a purchasing power sense). Taxpayers will be able to pay taxes from total cash receipts unless:

$$I_t > c[G(t, N) - \tau]/\{\tau - c[G(t, N) - t]\} \qquad (8)$$

where the function $G(t, N) = [1 - (1 + c)^{t-N-1}] - 1$ ranges between 1 and $(1 + c)/c$, and the denominator on the right-hand side of the inequality is assumed to be positive.

Neither the floater structure nor the ordinary nominal bond produces phantom income unless they are purchased at a discount.

Taxes and Pre-Tax Yields

If there are viable alternatives, no taxable investor would knowingly buy an indexed bond with an expected negative after-tax real yield. But Exhibit 2 shows that pre-tax coupons of 3% will not produce positive real returns unless the tax rate is fairly low or the inflation rate is relatively modest.

This implies two phenomena. First, as inflationary expectations increase, the indexed bond market will become increasingly segmented; only tax-exempt institutions or taxpayers with low rates will still be investors. Second, the coupons on indexed bonds will probably increase with inflationary expectations. This second effect follows from the first. As highly taxed investors flee the market under inflationary pressure, total demand and prices will fall.

It will, therefore, appear to the Treasury that index-linked bonds have higher borrowing costs during periods of high expected inflation. This has, of course, always been notable for nominal bonds — their yields increase with expected inflation. But here we are arguing that both types of bonds should display yields that increase *disproportionately* with inflation. This is necessary to offset what would otherwise be increasingly negative *after-tax* real returns.

To quantify this idea, let the demand for bonds be a function only of their expected after-tax real yield, ρ. To maintain the same level of demand, pre-tax yields must respond to changes in inflationary expectations to keep ρ constant. This implies the response function for pre-tax real yields to expected inflation:

$$\partial r/\partial I^e = \tau/[(1 - \tau)(1 + I^e)^2] \tag{9}$$

which is strictly positive. For a tax rate of 38% and expected inflation of 5%, $\partial r/\partial I^e = 0.556$, and we should expect about a 56 basis point increase in the pre-tax real yield for every 1% increase in expected inflation.

The response of the pre-tax nominal yield should be even larger:

$$\partial y/\partial I^e = (1 + \rho)/(1 - \tau) \tag{10}$$

where $y \equiv (1 + r)(1 + I^e) - 1$. Using the same numerical example, $\partial y/\partial I^e = 1.614$. The nominal pre-tax yield would have to respond with amplitude about 60% greater than the change in expected inflation to maintain constant investor demand.

IV. TWO TECHNICAL PROBLEMS: DEFLATION AND LAGGED INDEXATION

The Question of Deflation

Since deflation is so rarely observed in any country, and has not occurred in the U.S. for the past half-century, one might be prone to ignore its consequences for indexed bonds. It does, however, have a potential influence on the choice of structure. The floater structure is particularly susceptible to deflation over even a single coupon period because its inflation accrual could be negative and possibly exceed the coupon. In such a case, the investor would owe cash to the Treasury.

Of course, the Treasury could agree to renounce any negative principal accrual in excess of the currently payable coupon during a deflationary subperiod. The total cash payment by the Treasury, coupon plus principal accrual, could fall to zero but not become negative.

This provision would confront the difficulty that both deflationary and inflationary episodes might occur over a given floater bond's life. The Treasury would be paying out cash during the inflationary periods and perhaps not receiving enough cash during the deflationary periods. Conceivably, there might even be zero or negative average inflation over the bond's total life, yet the Treasury would have paid out some cash in addition to the ex ante real coupon.

Assuming that an appropriate tax treatment could be arranged, this problem would be eliminated by "escrowing" any uncollected negative accrual arising during a deflationary period and using it to offset a later positive accrual. Only a positive escrow balance would then be currently payable by the Treasury. At maturity, any remaining negative escrow balance would be absolved. Such a design has operational difficulties. Considering the exceedingly low probability of a deflation larger than the bond's coupon, the simplest and perhaps the best design would be to forgive a negative accrual, if one should ever come to pass.

The difficulties that the floater bond could encounter during even a brief period of deflation might be one reason the Treasury decided against it for the first issue, especially now with inflation modest by historical standards. The Canadian, zero-coupon, and mortgage structures have fewer difficulties under deflation; their cash payments are always positive. If deflation should be negative over the entire life of the bond, however, principal repayments could be less (nominally) than the initial par amount.

For the inaugural ten-year bond, the Treasury has agreed to repay the full nominal par at maturity, even if there has been deflation over its decade of life. Of course, this is not much of a concession; we have not experienced a decade-long deflation this century.

Lagged Indexation due to Publication Delay

Some lag in indexation is inevitable because of the time required to collect price information, calculate the index, and publish the updated value. In Britain, this has caused an eight-month delay between the index value and its associated coupon. British indexed "gilts" pay coupons semiannually. Immediately after an ex-coupon date, accrued interest must be calculated by reference to the next coupon, scheduled for six months hence. To fix that next coupon and thus determine accrued interest, the most recently available inflation number refers to the calendar month two months prior to the ex-coupon date, i.e., eight months before the next coupon.

The Canadian method approximates the index by interpolating, substantially reducing the lag. The interpolation provides an inflation index number every calendar day according to the formula:

$$\text{Index}_{\text{Date}} = PI_{M-2} + [(t-1)/D][PI_{M-1} - PI_{M-2}] \quad (11)$$

where the subscript, Date, refers to a settlement date t days into month M, and D is the total number of days in that month. PI_{M-1} and PI_{M-2} refer, respectively, to the value of the published index as announced in the previous month and the second previous month; these are averages of prices two and three months prior to the payment date.

For price indexes reported monthly, the latest available index value on the first day of any calendar month is published during the previous month and applies to prices sampled in the second-preceding month. For example, the latest available CPI on July 1, 1996, was the number announced June 12, which measured prices during May 1996. Under the Canadian method, a bond payment during July would have been based on the CPI value reported during the second week of May (for April prices), extrapolated on a daily basis by the difference between the latest two reported values.

