Capital Structure Effects on Prices of Firm Stock Options: Tests Using *Implied Market* Values of Corporate Debt

By

Robert Geske† and Yi Zhou*

November, 2007
This revision November, 2010

JEL Classification: G12

Keywords: Derivatives, Options, Leverage, Stochastic Volatility

We appreciate the comments and support from Walter Torous, Richard Roll, Mark Grinblatt, Michael Brennan, Mark Garmaise, Liu Yang, Geoffrey Tate, Charles Cao, Mark Rubinstein, Hayne Leland, Jurij Alberto Plazzi, Bernd Brommundt, Xiaolong, Cheng and Michael Nowotny and all seminar participants where this paper was presented. All remaining errors are ours.

† Robert Geske is the corresponding author at The Anderson School at UCLA, 110 Westwood Plaza, Los Angeles, California 90095, USA, 310-825-3670,
e-mail: rgeske@anderson.ucla.edu.
*Yi Zhou: The Michael F. Price College of Business at University of Oklahoma, Division of Finance, 307 W. Brooks, Norman, Oklahoma 73019, 310-866-3165,
e-mail: yizhou@ou.edu.

Abstract

This paper introduces a new methodology for measuring and analyzing capital structure effects on prices of call options on stocks of individual firms in the economy. By focusing on individual firms we examine the cross sectional effects of leverage on option prices. Our methodology allows the *market value* of each firm and thus the firm's debt to be *implied* directly from contemporaneous, liquid, nearest to at-the-money option prices without the use of any historical price data. We compare Geske's parsimonious model to the alternative models of Black Scholes (BS) (1973), Bakshi, Cao, and Chen (BCC, 1997) (stochastic volatility (SV), stochastic volatility and stochastic interest rates (SVSI), and stochastic volatility and jumps (SVJ)), and Pan (2002) (no-risk premia (SV0), volatility-risk premia(SV), jump-risk premia (SVJ0), volatility and jump risk premia (SVJ)) which allows state-dependent jump intensity and adopts implied state-GMM econometrics. These alternative models do not directly incorporate leverage effects into option pricing, and except for Black-Scholes these model calibrations require the use of historical prices, and many more parameters which require complex estimation procedures. The comparison demonstrates that firm leverage has significant statistical and economic cross sectional effects on the prices of individual stock options. The paper confirms that by incorporating capital structure effects using our methodology to *imply the market value* of each firm and thus the firm's debt, Geske's model reduces the errors pricing options on individual firms by 60% on average, relative to the models compared herein (BS, BCC, Pan) which omit leverage as a variable. However, we would be remiss in not noting that after including leverage there is still room for improvement, and perhaps by also incorporating jumps or stochastic volatility at the firm level would result in an even better model.

1. Introduction

Ross (1976) demonstrated that almost all securities and portfolios of securities can be considered as options. Black and Scholes (BS) showed that all options are actually levered investments in the underlying optioned security or asset. It is well known that most corporations have some form of direct or indirect leverage.[1] Thus, it seems puzzling that in the asset pricing literature, there have been few detailed examinations or tests for leverage effects using a model which directly incorporates leverage, based on economic principles.

In three recent papers, Geske and Zhou (2006, 2007a,b) have demonstrated that by including a new measure of implied market leverage in a parsimonious methodology using contemporaneous equity and equity option prices, they can significantly improve on the pricing of individual stock options and index options. Furthermore, they also show (2007) that their methodology allows an implied equity volatility measure that dominates the CBOE's VIX, and several GARCH techniques for forecasting future volatility.

Empirically, researchers have documented a negative correlation between stock price movements and stock volatility, which was first identified by Black (1976) as the "leverage" effect. A few papers have confirmed that debt is related to the observed negative correlation (Christie (1993), and Toft and Pruyck (1998)). Toft and Prucyk (TP) (1997) adapt a version of Leland and Toft (1996) to individual stock options, and using ordinary regression in cross-sectional tests they demonstrate significant *correlations* between their model's debt variables and the volatility skew for a 13 week period in 1994 for 138 firms in their final sample. However, TP do not investigate the extent of option pricing improvement attributable to leverage by comparison to more complex models

[1]A firm not *directly* issuing bonds has many *indirect promised* payouts (loans, receivables, taxes, etc).

which omit leverage. Instead they examine the cross-sectional correlations between volatility skew for individual stocks and their model debt variables which are: (i) LEV, the ratio of *book value (not implied market value)* of debt and preferred stock to debt plus all equity, and (ii) CVNT, ratio of short maturity debt (less than 1 year) to total debt, as a proxy for a protective covenant.

Some option pricing papers have modeled and tested this negative correlation between a stock's return and its stochastic volatility. Among these papers are the stochastic volatility models of Heston (1993), Bakshi, Cao, Chen (BCC) (1997) and Pan (2002), which is a more complex extension of Bates (2000). However, these papers all assume arbitrary functional forms for the correlation between a stock's return and changes in the stock's volatility. None of them provides the economic motivation of leverage for this correlation. If this negative correlation is partially caused by debt as identified first by Black, then the variations in actual market leverage should be both statistically and economically important to pricing equity options. Thus, it is important to isolate and analyze the magnitude of the leverage effect independent of other assumed possible complexities such as stochastic volatility, stochastic interest rates, and stochastic jumps. Otherwise, these additional assumed stochastic parameters may be estimated with error because of a relevant omitted variable. In order to incorporate debt into asset pricing, we adopt Geske's (1979) no arbitrage, partial equilibrium, compound option model.

Geske's model provides a unique method to *imply* the *market* value of debt. His leverage based stochastic equity volatility model does not assume any arbitrary functional form, and it provides the economic reason for the negative correlation between volatility and stock returns. The stock return volatility is not a constant as assumed in the Black and Scholes theory, but is a function of the level of the stock price, which also depends on the value of the firm. As a firm's stock value declines, the firm's leverage ratio increases. Hence the equity becomes more risky and its volatility increases. This model can explain the negative correlation between changes in a stock's return and changes in the stock's volatility. Geske's option model also results in the observed fatter (thinner) left (right) tail of the stock return distribution.

By incorporating the *implied market* value of each firm and thus each firm's debt directly and modeling its economic impact, Geske's option model uses Modigliani and Miller (M&M) to take the option pricing theory deeper into the theory of the firm.[2] His model incorporates the differential *implied market* value of stochastic debt, differential default risk, and differential bankruptcy. Thus, the Geske approach gives rise to stochastic equity volatility naturally, and this has the advantage of a direct economic interpretation for the stochastic volatility. This paper demonstrates the parsimonious Geske model performs much better with far fewer parameters and less difficult estimation than the more complex parameterized models of BCC and Pan which omit debt but include parameters for stochastic equity volatility, stochastic interest rates, and stochastic equity jumps. Geske also is shown to dominate Black-Scholes.

Both the size of the *implied market* value of debt and the duration of debt effect the stochastically changing shape of each firm's stock return distribution. It is the shape of the conditional equity return distribution at any point in time that determines the model values for options with different strike prices and different times to expiration. Thus, the omission of an important and measurable economic variable, debt, causes the return distribution to be mis-specified. This paper shows that the omission of debt is partially responsible for options valued with either BS or the more complex models of BCC and Pan to exhibit

[2]Since the stock price is known input, given M&M, the solution is actually for the market value firm debt.

greater errors. However, after including leverage there is still room for improvement, and perhaps by also incorporating jumps or stochastic volatility at the firm level would result in an even better model.

This is the first paper in the existing literature to empirically examine capital structure effects on the pricing of individual stock options by using Geske's closed-form compound option model. In a related papers, Geske and Zhou (2007) present the first evidence of the time series effects of leverage on prices of S&P 500 index put and call options. Since an index has no cross sectional variation in leverage, the paper examines the changes in aggregate index debt with time. The index paper shows that by including the time variations in leverage as a variable, Geske's model is superior for pricing index put and call options to the models of BS (1973) and BCC (1997) which omit leverage. Furthermore, the advantage of including debt is monotonic in the changing amount of leverage over time, and in time to option expiration.

This paper is related to many papers in the option pricing literature. For example, an implied bimonial tree lattice approach was developed (by Rubinstein (1994) and others) to better fit the cross-sectional structure of option prices wherein the volatility can depend on the asset price and time. This lattice approach to an implied binomial tree produces a deterministic volatility function (DTV), and these implied tree lattice approaches have been shown to work no better than an ad hoc versions of Black-Scholes where the implied volatility is modified for strike price and time.

The negative correlation between equity return and volatility has been modeled by Heston (1993) and others. Heston develops a closed-form stochastic volatility model with arbitrary correlation between volatility and asset returns and demonstrates that this model has the ability to improve on the Black-Scholes biases when the correlation is negative. [3] Heston and Nandi (2000) develop a closed-form GARCH option valuation model which exhibits the required negative skew and contains Heston's (1993) stochastic volatility model as a continuous time limit. They demonstrate that their out of sample valuation errors are lower than the ad hoc modified version of Black-Scholes which Dumas, Fleming and Whaley (1998) developed. Liu, Pan and Wang (2005) attempt to further disentangle the rare-event premia by separating the premia into diffusive and jump premia, driven by risk aversion, and then adding an intuitive component driven by imprecise modeling and subsequent uncertainty aversion. All of the latter three papers test their models on S&P 500 index options. In all cases, these more generally specified models with many more input parameters outperform the (ad hoc) Black-Scholes solutions.

In this paper we focus primarily focus on the following three papers: Black-Scholes, Bakshi, Cao and Chen (1997), and Pan (2002).[4] BCC (1997) formulate a series of nested models which include stochastic volatility (SV), and additionally either stochastic interest rates (SVSI), or jumps with constant jump intensity (SVJ). BS model is a special case of both Geske and BCC models. BCC test their model by comparing the implied statistical parameters to those of the underlying processes, as well examining out-of-sample pricing and hedging performance for S&P 500 index options. Pan (2002) examines the joint time series of the S&P 500 index and near-the-money short-dated option prices with a no-arbitrage model to capture both stochastic volatility and jumps. She introduces a parametric pricing kernel to analyze the three major risk factors which she assumes effect

[3]See Scott (1987), Stein and Stein (1991) and Wiggins (1987). With respect to Heston (1993), Pan (2002) says "Our first set of diagnostic tests indicates that the stochastic volatility model of Heston (1993) is not rich enough to capture the term structure of volatility implied by the data."

[4] Eraker, Johannes and Polson (2003) extend Pan(2002) by assuming uncertainties in both the jump timing and jump size in both the volatility and the returns, with either simultaneous arrivals with correlated jump sizes or independent arrivals with independent jump sizes.

the S&P 500 index returns: the return risk, the stochastic volatility risk and the jump risk. Pan (2002) extends Bates (2000) by allowing the jump premium to depend on the market volatility by assuming that the jump intensity is an affine function of the volatility for a state-dependent jump-risk premium so that the jump risk premium is larger during volatile periods. She also indicates that this jump risk premium dominates the volatility risk premium.

However, by omitting debt as a variable and instead assuming arbitrary functional forms for volatility, correlation and jump processes, the existing literature fails to address directly the importance of capital structure in asset pricing. This paper directly tests the extent of a leverage effect in individual stock options by measuring and using the actual daily *implied market* firm value and thus debt for each individual firm. The Geske model requires the current total market value of the firm's debt plus equity, and the instantaneous volatility of the rate of growth of this total market value, neither of which are directly observable. This problem is parsimoniously circumvented by observing contemporaneous, liquid market prices, one for the individual stock price and the second for the price of a call option on the individual stock. Then solving three simultaneous equations for the total market values, V = S + D, market return volatility, σ_V and the critical total market value, V*, for the option exercise boundary.

We first show that Geske's model improves the net option valuation of over 2.5 million listed in-the-money and out-of-the-money individual stock call options on over 11,500 firms by on average by about 60% compared to other models. Furthermore, we show for each firm's options this improvement is directly and monotonically related to both the firm's debt and the time to expiration of the option. The pricing improvement is monotonic with respect to time to expiration because leverage has a longer time effect. It may not be completely surprising that Geske dominates simple Black-Scholes when pricing equity options if the data quality for measuring leverage is good. However, when we compare Geske's model with more complex competing models which require more parameters (Bakshi, Cao and Chen (1997) and Pan (2002)), we find that Geske's model produces the best performance in both absolute and relative pricing error measures.

The rest of the paper proceeds as follows. Section 2 describes the Geske model and its relatively parsimonious implementation. Section 3 describes the data and explains in detail how the necessary data inputs are calculated. Section 4 compares the Geske results with the BS model and reports both statistical and economic significance. Section 5 describes and compares the three BCC model versions, SV, SVSI and SVJ with Geske. Section 6 describes and compares Pan's four model versions, SV0, no-risk premia, SV, volatility-risk premia, SVJ0, jump-risk premia, and SVJ, the volatility and jump risk premia model with Geske. Section 7 concludes the paper.

2. Compound Option Model

In this section, we briefly review the model of Geske (1979), and in later sections we review BS, BCC, and Pan. Recall that Geske's option model, when applied to listed individual equity options, transforms the state variable underlying the option from the stock to the total market value of the firm, V, which is the sum of market equity and market debt. In this case the volatility of the equity of the individual stock will be random and inversely related to the value of the individual stock equity. This interpretation of the Geske's model introduces a method which enables the measurement of each individual firm's *implied* market debt value from both option and equity prices, which permits better measurement of individual firm's debt value and credit risk.[5] Geske's model is consistent with Modigliani and Miller, and allows for default on the debt and bankruptcy. In Geske's model the partial equilibrium, self-financing, risk free no arbitrage portfolio is formed with the option, the firm, and a risk free bond. This differs from the Black-Scholes model where the partial equilibrium, self-financing, risk free no arbitrage portfolio is formed with the option, the stock, and a risk free bond. Black-Scholes is a special case of Geske's model which will reduce to his equation when either the dollar amount of leverage is zero or when the leverage is perpetuity. In Geske the boundary condition for exercise of an option is transformed from depending on the stock price and strike price to depending on the value of the firm, V, and on a specific exercise critical firm total market value, V*. In implementation we allow both Geske and Black-Scholes to have a term structure of volatility.[6] Given the above, if the firm value is described by a relative diffusion process, the following equations result for pricing individual stock call and put options:

$$C = VN_2(h_1 + \sigma_{vT1}, h_2 + \sigma_{vT2};\, \rho) - Me^{-r_{T2}(T_2-t)}N_2(h_1, h_2;\, \rho) - K\, e^{-r_{T1}(T_1-t)}N_1(h_1) \quad (1)$$

$$P = Me^{-r_{T2}(T_2-t)}N_2(-h_1, h_2; -\rho) - VN_2\big(-(h_1 + \sigma_{vT1}), h_2 + \sigma_{vT2}; -\rho\big) + Ke^{-r_{T1}}N_1(-h_1) \quad (2)$$

where

$$h_1 = \frac{\ln(V/V^*) + (r_{FT1} - 0.5\sigma_{vT1i}{}^2)(T_1 - t)}{\sigma_{vT1i}\sqrt{T_1 - t}}$$

$$h_2 = \frac{\ln(V/M) + (r_{FT2} - 0.5\sigma_{vT2d}{}^2)(T_2 - t)}{\sigma_{vT2d}\sqrt{T_2 - t}}$$

$$\rho = \sqrt{(T_1 - t)/(T_2 - t)}\;.$$

[5] We thank Leland for pointing out that Toft-Prucyk (1997) and other models which take option valuation to the firm level are also able to imply the market value of each firm's debt.