Evidently, the sole reason for basing July's payments on a CPI value as stale as April's is the desire to interpolate a linear trend day-by-day within July. But clearly, the CPI value for May (announced during June) is closer to the true price level in July.

The formula above is attractive because it is consistent at the beginning and end of each month. At the beginning of the month, t = 1, and the index level is exactly PI_{M-2}. On the last day of the month, the index value is converging to PI_{M-1}, which will in fact be its exact value on the first day of the following month. The interpolation traces a smooth transition across months.

But this arithmetic elegance incurs the cost of an extra month's lag. The Canadian formula could be improved in several ways. One simple alteration would base the calculation on PI_{M-1} instead of PI_{M-2}. The formula would be

$$\text{Index}_{\text{Date}} = PI_{M-1} + [(t-1)/D][PI_{M-1} - PI_{M-2}] \quad (12)$$

On the first day of any month, payments would be linked to PI_{M-1}, the *latest* reported index value. On later days during that month, there would be an increment extrapolated from the difference in the last two reported values. Admittedly, this technique would be subject to a jump on the first day of every month, but if inflation were steady, the jump would not amount to much.

Both formulas above are susceptible to seasonality in the index. If, for instance, December prices are typically seasonally higher than November's or January's, and a non-seasonally adjusted index is employed, the interpolation will be slightly biased. This suggests an extrapolation of the most recently observed ***annual*** inflation for interpolating within days of a month; e.g., the formula could be

$$\text{Index}_{\text{Date}} = PI_{M-1} + [(t-1)/N_Y][PI_{M-1} - PI_{M-13}] \quad (13)$$

where N_Y is the number of days in the spanned year.

In the U.S., the CPI is usually announced during the second week of the month following price collection. Why not revise the payment index immediately, the day after publication, rather than wait until the beginning of the subsequent month? Unless the index is being manipulated, there is no reason to postpone its adoption *whenever* it is announced.

Finally, simplicity suggests another scheme. Forsaking interpolation, simply set the inflation accrual constant and equal to its latest value between index publication dates. The accrual would then change only upon the announcement of a new index number. Any difference between the last reported value and the market's assessment of its "true" level should be quite small and reflected in the market price anyway.

V. VALUATION AND VOLATILITY

All the proposed indexed bond structures enjoy outstanding protection against inflation and default risks, but they are not homogeneous in their sensitivities to movements in real interest rates. To elucidate their differences, we use the additional notation:

- d = nominal discount factor;
- $d \equiv 1/[(1 + r) \times (1 + I^e)]$;
- V_j = net present value for bond j;
- $D_j(i)$ = duration (Macaulay) of bond j with respect to changes in interest rate i;[9] and

Σ = summation operator.

For simplicity, partial payment periods will be neglected; all the results are exactly valid only on coupon anniversary dates. Usually, only par coupon cases will be considered. There will be no lag in the inflation index. Sometimes, we shall consider a single inflation rate, I^c, expected for all future periods. In reality, of course, expected inflation might vary dramatically over the future, but this just complicates the formulas without offering any key insights. Cash flows are as shown in Exhibit 1.

Each bond can be valued by the standard method of discounting expected cash flows with a nominal discount rate, i.e., a "yield." A first approximation to interest sensitivity can be obtained by computing "duration."

Canadian Structure

$$\begin{aligned} V_{CDN} &= \Sigma_t[CF_t(d^t)] \\ &= c\Sigma_t\{(1 + I^e)^t/[(1 + r)(1 + I^e)]^t\} + 1/[(1 + r)(1 + I^e)]^N \\ &= c\Sigma_t[1/(1 + r)^t] + 1/(1 + r)^N \end{aligned} \tag{14}$$

The expected inflation rate does not appear anywhere in the net present value expression for the Canadian structure. Consequently, changes in inflationary expectations have no direct influence on its market value. As nominal interest rates vary due to expected inflation alone, the Canadian bond will display an effective duration of zero, provided that real interest rates do not respond to expected inflation, as they might because of time. If they do respond, the effective duration would not be zero, but this is an indirect influence subordinate to the change in real rates.[10]

The Canadian bond is definitely sensitive to changes in real interest rates, whatever their cause. Macaulay duration with respect to the real yield can be computed from Equation (14). Taking the appropriate derivatives and simplifying:

$$\begin{aligned} D_{CDN}(r) &\equiv -[\partial V_{CDN}/V_{CDN}]/[\partial r/(1 + r)] \\ &= \Sigma_t\{t\ [PV(CF_t)/V_{CDN}]\} \end{aligned} \tag{15}$$

where $PV(CF_t)$ is the present value of the real cash payment in period t discounted at the real rate, r. For example, the present value of the payment scheduled for coupon date t is $PV(CF_t) = c/(1 + r)^t$.

Macaulay duration is the familiar weighted average of the times until payment, each weight being the proportion of total net present value represented by that particular payment. The only difference from the usual formula is the discount rate; it is real rather than nominal.

This duration can be atypically long because the real interest rate is generally lower than the nominal rate.

Zero-Coupon Structure

The Canadian formulas apply fully to the indexed zero-coupon bond. Merely set the nominal coupon, c, to zero. Like the Canadian bond, the zero-coupon bond has no direct sensitivity whatsoever to changes in inflationary expectations. Its duration with respect to real interest rates is its term until maturity.

Mortgage Structure

For a constant inflation rate equal to I^c, the cash flows from the mortgage structure can be simplified to

$$CF_t = c(1 + I^e)^t/[1 - (1 + c)^{-N}] = A(1 + I^e)^t \tag{16}$$

where $A \equiv c/[1 - (1 + c)^{-N}]$ is the real value of the annuity (assuming an original par value of 1.0).

The present value is

$$\begin{aligned} V_{Mor} &= \Sigma_t[CF_t(d^t)] \\ &= A\Sigma_t\{(1 + I^e)^t/[(1 + r)(1 + I^e)]^t\} \\ &= A\Sigma_t(1 + r)^{-t} \end{aligned} \tag{17}$$

Like the Canadian and zero-coupon structures, the mortgage structure is immune to changes in inflationary expectations that do not affect the real yield.

Its duration with respect to changes in real interest rates has the standard form:

$$\begin{aligned} D_{Mor}(r) &\equiv -[\partial V_{Mor}/V_{Mor}]/[\partial r/(1 + r)] \\ &= \Sigma_t[t(1 + r)^{-t}]/\Sigma_t(1 + r)^{-t} \end{aligned} \tag{18}$$

Compared to the Canadian or zero-coupon structures with the same final maturity, the mortgage structure has a considerably shorter real rate duration, because there is no final large payment. For example, with a real yield of 3% and final maturity of thirty years, the zero-coupon, Canadian, and mortgage structures have real rate durations of thirty years and approximately twenty and thirteen years, respectively. For a final maturity of ten years, their real rate durations are 10, 8.71, and 5.00 years, respectively.

Floater Structure

$$V_{Flot} = \Sigma_t\{[(1 + c)(1 + I^e) - 1]d^t\} + d^N \quad (19)$$

After some algebraic manipulation, the summation can be eliminated and the expression simplified to

$$V_{Flot} = 1 + [(1 - d^N)/(1 - d)][(c - r)/(1 + r)] \quad (20)$$

When the coupon and the real rate are the same ($c = r$), the bond sells at par (= 1). Unlike the other indexed structures, the expected inflation rate still appears in this valuation equation embedded in d, the nominal discount factor. Consequently, the floater bond will have two distinct "durations," one for movements in real interest rates, and a different one for movements in inflationary expectations.

To compute the duration associated with changes in the real interest rate, the simplest procedure is to take the derivative directly from the fully expanded expression for discounted cash flows.[*] The result has the usual form:

$$D_{Flot}(r) \equiv -[\partial V_{Flot}/V_{Flot}]/[\partial r/(1 + r)]$$
$$= \Sigma_t\{t\ [PV(CF_t)/V_{Flot}]\} \quad (21)$$

but the present values of the nominal cash flows are calculated using the nominal discount factor. For example, the payment in period t has a present value:

$$PV(CF_t) = [(1 + c)(1 + I^e) - 1]d^t$$
$$= \{[(1 + c)(1 + I^e) - 1]/(1 + I^e)^t\}/(1 + r)^t \quad (22)$$

Equivalently, the term in braces is recognizable as the expected real cash flow in t, so it is appropriately discounted at the real yield. When $I^e > 0$, the expected real cash flow exceeds c, which implies that the floater structure has a shorter real rate duration than the Canadian structure.

The floater's duration can be either longer or shorter than the mortgage structure's duration, depending on coupon and maturity. The mortgage's real annuity is a constant, while the real value of the floater's pre-maturity cash flow, $[(1 + c)(1 + I^e) - 1]/(1 + I^e)^t$, declines

EXHIBIT 3 ■ Real Rate Durations of Indexed Bonds

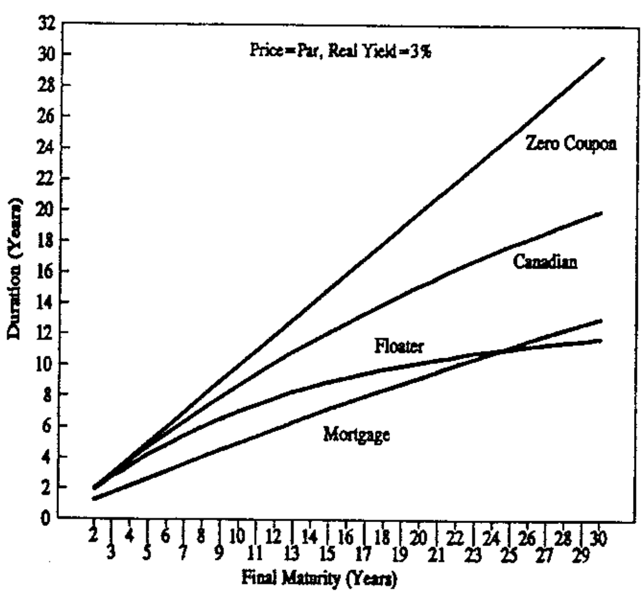
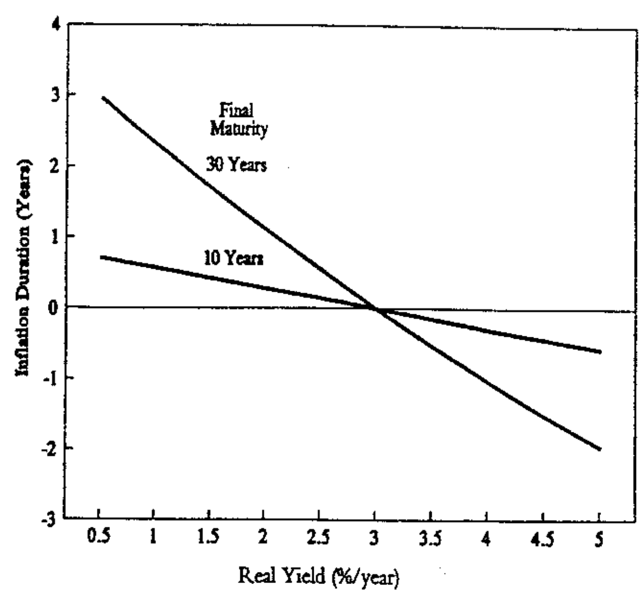


EXHIBIT 4 ■ Sensitivity to Inflationary Expectations — Floater Structure (3% coupon)



with t, and both are discounted at r. The final (nominal) maturity payment of the floater outweighs this effect for shorter maturities, but the final payment decreases in relative importance for longer maturities. Hence, floater duration is greater than mortgage duration out to a particular maturity, beyond which it is smaller.