[6] A term structure of volatility is known to exist in the equity option market. Implementing Geske and Black-Scholes with a volatility term structure allows their models to have a similar number of parameters as the more complex models of BCC and Pan which cannot accommodate a volatility term structure.

Here V^*, the critical firm value for option exercise, depends on each option's strike price, K_j, and each options expiration date, T_1, for all strikes j and option expirations T_1, and can be more fully described as $V^*(K_j, T_1)$. All options expire at specific dates T_1 which occur before the debt matures at T_2 , and options with specific days to expiration are valued with the relevant implied volatility from term structure bucket *i*. Thus, all call and put options depend on four unknowns, $C[V, V^*(K_j,T_{1i}), \sigma_{vT1i}, \sigma_{vT2d}]$ and $(P[V, V^*(K_j,T_{1i}), \sigma_{vT1i}, \sigma_{vT2d}])$. At each specific option expiration T_1, if $V < V^*$, $C = 0$ ($P = K-E$), and if $V > V^*$, $C = E-K$ ($P = 0$). The firm implied volatility to each option expiration date T_{1i} in the relevant volatility term structure bucket *i*, is $\sigma_{v\,T1i}$, and the firm implied volatility to the debt maturity date, T_{2d} , is $\sigma_{v\,T2d}$. The face value of a firm's debt outstanding is M and T_{2d} is the maturity of this debt.[7] The events of exercising the call option and the firm defaulting are correlated. If a firm is more likely to default at T_{2d}, where V is less than M at T_{2d}, then V will also be more likely to be less than V^* at T_1, which makes call options expiring at T_1 less likely to be exercised. For Geske's compound option there are two correlated exercise opportunities at T_1 for the call option expirations and at T_2 for the debt maturity. This correlation is measured by $\rho = \sqrt{(T_{1i} - t)/(T_{2d} - t)}$ where individual stock option expiration T_{1i} is always less than or equal to debt maturity, T_{2d}.

In order to solve for these four unknowns, $V, V^*(K_j,T_{1i}), \sigma_{vT1i}, \sigma_{vT2d}$, we utilize equations (1) and (2) above, and we use Merton's (1974) equation for stock as an option on the firm assets V in order to solve for the firm implied volatility over the debt maturity, T_{2d}.

$$S = VN(d_1) - Me^{-rT2d} N(d_2) \qquad (3)$$

[7] We follow the standard practice from Merton (1974) implementing corporate debt as a zero coupon bond maturing at the duration of all promised payments outstanding. This is more explicitly described and detailed in section 3.

where

$d_1 = [\ln(V/M) + (r + ½\,\sigma^2_{v\,T2d})\,T_{2d}] / (\sigma_{v\,T2d}\sqrt{T_{2d}})$ and $d_2 = d_1 - \sigma_{v\,T2d}\sqrt{T_{2d}}$.

Equation (3) does not depend on V* or σ_{vT1i}, but does depend on V and $\sigma_{v\,T2d}$.[8]

The notation for these three equations is summarized as follows:

C = current market value of an individual stock call option,

P = current market value of an individual stock put option,

S = current market value of the individual firm stock,

D = current market value of the individual firm debt,

V = current market value of the firm's securities (debt B + equity S),

V* = critical total market value of the firm where $V \geq V^*$ implies $S \geq K$,

M = face value of market debt (debt outstanding for the firm),

K = strike price of the option,

r_{Ft} = the risk-free rate of interest to date t,

$\sigma_{v\,T1i}$ the instantaneous volatility of the firm return at expiration T_1 and volatility bucket *i*,

$\sigma_{v\,T2d}$ the instantaneous volatility of the firm return at expiration T_2 and volatility bucket *d*,

t = current time,

T_1 = specific expiration date of the option,

T_2 = maturity of the market debt,

$N_1(.)$ = univariate cumulative normal distribution function,

$N_2(\ldots)$ = bivariate cumulative normal distribution function,

ρ = correlation between the two option exercise opportunities at and T_1 and T_2.

[8] If we set the firm volatility over the maturity of the debt equal to the implied volatility from the last expiration bucket, $\sigma_{vT2d} = \sigma_{v\,T1i=4}$, instead of using the Merton (1974) equity option equation, this does not change our results. Either method uses the market price of equity, S, in the solution, so our numbers are always consistent with the equity market prices.

The solution involves solving Equation (1) twice while equations (2) and (3) are used once to give us four equations for four unknowns, and all equations are used at the same point in time for options with the same time to expiration. At any date t, Equations (1) is used with a most at-the-money (MATM) call option, C_t $(V_t, V_t*(K_2, T_1))$. Then equations (1) is used again with equation (2) for an option pair, C_t $(V_t,V_t*(K_1, T_{1i})$ and P_t $(V_t,V_t*(K_1, T_{1i})$, of a slightly in-the-money (ITM) call ($K_2 > K_1$) and thus out-of-the-money (OTM) put at the same expiration T_1 as the first call option. [9] Thus, at any time t, all three options expiring at the same T_1, are subject to the *same* firm value V_t, the *same* firm implied volatility for option expiration at T_1 from volatility bucket *i*, σ_{vT1i} , the *same* firm implied volatility for debt maturing at T_{2d} , σ_{vT2d} , but *different* critical firm expiration values, V*, which changes only because the strike price K_j changes. The critical firm values for option exercise, V*, are comprised of two components, S* and D*, where V*=S*+D*. At each option expiration, T_1, S_j* must equal to the known relevant strike price, K_j, while D* is the same across strikes. Thus at any t, $V_t*(K_1, T_{1i}) = S_{t\,K1}^* + D_t^* = K_1 + D_t^*$, and $V_t*(K_2, T_{1i}) = S_{t\,K2}^* + D_t^* = K_2 + D_t^*$. So the solution for critical V* is actually a solution for D* since S* is known to equal to the relevant strike, K. Thus, the four equation and four unknowns are:

$$(1)\ C_t\,(V_t,V_t*(K_1, T_{1i}), \sigma_{v\,T1i}, \sigma_{v\,T2d}) = C_t\,(V_t,V_t* = K_1 + D_t^*, \sigma_{v\,T1i}, \sigma_{v\,T2d})$$

$$(2)\ C_t\,(V_t,V_t*(K_2, T_{1i}), \sigma_{v\,T1i}, \sigma_{v\,T2d}) = C_t\,(V_t,V_t* = K_2 + D_t^*, \sigma_{v\,T1i}, \sigma_{v\,T2d})$$

$$(3)\ P_t\,(V_t,V_t*(K_2, T_{1i}), \sigma_{v\,T1i}, \sigma_{v\,T2d}) = P_t\,(V_t,V_t* = K_2 + D_t^*, \sigma_{v\,T1i}, \sigma_{v\,T2d})$$

$$(4)\ S_t\,(V_t, \sigma_{v\,T2d}) = S_t\,(V_t, \sigma_{v\,T2d})$$

The four unknowns in the above equations are V_t, D_t^*, $\sigma_{v\,T1i}$, $\sigma_{v\,T2d}$. The critical stock prices, $S_{t\,K1}^*$ equals known strike K_1 while $S_{t\,K2}^*$ equals known strike K_2. V_t and σ_{vT2d} are in

[9] While both the call and put options are American, the call options are valued only for time periods when there is no dividend. We set the put options to be slightly out-of-the-money and valued them with both the Geske-Johnson (1984) approximation for the American put for two exercise possibilities and with equation (2). We implement the simple European approach which adds noise but our results remain very significant.

all four equations, while V* and σ_{vT1i}, are only relevant to the three option equations. Note now that the Geske model has four explicit parameters at each option expiration and additional implicit parameters in the imbedded stochastic process for the stock stochastic volatility. At any point in time the parameters V and σ_{vT2d} are the same across *all* options, while the parameters V* and σ_{vT1i} are the same only across options of the same expiration.[10]

There are other ways to implement the Geske model which we discuss in a footnote below. We have used these alternate implementation methods and they do not change our results at all. [11] Because the Geske model constructed using a portfolio of three securities, the option, the firm, and a bond, that is both risk neutral and self-financing, the stock does not enter the argument. The firm value critical for exercise, V*, must be constrained so that the equity component of V* is set equal to the know strike price K (K=S*). In the next section we describe the sources and data necessary implement BS and G with a term structure of volatility and to test for the presence of any leverage effects in individual stock call option prices.

3. Data Collection and Variable Construction

3.1. Option Data

The Ivy DB OptionMetrics has the Security file, the Security_Price file and the Option_Price file. The OptionMetrics data was collected in June 2007. It contains option

[10] Here, BS has 4 parameters (not 1 as in BCC (1997) and G has 6 unconstrained parameters across options. The maturity of the debt is also an important variable that differs across firms and contributes to the impact of leverage which has much stronger effects on long dated options.

[11] First, we could omit the use of the Merton equation (3) and use another option equation. Then we must use the longest expiring option implied volatility as the volatility to the debt maturity. We could also minimize the sum of squared option errors as the method of solving for the parameters while still using the MATM term structure of volatility to price all the options in a given expiration bucket. The term structure buckets can be varied both in number and in time intervals.

data from January, 1996 through December, 2005. This 120-month sample period covering 10 years has about 2500 observation days.

From the Security file, we obtain Security ID (The Security ID for the underlying securities. Security ID's are unique over the security's lifetime and are not recycled. The Security ID is the primary key for all data contained in Ivy DB.), CUSIP (The security's current CUSIP number), Index Flag (A flag indicating whether the security is an index. Equal to '0' if the security is an individual stock, and '1' if the security is an index.), Exchange Designator (A field indicating the current primary exchange for the security: 00000 - Currently delisted, 00001 - NYSE, 00002 - AMEX, 00004 - NASDAQ National Markets System, 00008 - NASDAQ Small Cap, 00016 - OTC Bulletin Board, 32768 - The security is an index.). We choose all the securities that are equities and we exclude all indices. An exchange-traded stock option in the United States is an American-style option. We further select the securities that are actively traded on the major exchanges. Now we have a sample of 11,539 securities whose stock options are American-style options.

From the Security_Price file, we obtain Security ID, Date (The date for this price record) and Close Price (If this field is positive, then it is the closing price for the security on this date. If it is negative, then there was no trading on this date, it is the average of the closing bid and ask prices for the security on this date.). We select the security price records when there are definitely trades on the dates.

From the Option_Price file, we obtain Security ID, Date (The date of this price), Strike Price (The strike price of the option times 1000), Expiration Date (The expiration date of the option), Call/Put Flag (C-Call, P-Put), Best Bid (The best, or highest, closing bid price across all exchanges on which the option trades.), Best Offer (The best, or lowest, closing ask price across all exchanges on which the option trades.), Last Trade Date (The date on which the option last traded), Volume (The total volume for the option), and Open Interest (The open interest for the option).

We merge the selected datasets from the Option_Price file and the Security_Price file, and we further merge the newly generated dataset with the selected dataset from the Security File. We keep all the options on the securities that are present in both files. In order to minimize non-synchronous problems, we keep the options whose last trade date is the same as the record date and whose option price date is the same as the security price date. Next we check to see if arbitrage bounds are violated ($C \leq S - K\, e^{-r_T T}$) and eliminate these option prices. If non-synchronicity occurred because the stock price moved up after the less liquid in or out of the money option last traded, then option under-pricing would be observed, and some of these options would be removed by the above arbitrage check. If non-synchronicity occurred because the stock price moved down after the less liquid in or out of the money option last traded, then option over-pricing would be observed. Because we cannot perfectly eliminate non-synchronous pricing for the in and out of the money options with this data base we keep track of the amount of under and over-pricing in order to relate this miss-pricing to the resultant under (over) pricing of in (out of) the money individual stock call options.

3.2. Dividends

The dividend information is obtained from CRSP. From CRSP, we collect the following dividend information: CUSIP, Closing Price (to cross check with the security price from OptionMetrics), Declaration Date (the date on which the board of directors declares a distribution), Record Date (on which the stockholder must be registered as holder of record on the stock transfer records of the company in order to receive a particular distribution directly from the company) and Payment Date (the date upon which dividend checks are mailed or other distributions are made).

A dividend paid during the option's life reduces the stock prices at the ex-dividend instant and reduces the probability that the stock price will exceed the exercise price at the option's expiration. Because of the insurance reason and time value of the money, it is never optimal to exercise an American call option on a non-dividend-paying stock before the expiration date. Therefore, we use the collected dividend information to restrict the sample to be all the eligible call options on stocks with no dividend prior to the option expiration.

Thus, all the stocks in the sample can be separated into two groups: the first group of stock never pays any dividend between January 4, 1996 and December 30, 2005; the second group of stock pays dividends in that period at least once. For the first group of stock, we use all the options written on these stocks in the whole sample period; for the second group of stocks, we use all the options whose expiration dates are before the first ex-dividend date and all the options whose expiration dates are after the previous ex-dividend dates and before the next ex-dividend dates. There are typically four days between the ex-dividend day the record date for the individual stocks in U.S. As we cannot obtain the ex-dividend dates directly from CRSP but we can obtain the record date from CRSP, we assume that the ex-dividend date occurs 4 trading days prior to the record date to get the ex-dividend dates. For the options on the second group of stocks, the options selected are not subject to dividend payment and can be taken as the American call option on non-dividend-paying stocks; the underlying security prices are the daily closing prices of the securities and we do not need to take into account of dividends.

3.3. Balance Sheet Information

From the COMPUSTAT Annual database (collected as of June 10, 2007), from year 1996 to 2005, by CNUM (CUSIP Issuer Code), there are 95,769 single firm-year observations and 293 duplicate firm-year observations due to mergers. These duplicate firm-year observations have different values for each data item because they are different firms before the merger and acquisition. CNUM (CUSIP) is the only way to merge the COMPUSTAT database with IVY OptionMetrics. If firms are duplicates on CNUM, we cannot differentiate two (or more) firms by CNUM, we am not able to know which options belong to which firms. Therefore, we excluded those 293 duplicate records from the COMPUSTAT sample and the options written on these firms from the IVY OptionMetrics data sample. The 95,769 single firm-year observations from COMPUSTAT is composed of the following records: 1996: 10,604; 1997: 10,328; 1998: 10,654, 1999: 10,685, 2000: 10,221, 2001: 9,645, 2002: 9,192, 2003: 8,899, 2004: 8,411, 2005:7130.

The balance sheet information we collect from COMPUSTAT is the book debt outstanding. The debt to be matured in one year is defined as the sum of debt due in one year (Data 44: not included in current liabilities Data 5), the current liabilities (Data 5), the accrued expense (Data 153), the deferred charges (Data 152), the deferred federal tax (Data 269), the deferred foreign tax (Data 270), the deferred state tax (Data 271) and the notes payable (Data 206). The debt of maturity of the 2nd years is Data 91. The debt maturing in the 3rd year is the total of the reported debt maturing in the 3rd year (Data 92) and the capitalized lease obligation (Data 84). The debt of maturity of the 4th years is Data 93. The debt to be matured in the 5th year is the total sum of the reported debt maturing in the 5th year (Data 92), the consolidated subsidiary (Data 329), the debt of finance subsidiary (Data 328), the mortgage debt and other secured debt (Data 241), the notes debt (Data 81), the other liabilities (Data 75) and the minority interest (Data 38). The debt categorized to be due in the 7th is either zero or the total of debentures (Data 82), the contingent liabilities (Data 327), the amount of long-term debt on which the interest rate fluctuates with the prime interest rate at year end (Data 148), and all the reported debt with maturity longer than 5 years (Data 9 - Data 91 - Data 92 - Data 93 - Data 94).[12] In addition, we delete firms whose convertible debt is (Data 79) more than 3% of total assets (Data 6) and/or finance subsidiary (Data 328) is 5% of total assets. Among all these annual data items, Data 5, 75 and 9 are updated quarterly from the COMPUSTAT quarterly data file as Data 49 (Q), 54 (Q) and 51 (Q). This structure of debt outstanding permits the computation of the daily duration of the corporate debt and the daily amount due at the duration date.