Exhibit 3 plots all the indexed bonds' real rate durations as functions of final maturity. The zero-coupon's duration is, of course, a straight 45-degree line. The mortgage's duration appears to be a straight line, but it is actually concave downward. It reaches a maximum value of $(1 + r)/r \cong 34$ years at $r = 3\%$. The crossover point where the floater duration falls below the mortgage duration decreases with the increasing yield.

With respect to changes in inflationary expectations, i.e., holding real rates constant, the effective duration of a floater bond can be computed from the partial derivative:

$$\partial V_{Flot}/\partial I^e =$$

$$[(c - r)/(1 + r)]\{\partial[(1 - d^N)/(1 - d)]/\partial I^e\} \quad (23)$$

Inspecting this result, we notice that the inflation sensitivity of a floater will be zero when it is selling at par, $c = r$. Moreover, the inflation sensitivity will change signs as a floater bond moves from a discount to a premium, or vice versa. The only question is whether the inflation sensitivity will be negative at a premium or discount; that depends on the sign of the partial derivative on the right-hand side of Equation (23).

Although it is somewhat tedious, a straightforward calculation reveals that the derivative is negative for positive interest rates. Consequently, a floater's inflation duration will be negative (positive) when the bond is selling at a discount (premium.) When the floater bond is selling at a discount, an increase in expected inflation will actually *increase* its market value.

A simple expression for the inflation duration of the floater structure is elusive, but here are two formulas, neither very obvious:

$$D_{Flot}(I^e) = D_{Flot}(r) - \{1 + [(\Sigma_t d^t) - d^N]/V_{Flot}\}$$

$$= (1 - 1/V_{Flot})[d/(1 - d) - Nd^N/(1 - d^N)] \quad (24)$$

It's not immediately obvious, but these expressions are exactly zero when the bond is selling at par ($V_{Flot} = 1$), and they are negative (positive) when the bond is selling at a discount (premium); this is somewhat more discernible in the second expression, given the recognition that the term in brackets is strictly positive for $d < 1$ (a positive discount rate) and more than a single remaining payment ($N > 1$).[11] Even when the bond sells at a premium, the second term of the first formula is positive, which implies that the inflation duration of the floater structure is strictly less than the real rate duration.

Exhibit 4 shows inflation duration for two cases, final maturities of ten and thirty years, over a range of real yields holding the coupon constant at 3%.

What is the intuition for the negative value of the inflation duration of a floater bond selling at a discount? To elucidate this, remember that a discounted bond derives more of its total value from earlier payments than does the same bond selling at a premium. Also, it turns out that inflationary expectations have a differential impact on the present values of earlier- and later-scheduled payments.

To be specific, let's examine the present value of the floater payment for date t and find its response to a change in expected inflation:

$$\partial PV(CF_t)/dI^e =$$

$$\partial\{[(1 + c)(1 + I^e) - 1]/[(1 + r)(1 + I^e)]^t\}/\partial I^e \quad (25)$$

The calculation is a bit messy, but it's not hard to prove that the present value increases with inflation if, and only if:

$$t < [(1 + c)(1 + I^e)]/[(1 + c)(1 + I^e) - 1] \quad (26)$$

For larger t, the present value of the payment *decreases* as expected inflation increases. For example, with a real coupon of 3% per year, expected inflation of 5% per year, and semiannual payments, the present values of cash flows scheduled sooner than 12.88 years *increase* with inflationary expectations. Moreover, the present value of the final total payment always decreases with expected inflation when $N > 1$, i.e., always until there is a single remaining payment.

This explains the puzzle. As inflationary expectations increase, the present values of early (late) cash flows increase (decrease). Since early cash flows are weighted proportionately more (less) heavily for a discount (premium) bond, the bond's price increases (decreases.) As we have already noted, par is the crossover point where this phenomenon reverses itself.

Nominal Bond

Valuation of the nominal bond is familiar. Yet there is an interesting contrast between the interest rate sensitivities of indexed bonds and the duration of an ordinary nominal Treasury. To compare apples with apples, let's assume that the coupon of the nominal Treasury is equivalent, on an ex-inflation basis, to those of the indexed bonds, i.e.:

$$c_{Nom} = (1 + c_{Indexed})(1 + I^e) - 1 \tag{27}$$

so that the nominal bond would also sell at par if $r = c_{Indexed}$. We are assuming, strictly for purposes of illustration, that the inflation risk premium, π, is zero. In this case, the actual nominal cash payments of the nominal bond will be identical to the *expected* nominal cash payments of the floater. Consequently, when a floater is selling at par, its real yield duration will be identical to the familiar (nominal yield) duration of the nominal bond.

It is straightforward to show that the nominal bond has the same duration with respect to changes in real interest rates, to expected inflation rates, or to its nominal yield; i.e.:

$$[\partial V_{Nom}/V_{Nom}]/[\partial r/(1 + r)] =$$

$$[\partial V_{Nom}/V_{Nom}]/[\partial I^e/(1 + I^e)] =$$

$$[\partial V_{Nom}/V_{Nom}]/[\partial y/(1 + y)] \tag{28}$$

where V_{Nom} is the value of the nominal bond, and $y = (1 + r)(1 + I^e) - 1$.