In order to make sure that the key debt information is not missing from the COMPUSTAT data, we check Data 44, Data 9, Data 91 to Data 94. If all of the six data items are missing, then we do not include this company's record. If only some of the data items are missing while others have positive values, then we set the missing items as zero and keep this company's record. For the other data items besides the above six ones, if they are missing, we set them as zero. We also need to make sure that Data 25 (Common Shares Outstanding) is not missing, as the market leverage will be calculated on a per share basis. We exclude all utility firms (DNUM=49), financial and non-profit firms (DNUM60).

3.4. Interest Rate and Discount Rate

Estimating the present value of debt and duration requires estimates of the riskless interest rates and the discount rates. The riskless rate and discount rate appropriate to each option were estimated by interpolating the effective market yields of the two Treasury Bills of U.S. Treasury securities at 6-month, 1-, 2-, 3-, 5-, 7- and 10-year constant maturity from the Federal Reserve for government securities. The interest rate for a particular maturity is computed by linearly interpolating between the two continuous rates whose maturities straddle.

[12]The mean duration of issued US corporate debt was 7 years (1982–1993). See Guedes and Opler (1996).

3.5. Characteristics of the Final Sample

We divide the option data into several categories according to either term to option expiration or moneyness. Five ranges of time to expiration are classified:

1. Very near term (21 to 40 days)
2. Near term (41 to 60 days)
3. Middle term (61 to 110 days)
4. Far term (111 to 170 days)
5. Very far term (171 to 365 days)

Options with less than 21 days to expiration and more than 365 days to expiration were omitted.[13] The five ranges of option maturity classification are set such that the numbers of each category are relatively even.

The ratio of the strike price to the current stock price is defined as the moneyness measure. The option contract can then be classified into seven moneyness ranges:

1. Very deep in-the-money (0.40 to 0.75)
2. Deep in-the-money (0.75 to 0.85)
3. In-the-money (ITM) (0.85 to 0.95)
4. At-the-money (ATM) (0.95 to 1.05)
5. Out-of-the-money (OTM) (1.05 to 1.15)
6. Deep out-of-the-money (1.15 to 1.25)
7. Very deep out-of-the-money (1.25 to 2.50)

We omit options with a ratio less than 0.40 or larger than 2.5 because their light trading frequency and thus possible non-synchronicity of trading. The coverage of the term to expiration and moneyness is the largest in all the literature on individual stock options.

[13] Rubinstein (1985) also used this practice.

After the dividend restrictions, the final sample is composed of nearly 3.5 million eligible individual stock call options on 1,683 firms.

Table 1 describes the sample properties of the eligible individual stock call option prices. we report summary statistics for the average bid-ask mid-point price, the average effective bid-ask spread (i.e., the ask price minus the bid-ask midpoint), the average trading volume and the total number of options, for all categories partitioned by moneyness and term of expiration. Note that there are a total of 3,487,894 call option observations. ITM consists 26.5% of the sample; ATM takes up 27.8% of the total sample and OTM consists 45.7% of the sample. There are almost twice as many OTM as ITM or ATM individual stock call options. The very near term ATM has the largest number per category (272,856).

With the longer term to expiration, the average call option prices in all moneyness categories increase monotonically. With the larger ratio of K/S, the average call option prices in all terms of expiration categories decrease monotonically. The most expensive average option price is in the category of the very deep in-the-money and the very far expiration term options. The least expensive average option prices are from the deep and very deep out-of-the-money options and of the very near terms of expiration. Very deep in-the-money options (0.40 <= K/S<0.75) are the most expensive with the average price across all terms to expiration around $17.11 while very deep out-of-the-money (1.25 <= K/S<2.50) are the least expensive with the average price across all terms to expiration around $0.25. The average price of ATM options is $3.45.

The average effective bid-ask spreads also decrease monotonically with the increase of from $0.22 to $0.08. The average effective bid-ask spreads are about $0.12 for all the terms of expiration. In fact, they do not vary too much across terms to expiration given any level of moneyness.

The very near term ATM options have the highest average trading volume 253.01 in contracts (on 100 shares). Across all terms to expiration, the ATM options have the average trading volume 150.61. ITM options' average trading volumes are from 31.28 to 80.00 and OTM options' average trading volumes are from 68.89 to 132.47. The deeper the moneyness and the further the expiration terms are, the less the average trading volumes of the options are, which has been reported by the previous papers.

Table 2 describes the distribution of options in each moneyness and term to expiration category for each year covered by the sample. From 1996 to 2003, the average number of options is around 320,000 per year. In 2004 and 2005, the average number is 450,000 per year. At the money options contain almost 30% the total options. The numbers of options decrease with respect to time to expiration and moneyness. This table also shows that in each category, we have sufficient amount for data to draw statistical conclusions.

3.6. Final Inputs and Implementation

As previously mentioned, we want to implement the models in a way that i) does not give a model more parameters and thus an unfair advantage, and is ii) consistent with the data. These two implementation goals are related. First, without a term structure of volatility, BS must fit all available options on any specific day for each strike price K_J and time to expiration T_{1i}, with only 1 parameter estimate, the MATM equity implied volatility, σ_S.[14] Similarly, G must fit all the available option prices with two parameter estimates, V and σ_V.

[14] See Appendix II, Table 3 from BCC where BCC dominates BS.

However, the more complex BCC models can fit these prices with either 9 parameter estimates for BCC's best model, SVJ, 8 parameters for SVSI, and 5 parameters for SV. It would be a surprise if a model with far fewer parameters could compete with a model with many more parameters. It is not possible to implement BCC (or Pan) with a volatility term structure, but if it were possible BCC's models would have an even greater parametric advantage. Second, if the model is intended to value options of different maturities and the implementation is to be consistent with the data, then the option data and literature is unambiguous on the importance of a term structure of volatility. Pan (2002) suggests that "to accommodate a richer term structure of volatility, one solution is to allow for multiple volatility factors", which she (and many others, Duffie (2000)) have argued is necessary "if one is trying to price both short and long dated options". [15]

Thus, on each day we estimate only four volatilities to accommodate the term structure of volatility, using the MATM options with expiration closest to 25 days, 50 days, 100 days, and 160 days. Since index options expire monthly on the third Friday of each month, this generally means the term structure of volatility will be constructed from options that have one month, two months, four months, and six months to expiration. In the matched pair comparisons of the models each valuing the same matched options, we analyze the pricing errors by grouping all the options into the previously mentioned time buckets. We use the four volatilities estimated for this term structure for all the options in each relevant group, and for the two longest expiration time buckets we use the same volatility parameter. [16]

BCC choose to model the underlying equity distributional complexity by adding additional

[15] The numbers of articles are numerous and growing which show that both option price and volatility data suggest the importance of a term structure of volatility. See Pan (2002), p. 32, especially footnote 29, for more references and details regarding the necessity of a term structure of volatility when pricing both short and long dated options. The volatility term structure, like other term structures (c.f. interest rates or default probabilities), contains important information. A term structure of model implied volatility is consistent with the market belief that the relevant risk is different for future time periods of exposure to different option expirations. While this idea is quite intrinsic, the notion that risk exposure is different for options on the same underlying and same expiration but different strikes is neither intuitive nor consistent.

[16] The grouping follows Rubinstein (1985). Longer expiration options have less daily volume across strikes.

stochastic processes, which increases the data requirements to test their model. For example, BCC's stochastic equity volatility process requires four additional parameters, while G obtains an implied stochastic equity process implicitly from the economics of leverage without any additional stochastic equity volatility process parameters.[17] Thus, G's model performs well with fewer explicit parameters because of the implicit ones.

Now the importance of including BS in these model comparisons becomes more evident, because this allows us to conclude that it is leverage and not only the term structure of volatility that is important for the model differences. We will see this clearly when we examine the model errors relative to the market prices for matched pairs of options for these models together on the same graphs and in tables. Any observed differences between BS and G cannot be attributed to a term structure of volatility since both BS and G models have the exact same implementation, both using the same term structure buckets for volatility. Thus, the observed differences between BS and G must be attributed to leverage, because if there is no leverage the two models would be identical. In the same graphs, the comparison of BCC and BS shows that BCC is closer on average to the market prices, even though BS is using a volatility term structure. So when leverage is added to the BS model already using a volatility term structure, and then BS becomes the G model, we now observe that BS with leverage (i.e. G) is on average much closer to the market prices than BCC. Thus, it cannot be the term structure of volatility that causes the BS model to improve when leverage is added, and then BS becomes the G model and improves relative to BCC. Instead this improvement must be due to the addition of leverage.

[17] In Section 4's discussion of the reasons for G's improvements with so few additional parameters, recall that G has an implied stochastic equity process with stochastic equity volatility, negative volatility-return correlation, and the equity distribution has a fat left tail and thin right tail indicative of asymmetry (skew) and different kurtosis [more (less) mass left tail (right tail)] relative to the normal distribution, and the tail mass differentials change as the leverage changes. Thus, for a given equity value, E, the probability of being above or below any give strike price is different for G relative to BS, and as E changes, for a fixed F, the leverage changes, and the volatility and shape of G's implied equity distribution changes.

As previously mentioned, the three versions of BCC models, SV, SVSI, and SVJ, and the four versions of Pan models, SV, SV0, SVJ0 and SVJ, have many additional parameters to be estimated for the stochastic processes assumed. To estimate these additional parameters it is necessary for BCC to use most of the options present on each day in order to find volatility that day that minimizes the sum of squared errors across all those options. Thus, in order for BCC's parameter estimates to remain "out of sample", researchers typically estimate the required parameters from prices lagged one day, and then use the parameter estimates to price options the next day. To estimate all the parameters for Pan's model, one option per day is chosen for all the days in the sample and all options are pooled as one single set. The option series is combined with a daily stock return set to set up the optimal moment conditions of return and volatility. The daily volatilities are implied from the daily options chosen. Pan specifically mentioned that by using her method, the complexity of a time dependency in the option-implied volatility due to moneyness and expiration is compromised. To compare Geske's model with BCC and Pan's models, we implement Geske's model using the MATM term structure of volatility, we follow the BCC's estimation technique by minimizing the sum of squared errors as described in Section 5, and we follow Pan's estimation technique by using implied state–GMM as described in Section 6. Given the data and estimates described, we can now examine what improvement, if any, Geske's leverage based option model may provide.

4. Comparison with the Black-Scholes Models

In this section, we start with Black-Scholes and present more details about the model comparison methodology, graphs of the model errors with respect to the option's time to expiration and moneyness. Also presented are tables illustrating both the statistical and economic significance of the Black-Scholes errors and Geske's improvements with respect to moneyness and time to expiration by calendar year and by leverage.

4.1. Model Pricing Error Comparison

Figure 1 presents a graph of individual stock call option market prices, Black-Scholes model values, and moneyness, *K/S*, which is representative of most research findings for the individual stock call options.

Black-Scholes model under values most in the money call options (low *K*) and overprices most out of the money call options (high *K*) on the individual stock. Since the individual stock level, *S*, is the same for all at any point in time during or at the end of any day, as varies in Figure 1, ITM individual stock call options (low *K*) are shown to be under-valued and OTM individual stock call options (high *K*) are shown to be over-valued by the Black-Scholes model relative to the market prices.[18]

Figure 1 shows that Geske's compound option model has the potential to improve or even eliminate these Black-Scholes valuation errors because of the leverage effect. Leverage creates a negative correlation between the individual stock level and the individual stock volatility. This interaction between the individual stock level and individual stock volatility implies that the individual stock volatility is both stochastic and inversely related to the level of the individual stock, and that the resultant implied individual stock return distribution will have a fatter left tail and a thinner right tail than the Black-Scholes assumption of a normal return distribution. Thus, Geske's compound option model produces option values that are greater (less) than the Black-Scholes's values for in (out of) the money European individual stock call options, and could potentially eliminate the

[18]Figure 1 presents the most ubiquitous result. There are 15 different model distance comparisons: both over market, both under, one over while the other is under, one equal to the market while the other is either over or under, both equal to each other but either over or under, both equal to each other and equal to the market, and there are multiple cases for each situation when the models are not equal to each other.

known Black-Scholes bias.

Figure 1 presents how we measure the amount of improvement Geske's model provides for stock individual stock call options during this sample period. For each option, we calculate the compound model value and the Black-Scholes model value. The improvement of Geske's compound option model compared to the Black-Scholes is calculated with the following formula:[19]

$$\frac{\text{BS error} - \text{CO error}}{\text{BS error}} = \frac{(\text{Market - BS}) - (\text{Market - CO})}{(\text{Market - BS})} \qquad (7)$$

We present this analysis for all matched pairs of options for a variety of categories with different times to expiration, different moneyness, and for the different market leverage exhibited during the sample time period. This is the first paper to report on Geske's compound option model and its potential to correct these errors when used to price individual stock call options.

4.2. Error Significance by Year, Leverage, Expiration and Moneyness

In the following tables, we present a more detailed analysis of the above results relating these ITM and OTM Black-Scholes pricing errors and Geske's improvements to the option's time to expiration by calendar year and by leverage. We also present the number of options available in these categories during this time period, and examine both the statistical and economic significance of Geske's model relative to Black-Scholes. The ATM option region is considered to be within 5% of the individual stock price.

Consider the number of matched pairs of traded ITM call options presented in Table 3 Panel A. Year 1999, 2000, 2004 and 2005 contain 451,100 out of 923,353 total options,

[19] Care must be taken with the sign of the variety of matched pair errors, especially if one model value distance is above and the other distance is below the market, when computing the average error across all matched pairs. However, the result depicted in Figures 1 is found for the vast number of all options.

which is about 50%. As expected, the table shows that ITM very near term to expiration category is traded more heavily than the far expiration ones in every year. The very near term to expiration category (21-40 days) contains 223,509 of the 923,353 total options, about 24%.

Table 3 Panel B presents the net pricing error improvement of Geske's model relative to Black-Scholes by calendar year for the various times to expiration for all ITM individual stock option matched pairs. The improvement of Geske's model with respect to time to expiration varies on average across all leverages from 14% for shortest expirations to 47% for longest expirations, and is strictly monotonic across all years. The leverage effect is greater the longer the leverage has to act on the option.

Next, consider the number of matched pairs of traded ITM call options presented in Table 4. Panel A presents the ITM individual stock call options by time to expiration and by debt/equity (D/E) ratio.[20] The D/E ratio during this time period ranges between 0% and 200%. Panel A shows that about 50% of this sample of ITM options traded when the D/E ratio ranged from 30% to 200%. Each option expiration category has at least 20% of the total options.