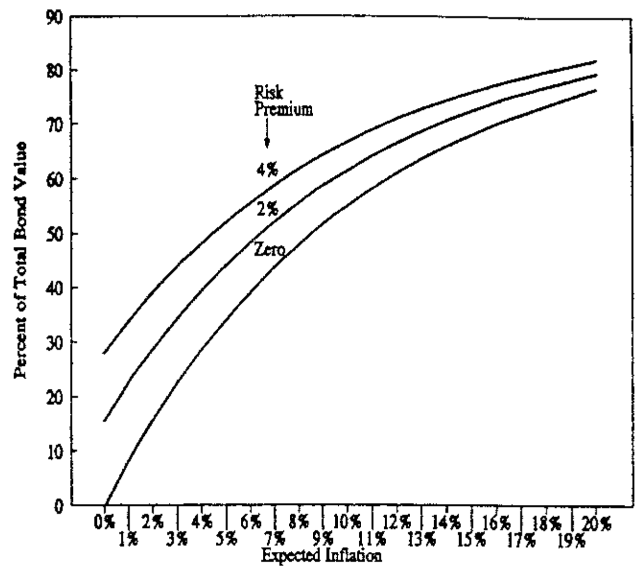
Conclusions About Volatility

The major risk advantage of indexed bonds is their low price sensitivity to inflationary expectations.

EXHIBIT 5 Cash Flows from a Stripped Canadian Structure

Strip Component	From Coupons, $t \le N$	From Principal $t = N$ only
Fixed Nominal	c	1
Inflation Accrual	$c[\Pi_{\tau\le t}(1 + I_\tau) - 1]$	$\Pi_{\tau\le N}(1 + I_\tau) - 1$

EXHIBIT 6 ■ Aggregated Relative Value of Inflation Accrual Strips from Canadian Structure, Ten-Year Final Maturity



It is either zero or, for the floater, much lower than an otherwise-equivalent nominal bond because indexed bonds' cash flows respond to *actual* inflation, unlike the nominal bond whose cash flows are locked permanently to the inflation expected at origination.

Comparing indexed structures (with given final maturities), the floater and mortgage structures have an advantage over the zero-coupon and Canadian structures by having substantially lower real rate sensitivities. When selling at par, the floater has zero inflation duration, and it has only a modest inflation duration even when selling at a premium. At a discount, the floater actually has an advantage over the other indexed bonds, with its negative inflation duration. Overall, the floater and mortgage structures have much to admire from a volatility perspective.

VI. STRIPS FROM INDEXED BONDS

The strippability of indexed bonds will be an important determinant of their popularity, because the entire bond is unlikely to appeal to every clientele group. Stripping individual cash flows and then recombining them into more palatable packages has the potential to satisfy many more investors.

Indexed bonds can be stripped down two levels. The first level converts each *total* payment into a distinct security. The second level further separates the nominal and inflation-linked components of each total payment. Whether the market will develop to the second level is an intriguing question. The first level of strippability is probable, given public statements by the Treasury and by other interested parties.

Canadian Structure

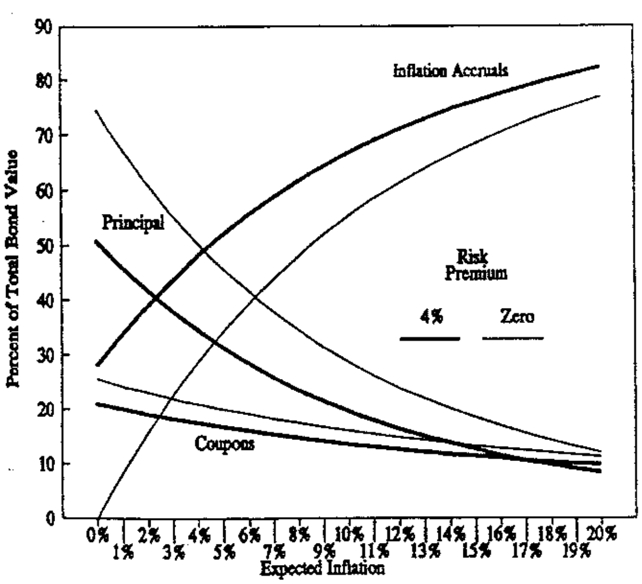
The Canadian structure stripped to a fixed nominal payment and an inflation accrual payment on each scheduled payment day would produce the cash flows shown in Exhibit 5.[12] Stripping to only the first level would provide a zero-coupon index-linked bond for each scheduled payment date, with cash flows equal to the column sums. In this case, the term structure of real interest yields could be computed easily from the sequence of strip prices.

If stripping proceeds to the second level, the market will determine the values of the fixed nominal strips by discounting cash flows at nominal rates that include expected inflation plus a risk premium. Let this discount rate be denoted

$$y = (1 + r + \pi)(1 + I^e) - 1 \qquad (29)$$

where r is the real discount rate and π is the risk premium. (For simplicity of illustration, we assume temporarily that $r + \pi$ and I^e are the same for all the nominal strips.)

EXHIBIT 7 ■ Nominal Coupons and Principal, and Inflation Accruals — Relative Values from Canadian Structure, Ten-Year Final Maturity



The aggregate market value of the inflation accrual strips will be determined at origination by the arbitrage requirement that all components aggregate to par.

Exhibit 6 shows the proportion of market value represented by *all* the inflation accrual strips under a particular set of assumptions: The real coupon is 3%, final maturity is ten years, $I_t = I^e(\forall t)$, and the bond's total price is par. Inflation strip aggregate value is shown over a range of expected inflation and for three different levels of the inflation risk premium.

It is interesting to note that inflation accruals are worth, in aggregate, at least 50% of the indexed bond's total value for many inflation rates and risk premiums. Although not depicted in this graph, it is straightforward to prove also that their relative value increases with the bond's final maturity.

Unless the risk premium is zero, inflation accrual strips represent a significant fraction of market value even when inflation is expected to be zero. To understand why this must be true, think about the valuation of a pure nominal bond when inflation is *expected* to be zero but there is some possibility that it will not *actually* be zero. To the extent that the market requires an inflation risk premium, a pure nominal bond with the same coupon as an index-linked bond will sell at a lower price. If the indexed bond sells at par with a coupon of c, a nominal bond with an identical coupon will sell below par.