Panel B presents the net pricing error improvement of Geske's model relative to Black-Scholes by D/E ratio for the various times to expiration for all ITM call individual stock option matched pairs during this sample period. As in Table 3 Panel B, the improvement of Geske's model with respect to time to expiration varies from 14% for shortest expirations to 47% for longest expirations, and is strictly monotonic across all

[20] Note leverage is D/E, not D/V, and the model numbers are consistent with empirical data. Also note that D is a pure discount bond and can never exceed the face value M. We confirmed this empirically.

ranges of leverage. Relative to Black-Scholes, the improvement of Geske's model's increases with the D/E ratio monotonically across time to expiration. From the lowest D/E category to the highest D/E category, the improvement increase from an low of 4% for the shortest expiration lowest leverage category to a high of 96% for the longest expiration highest leverage category.[21]

Table 5 presents similar data to Table 3 for out of the money (OTM) individual stock call options. First consider the number of traded individual stock calls presented in Table V Panel A for OTM options. Panel A shows the most active trading years for OTM individual stock options during the sample period are 2000, 2001, 2004 and 2005. Each option expiration category has about 20% of the total options.

Table 5 Panel B demonstrates that Geske's compound option model's pricing error improvement for each year. Almost monotonically for every time to expiration, the improvement of Geske's model with respect to time to expiration varies from 49% for near term expirations to 65% for longest expirations, and is strictly monotonic across all years and ranges of leverage. Year 1996, 1997, 1999, 2000 and 2005 exhibit more than 70% pricing error improvement and the smallest yet substantial improvement around 30% happen in the year 2002 and 2003. Similar patterns also can be found in the Table 3 and 4's Panel Bs for ITM individual stock options.

Table 6 presents similar data to Table 4 for OTM individual stock call options. First consider the number of traded individual stock calls presented in Table 6 Panel A for OTM options. Panel A shows that about 50% of this sample of OTM options traded when the

[21] Thus, as the D/E ratio declines, G becomes similar to BS, especially for near term options. Also ratings and D/E ratios are not perfect substitutes since low (high) rated firms can have a low (high) D/E ratio.

D/E ratio ranged from 30% to 200%. 22% of options have D/E ratios from 30% to 60%, and 20% of options have D/E ratios higher than 60%. Each option expiration category has about 20% of the total options.

Table 6 Panel B demonstrates that Geske's compound option model's improvement also increases with the D/E ratio, almost monotonically for every time to expiration, the improvement of Geske's model with respect to time to expiration varies from 49% for shortest expirations to 65% for longest expirations, and is strictly monotonic across all years and ranges of leverage. Relative to Black-Scholes the improvement of Geske's model's increases with the D/E ratio almost monotonically for every time to expiration from 20% to 83%.

4.3. Alternative Testing

We also tried a different volatility methodology of basing the aggregate net pricing errors and improvement of Geske's model compared to Black-Scholes on the volatility that minimizes the sum of squared errors. We find that this does not change the characteristics of the results. This result is not surprising because finding the volatility that minimizes the sum of squared errors moves the "pricing volatility" away from ATM toward either the ITM or OTM and this will exhibit a more than off-setting effect.[22]

4.4. Statistical Significance

Here we use non-parametric statistics to test the significance of the differences between Black-Scholes and Geske's model. As can be seen in Table 3 and 4 Panel C for ITM

[22] This is discussed in more detail in Appendix I.

options and Table 5 and 6 Panel C for OTM options, we find Geske's model improvements are all significant at -value smaller than the 0.001% by rank-sum test.

The rank-sum test (also called Wilcoxon test or Mann-Whitney test) is a nonparametric or distribution-free test which does not require any specific distributional assumptions. We first list all observations from both samples in a increasing order, label them with the group number, create a new variable called "rank". For ties, we give them the same rank. Then we sum up the ranks for each group. The sum of the ranks is called *T*.

The test statistics is $Z - statistic = [T - Mean(T)]/SD(T)$, Where T : the sum of the ranks, Mean (T): n times the mean of the whole (combined) sample, SD(T): the standard deviation of Mean (T). A p-value is the proportion of values from a standard normal distribution that are more extreme than the observed Z -statistic. The p-values which are all 0 lead us to conclude that there is significant difference between Black-Scholes and Geske's model.

We also did other non-parametric tests: signed rank test, sign test and Kruskal-Wallis test (for two independent samples, i.e. Mann-Whitney U Test). All of them yield the same results that Geske's model improvements are all significant at p -value smaller than the 0.001% for all terms to expiration and calendar years and leverage ratios.

5. Comparison with Bakshi, Cao and Chen (1997)

In this section, we present more details about the model comparison methodology, graphs of the model errors with respect to the option's time to expiration and moneyness. Also included are tables of the statistical and economic significance of the Bakshi, Cao and Chen's SV, SVSI and SVJ errors and Geske's improvements with respect to moneyness and time to expiration by calendar year and by leverage.

5.1. BCC Description and Structural Parameter Characteristics

To conduct a comprehensive empirical study on the relative advantages of competing option pricing models, we further compare Geske's model with the three competing BCC models: the stochastic-volatility (SV) model, the stochastic-volatility and stochastic-interest-rate (SVSI) model, and the stochastic-volatility random-jump (SVJ) models (Bakshi, Cao and Chen (1997)). These models relax the log-normal stock return distributional assumptions and do correct some of the bias of the Black-Scholes model. The implicit stock return distribution is negatively skewed and leptokurtic.

To derive a close-form jump diffusion option pricing model, BCC specify a stochastic structure under a risk-neutral probability measure. Under this measure, the dynamics of stock return process, the volatility process and the interest rate process are:

$$\frac{dS(t)}{S(t)} = [R(t) - \lambda\mu_J]dt + \sqrt{V(t)}dw_S(t) + J(t)dq(t) \tag{8}$$

$$dV(t) = [\theta_v - \kappa_v V(t)]dt + \sigma_v \sqrt{V(t)}dw_v(t) \tag{9}$$

$$\ln[1 + J(t)] \sim N(\ln[1 + \mu_J]) - \frac{1}{2}\sigma_J^2, \sigma_J^2) \tag{10}$$

$$dR(t) = [\theta_R - \kappa_R R(t)]dt + \sigma_R \sqrt{R(t)}dw_R(t) \tag{11}$$

whereas $R(t)$ is the instantaneous spot interest rate; λ is the jump frequency per year; μ_J is the mean relative jump size; *V(t)* is the diffusion component of return variance (conditional on no jump occurring); $\omega_S(t), \omega_v(t)$ is standard Browning motion with correlation ρ; $q(t)$ is a Poisson jump counter with intensity λ; κ_v is the mean-reversion rate of the process; θ_v / κ_v is the long-run mean of the *V(t)* process; σ_v is the variation coefficient of the diffusion volatility *V(t)*; $J(t)$ is the percentage jump size (conditional on a jump occurring) that is the *iid* distributed with mean μ_J and variance σ_J^2; σ_J is the standard deviation of $\ln[1+J(t)]$; κ_R is the mean-reversion rate of the *R(t)* process; θ_R/κ_R is the long-run mean of the *R(t)* process; σ_R is the variation coefficient of the *R(t)* process.

Under the risk-neutral measure, the option price is a function of the risk-neutral probabilities recovered from inverting the respective characteristic functions. For detailed expression, please refer to Bakshi, Cao and Chen (1997).