Consequently, the nominal strips considered here, which essentially form an ordinary nominal coupon bond in aggregate, will be valued below par if the risk premium is positive. Since the entire bond is valued at par, the inflation accrual strip is worth the difference (even when no inflation is expected).

Exhibit 7 decomposes the Canadian payments into the aggregate of the inflation accrual strips, the aggregate of the nominal coupon strips, and the final nominal principal strip. Values are shown for a range of expected inflation and two levels of the inflation risk premium, zero and 4% per year.

For this ten-year bond, the nominal coupons as a group represent a smaller component of value than the nominal principal at low inflation rates but overtake principal at higher rates. The crossover point occurs at lower inflation rates for bonds with longer final maturities. Both nominal components, coupons and principal, decrease in relative value with increasing expected inflation and increasing inflation risk aversion, while the values of the inflation accrual components have the opposite reaction.

To value individual strips separately, it will be necessary to use the term structure of interest rates. Now, however, there will be both a nominal and a real term structure.

Suppose we wish to calculate the current market value of one of the nominal payments, say, the payment scheduled t periods from the present. The nominal term structure of zero-coupon Treasuries implies a discount rate d_t, which has the same form as above:

$$y_t = (1 + r_t + \pi_t)(1 + I^e_t) - 1 \tag{30}$$

where all the components, the real interest rate r, the inflation risk premium π, and the expected inflation I^e, now have subscripts denoting the fact that they can be different across scheduled payment dates. Note that y_t should be observable directly from existing nominal Treasury zero-coupon bond yields.

Calculating the present value of any nominal payment is then straightforward; e.g., the nominal coupon strip payable at t has a present value:

$$PV[\text{Nominal Strip}_t] = c/(1 + y_t)^t \tag{31}$$

Since the total payment from the indexed bond at t is inflation-protected, its present value is calculated by discounting its nominal expected value at a nominal discount rate that should not include an inflation risk premium. The *total* expected payment at t (a scheduled coupon date) thus has a present value[13]

$$PV[\text{Total Payment}_t] =$$

$$c(1 + I^e_t)^t/[(1 + r_t)(1 + I^e_t)]^t = c/(1 + r_t)^t \tag{31}$$

This implies the valuation of the inflation accrual strip payable at t:

EXHIBIT 8 ■ Cash Flows from a Stripped Mortgage Structure

Strip Component	Cash Payment t ≤ N
Fixed Nominal	A
Inflation Accrual	$A[\Pi_{\tau\le t}(1 + I_\tau) - 1]$

EXHIBIT 9 ■ Cash Flows from a Stripped Floater Structure

Strip Component	From Coupons t ≤ N	From Principal t < N	From Principal t = N only
Fixed Nominal	c	0	1
Inflation Accrual	cI_t	I_t	I_N

$$PV[\text{Inflation Strip}_t] =$$

$$c[1/(1 + r_t)^t - 1/(1 + y_t)^t] \tag{32}$$

An identical set of present value calculations applies to the final principal payments, nominal, total and inflation accrual; the only difference is that "c" is replaced by "1."

These formulas give a quantitative cast to an intriguing aspect of Canadian bond strips: The prices of nominal and inflation accrual strips respond in opposite directions to changes in inflationary expectations. Holding real interest rates constant, the *summed* value of nominal plus inflation accrual strips for any date is unaltered by a change in expected inflation, but the nominal strip will inexorably decline in value if expected inflation increases; and vice versa. This implies that the inflation accrual strip will actually *increase* in value with higher inflation (assuming no change in real yield).

As inflation varies, the market prices of the two strips will be negatively correlated. The same results hold with respect to changes in the inflation risk premium, π; whenever it increases, the nominal (inflation accrual) strip will decrease (increase) in value.

The initial difficulty of market makers in these new Treasury issues involves the absence of reliable information about the term structure of real interest rates, r_t. The nominal rates, d_t, will be directly observable on old nominal Treasuries, and should present no problem unless there is a substantial liquidity difference.

The real interest rates will be observable for the first time in U.S. history when the strips begin to trade. This should be an exciting time for all, especially traders.

Mortgage Structure

The only material distinction between the mortgage and Canadian structures is the Canadian's balloon payment at maturity. For the mortgage, cash flows to strips are given in Exhibit 8, where, as before, "A" denotes the real value of the mortgage annuity $A = c/[1 - (1 + c)^{-N}]$.

Stripping only to the first level, to the total cash payment, would provide a sequence of zero-coupon indexed bonds. Because the mortgage structure provides no balloon payment, the market values of all but the last such strip would be considerably larger than in the Canadian case. This might create a more liquid set of strips and a more reliable real term structure.

Floater Structure

The fixed nominal and inflation accrual strips from the floater structure are shown in Exhibit 9. The inflation accrual strips have a very different form from that under the Canadian model, yet they must in aggregate represent the same fraction of market value. This follows because the nominal strips are identical in the two structures, and therefore must have the same aggregate value, given a par coupon c. Of course, the par coupon might very well be different for the Canadian and floater structures, but the graphs for the Canadian structure apply equally to the floater structure because they depict only the *relative* values of fixed and inflation accrual components for a given coupon.

Valuing the individual inflation accrual strips from a floater bond is, however, a bit more difficult. There is no obvious real rate discounting formula for the *total* scheduled payment. Each payment is *not* the "real" value of a fixed number, but is instead related to the idiosyncratic inflation that happens to occur during the future coupon interval. This might seem a disadvantage, but it actually provides some interesting possibilities to the market.

Because floater bond inflation accrual strips pertain to separate individual periods, they represent a term structure of inflation forward contracts, with the Treasury as seller. They reveal the market's consensus forecast of the entire path of future inflation, period by period.

To obtain the market's inflation forecast for a given period, consider the trade following:

Buy: 1/c units of the floater bond's total payment (both strips) for period t, and

Sell: 1/[c(1 + c)] units of the floater's inflation accrual strip for period t.

This position could be called a "pure inflation strip." Its net cash flows are zero in every period other than t, when it pays

$$(1/c)[(1 + c)(1 + I_t) - 1] - \{1/[c(1 + c)]\}I_t(1 + c) = 1 + I_t \tag{33}$$

where I_t is the *actual* inflation in t (an unknown prior to t). At any date earlier than t, the market price of the inflation strip is the present value of a pure claim on inflation in t. This would provide a direct measure of the market's consensus belief about inflation in *every* future period covered by a floater bond payment.

VII. CONTINUOUS-TIME VALUATION

Modern continuous-time models of interest rates are widely employed to value fixed-income securities and their associated derivatives. To handle both nominal and indexed issues simultaneously, these models will have to admit at least three stochastic "state" variables: the (instantaneous) real interest rate, the expected inflation rate, and the actual inflation rate. In a rarely cited section of their otherwise well-known article, Cox, Ingersoll, and Ross [1985] (hereafter, CIR) present just such a model designed explicitly to capture the simultaneous movements of these three variables.

In the CIR framework, the real interest rate, r, obeys a mean-reverting, square root stochastic Itô process of the form

$$dr = \kappa_r(\theta_r - r)dt + \sigma_r\sqrt{r}\,dz_r \tag{34}$$

where dr is the instantaneous rate of change of the real interest rate, dz_r is a Wiener process, and κ_r, θ_r, and σ_r are parameters.[14] The parameter σ_r is the volatility of the process, and $\kappa_r(\theta_r - r)$ is the "drift," with θ_r the long-run mean and κ_r the speed of adjustment from the current level, r, to the long-run mean.

Given this specification, CIR derive prices for zero-coupon indexed bonds of all maturities along with the term structure of real yields:

$$R_N = r\beta_N + \alpha_N \tag{35}$$

where R_N is the real yield for a zero-coupon bond of maturity N, and β_N and α_N are functions of N and of the parameters κ_r, σ_r, and θ_r. Although real yields of every maturity are related linearly to the instantaneous real rate, r, the functions β_N and α_N are algebraically complex, and the real yield curve can assume a variety of shapes, including some that are non-monotonic.

CIR propose two alternative versions of the process describing expected inflation. The simpler version is:

$$dI^e = \kappa_e(\theta_e - I^e)dt + \sigma_e \sqrt{I^e}\, dz_e \qquad (36)$$

where I^e now denotes the *instantaneous* expected inflation rate.[15] Like the real interest rate, the instantaneous expected inflation can drift away from its long-term level θ_e, and it reverts with its own speed of adjustment κ_e and volatility σ_e; dz_e is a distinct Weiner process independent of dz_r.

Finally, the commodity price index, CPI, follows a geometric diffusion; the instantaneous actual inflation rate is thus

$$d(CPI)/CPI = I^e dt + \sigma_I \sqrt{I^e}\, dz_I \qquad (37)$$

where dz_I is another Weiner process, and $\sigma_I \sqrt{I^e} dz_I$ is the instantaneous unexpected rate of inflation.

CIR make the simplifying assumption that inflation has no real effects; consequently, there is no correlation between movements in inflation, either expected or unexpected, and movements in real interest rates. They do permit correlation between actual inflation and percentage changes in expected inflation, $Cov[dz_I, dz_e]$.

Given these preliminaries, CIR derive a formula for the price of a nominal bond, i.e., a zero-coupon bond that pays a real value of $\$1(CPI_t/CPI_{t+N})$ for sure at maturity, t + N. This allows us to obtain the expression for the term structure of *nominal* yields:

$$Y_N = R_N + \delta_N I^e + \gamma_N \qquad (38)$$

where Y_N is the nominal yield on a zero-coupon bond with maturity N, and the complicated functions δ_N and γ_N depend on N and on the parameters of the expected and unexpected inflation processes. The algebraic forms of β_N and δ_N are similar, as are the forms of α_N and γ_N.[16]

Once enough Treasury indexed bond strips begin simultaneous trading with nominal Treasury strips (both in zero-coupon form), it should be possible to determine the instantaneous real rate r, the instantaneous inflationary expectation I^e, and all eight parameters of the three stochastic processes by using the prices of nominal and indexed strips of various maturities (assuming, of course, that the CIR model depicts reality).[17]

This will allow traders to assess arbitrage possibilities directly; any bond or strip that does not conform to the pricing conditions is under- or overpriced. In general, continuous-time methods have several advantages in addition to rigor; they eliminate the Jensen's inequality problem that plagues traditional discounted cash flow valuation whenever cash flows are stochastic, and they provide more accurate measures of interest rate sensitivity than traditional duration, which is exact only for parallel shifts in the yield curve.

The CIR model will be an approximation if nature admits more complexity than three stochastic processes. Yet it already seems promising, for three-factor models have proved useful in empirically describing nominal yields (see Litterman and Scheinkman [1991], for example).

At this point, we simply do not know whether the empirical factors proxy for a single real rate and a couple of inflation variables, or whether real rates themselves, or inflation, require several factors. Indexed Treasuries will provide the opportunity to investigate this issue more precisely.

VIII. CONCLUSIONS AND PROSPECTS

What is the likely success of U.S. Treasury inflation-protection bonds? Several precedents raise some questions about their prospects. British and Canadian issues have not been notably liquid. Although British index-linked gilts constitute about 15% of government debt outstanding, trading is not active; many investors seem to have put them away until maturity. (Of course, this does not signify a poor reception by long-term investors.) Canadian indexed bonds are even less liquid.

It might be too early to judge the Canadian situation, but there are several reasons why the British market has not developed to the extent likely in the U.S. market. First, British aggregate debt has actually been declining; there have even been reverse auctions for outstanding bonds. This reduction in scale has been accompanied by a general decrease in bond market liquidity.

Second, the tax treatment of index-linked gilts motivates taxable investors to buy and hold. There is no taxation on the inflation accrual, so index-linked gilts are tax-advantaged, and their yields are correspondingly too low to attract much attention from tax-exempt institutions. This is similar to the U.S. municipal bond market. Pension funds do not buy municipals (except in rare and irrational cases).

The paucity of tax-exempt institutional indexed gilt investors in Britain has no doubt reduced liquidity to less than can reasonably be anticipated in the United States. Since the proposed U.S. indexed bonds will conspicuously *not* be tax-advantaged, they will offer attractive returns to most tax-exempt institutions, which will likely buy them in large amounts because of their low risk and inflation protection.

In other countries, moreover, indexed bonds are a resounding success. Where memories of rampant inflation are vivid, it is hard to find *non*-indexed bonds. In one period in Israel, for example, there were virtually no nominal bonds in existence.

Another contention often voiced is that American investors just don't care that much about inflation protection. This argument is frequently adumbrated in the scholarly literature (see Fischer [1983], for example) to explain why private American borrowers rarely seek indexed instruments. A counterargument is that floating-rate debt is extremely common, and possibly easier to hedge with swaps and other securities. Moreover, the aggregate quantity of fixed-rate debt contracting is correlated negatively with inflation volatility in the U.S., which is not too surprising theoretically; see Cornell [1978]. McCulloch [1980] offers a different and ingenious reason for the scarcity of private indexed bonds; he alleges they were illegal in the U.S. because of an unintentional but venerable Supreme Court decision.

Another doubt is raised by the failure of futures contracts on inflation. In mid-1985, a futures contract on the CPI-W (wage earners' component) began trading on the Coffee, Sugar, & Cocoa Exchange in New York. The contracts initially had quarterly maturities out to one year, then biennial maturities out to three years. Petzel and Fabozzi [1986], in an early investigation of these contracts, devised various hedging methods to protect against fluctuations in real interest rates. Koppenhaver and Lee [1987] investigated the CPI futures' correlation with nominal interest rates (it was surprisingly low), and noted that the CPI contracts were illiquid and had minimal open interest, fewer than 100 contracts over all maturities. The contract is no longer listed.

This lack of appeal is puzzling. Perhaps it means investors are not concerned with short-term inflation; the longest futures contract was only three years to maturity. Nonetheless it does make one wonder.

Despite these reservations, the experience of other countries strongly suggests that a well-designed inflation-protection government bond will generate strong investor demand. Its 1997 American appearance will doubtless be awaited with curiosity and perhaps with enthusiasm.

ENDNOTES

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[1]This is the estimated *risk premium*, not the nominal interest rate. Naturally, the level of the nominal interest rate is much higher during periods of high inflation.

[2]*USA Today*, September 26, 1996, p. 2B, reported, without citing a source, that "virtually none of the 800 people and organizations who commented...since May argued for a different index."

[3]Ignoring any lag in indexation.

[4]Another impetus toward shorter durations involves potential hedging instruments for the new indexed bonds. With no other available methods, hedgers might attempt to construct a portfolio of inflation-sensitive futures contracts from combinations of commodities. Since traded futures contracts are generally rather short-dated, such hedges would be most effective for a shorter-maturity bond.

[5]*USA Today*, September 26, 1996, p. 2B.

[6]Treasury bonds are exempt from state income taxes.

[7]Plus an additional tax on the risk premium, if any.

[8]In either case, the price accretion exactly offsets the difference between coupon payments and taxable income.

[9]As a convention, the units of duration match the bond's payment frequency; e.g., if the bond pays a semiannual coupon, duration will be expressed in half-years.

[10]In this section, we ignore Jensen's inequality. Actually, the expected cash flow in period t would be more precisely expressed as $E[c\Pi_{\tau\leq t}(1 + I_\tau)]$, which is not necessarily $c(1 + I^e)^t$. Thus, a real yield, solved from the internal rate of return of expected cash flows, can be a biased estimate of the real interest rate.

[11]When there is only one payment remaining ($N = 1$), the inflation duration of a floater bond is zero; this is obvious because the single cash flow remaining is $(1 + c)(1 + I^e)$.

[12]There is a knotty problem associated with stripping indexed bonds to the second level. What happens in

the case of *deflation*? In such a circumstance, the owner of the accrual strip should be paying cash to the owner of the nominal strip since the total payment from the Treasury would fall short of the bond's coupon. Perhaps a payments mechanism could be patterned after futures, where marking to market requires frequent cash infusions from both buyer and seller.

[13]This formula should be considered only an illustration. It is an approximation that ignores the influence of Jensen's inequality, for which a technical correction must be made in practice.

[14]Subscripts are added to parameters for clarity.

[15]Note that I^e differs from the inflation rate expected from now until some future date; it is the rate anticipated over the next instant. Consequently, it can be expected to change (and have a non-zero drift) without violating rationality.

[16]See Cox, Ingersoll, and Ross [1985], Equation (23), p. 393, and Equation (56), p. 403. Using their Equation (48), it can be readily seen that $\delta_N I^e + \gamma_N = -\ln[E(CPI_t/CPI_{t+N})]$. The continuously compounded expected inflation over N periods would be $E[\ln(CPI_{t+N}/CPI_t)]$ so the entire expression in the text above is analogous to, but not algebraically equivalent to, the Fisher relation, where the nominal yield is the real yield plus expected inflation.

[17]The eight parameters are three volatilities, two long-run means, two speeds of adjustment, and the correlation between actual and expected inflation. In principle, the CIR model also involves a risk parameter, which cannot be identified separately from κ_r and θ_r, nor can the latter two parameters be independently identified. Nonetheless, combinations of parameters are adequate for describing the stochastic processes. See Brown and Dybvig [1986].

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