The SV model is by setting λ=0 and $\theta_R=\kappa_R=\sigma_R$=0. The SVSI model is by setting λ=0. The SVJ model is by setting $\theta_R=\kappa_R=\sigma_R$=0. The SV model assumes that there exists a negative correlation between volatility and spot asset returns and the volatility follows a stochastic diffusion process. The negative correlation produces the skewness and the variation coefficient of the diffusion volatility controls the variance of the volatility–kurtosis. The SVJ model assumes that the discontinuous jumps causes negative skewness and high kurtosis. SVSI model assumes that the interest-rate term structure is stochastic to reduce the pricing error across option maturity. This is not related to the implicit stock return distribution, but to improve the valuation of future payoffs. All three models are implemented by backing out daily, the spot volatility and the structural parameters from the observed market option prices of each day.

In order to measure the latent structural parameters of the SV, the SVSI and the SVJ models, we adopt the Bakshi, Cao and Chen (1997)'s approach method of minimizing the sum of squared dollar pricing errors. We collect all the options for a firm in one day, for any option number greater or equal to one plus the number of parameters to be estimated.

For each option with a term to expiration and strike price, we calculate the model price. The difference between the model price and the market price is the dollar pricing error. Then we sum all the squared dollar pricing errors as the objective function to minimize to imply the latent structural parameters and the volatility. In implementing the above procedure, we first use all individual stock call options available for each firm on each given day, provided that the option number is greater or equal to the one plus the number of parameters to be estimated, regardless of maturity and moneyness, as inputs to estimate the latent structural parameters and the volatility.

Table 7 reports that daily average and the standard error of each latent parameter and volatility, respectively for the BS, and BCC's SV, SVSI and SVJ models. The first observation is that the implied spot volatility is quite different among the four models. The BS model has the highest implied volatility (55%), which is not so different from the second highest SV and SVSI implied volatilities (52%), while SVJ model has the lowest implied volatility (49%).

The second observation is that the estimated structural parameters for the spot volatility process differ across the SV, the SVSI, and the SVJ models. Note that all the three models have the similar estimated κ_v, the implied speed-of-adjustment θ_v, which is around 1.67. The SV, SVSI and SVJ models have estimates that are not significant, indicating the long-run mean of the diffusion volatility is ignorable. Recall that in the SV model, the skewness and kurtosis levels of stock returns are controlled by the correlation ρ and volatility variation coefficient σ_v. The variation coefficient σ_v is significant for all three models and is the highest for SV model, followed by SVSI model and the lowest for SVJ model. The magnitudes of correlation ρ are similar for all three models, around and significant. θ_R is significant for the SVSI models while the speed of adjustment of interest rate κ_R and the interest variation coefficient σ_R are not significant. For the SVJ models, none of the four parameters are significant: the jump frequency per year λ, the mean relative jump size μ_J, the standard deviation σ_J and the instantaneous variance of the jump components V_J. For the SV model, the variation coefficient σ_v and the correlation ρ seem to control the skewness and kurtosis levels of stock returns more strongly. For the SVSI model, the variation coefficient σ_v and the correlation ρ seem to control the skewness and kurtosis levels of stock returns, along with the additional flexibility provided by θ_R. For individual stock returns, the SVJ model allowing price jumps to occur, should absorb more negative skewness and higher kurtosis without changing the stochastic volatility parameters too much. It is true that the stochastic volatility parameters do not change too much for the SVJ model, but the jump parameters' insignificance has led us to conjecture that for the individual stock option pricing, the SVJ model may not perform as well as a stochastic equity volatility model based on the leverage as the economic reason for the negative correlation between the volatility and the individual stock price.

5.2. Pricing Error Analysis of G vs. BS, and BCC's SV, SVSI and SVJ

In order to show that the implied implementation method is not the only reason for dominance of the leverage model, Table 8 and 9 report the out-of-sample absolute and relative pricing errors using BCC's technique. To generate the out-of-sample result, for a given model, we compute the price of each option using the previous day's implied parameters.

To be more specific, for the BS model, we use the one-day-lagged volatility to calculate current day's price. For the SV, SVSI and SVI models, we lagged the set of parameters by one day for each day of each firm, and we use this lagged set of parameters to calculate current day's model prices. For the G model, we lagged σ_v –the volatility of the return of the market firm value by one day. In order to calculate the model price, given the σ_v, we obtain current day's firm value *V* by solving the Merton's equation in which *S* is an option on the firm value. We also solve for through the boundary equation. Then we further use the set of *V*, V^* and σ_v to calculate today's model price. (See Appendix 1) [23]

Out-of-sample Geske's model has the lowest absolute pricing errors and the lowest relative pricing errors for most of the moneyness and terms-to-expiration categories, indicating the best fit. The second best is the SVJ model overall, and the SV and SVSI are similar in terms of the absolute pricing errors, but the SV model has lower relative pricing errors than those of the SVSI models. The BS model has the worst absolute and relative pricing errors, indicating that incorporating stochastic volatility does produce the most significant improvement over the BS model, lending validity of the stochastic models. Averaging the whole sample, the absolute pricing error for G is \$0.04, for SV is around \$1.00, for SVSI and SVJ is around \$1.50 and for BS is around \$1.30. For the whole sample average, the relative pricing error for G is around 0.4%, for SVJ is around 50% and for SV and SVSI are around 100%, and for BS is higher than 150%.

For options on individual stocks, both pricing error measures rank the G model first and it is far better than the rest of the models, the SVJ as the second, the SV and the SVSI the third and the BS model the last. The SV, SVSI and SVJ model price OTM individual stock call options far worse than ITM individual stock call options, but SVJ does surpass SV and SVSI in pricing OTM options.

[23] Notice here the BS and G model parameters are not implied as preferred but are calculated by minimizing sum of squared errors, to show that G's superiority is independent of implementation.

5.3. Graphs of Errors with respect to Time to Expiration

Figure 2/ 3 presents the absolute/relative pricing errors for all models of in-the-money individual stock options. The average is across all strike prices for the same time to expiration. Here G is shown to be superior to the BS, SV, SVSI and SVJ models. For the absolute pricing errors, G is always less than $0.50, SVJ and SVSI is from $1.00 to $2.00, SV is from $1.00 to $3.00 and BS is from more than $1.00 to as high as $5.00. For the relative pricing errors, G is always less than 0.05, SVJ and SVSI is from 0.10 to 0.20, SV is from 0.10 to 0.30 and BS is from more than 0.20 to as high as 0.50.

Figure 4/ 5 presents the absolute/relative pricing errors for all models of out-of-the-money individual stock options. The average is across all strike prices for the same time to expiration. G is again shown to be superior to the BS, SV, SVSI and SVJ models. For the absolute pricing errors, G is always less than $0.50, SVJ and SVSI is from $1.00 to $2.00, SV is from $1.00 to $3.00 and BS is from more than $1.00 to as high as $6.00. For the relative pricing errors, G is always less than 0.25 (25%), SVJ and SVSI is from 1.5 to 2, SV is from 2 to 2.5 and BS is from more than 3.25 to 4.

5.4. Graphs of Errors with respect to Moneyness

Figure 6/ 7 presents the absolute/relative pricing errors for all models of in-the-money individual stock options. The average is across all strike prices for the same moneyness. Here G is shown to be superior to the BS, SV, SVSI and SVJ models. For both absolute/relative pricing errors, G is closest to the market price and is far below the rest, indicating the best fit. SV, SVSI and SVJ cluster in the middle while BS's line is far above. Figure 8/ 9 presents the absolute/relative pricing errors for all models of out-of-the-money individual stock options. The average is across all strike prices for the same moneyness. G is again shown to be superior to the BS, SV, SVSI and SVJ models. Similar to in-the-money options, but in even more prominent ways, for both absolute/relative pricing errors, G is closest to the market price and is far below the rest, indicating the best fit. SV, SVSI and SVJ cluster in the middle while BS's line is far above.

5.5. Economic Significance of G Improvements Compared to BS, and BCC's SV, SVSI and SVJ models

In this section, we report the economic significance[24] of G's improvements for ITM in Table 10 and Table 11. We report the economic significance of G's improvements for OTM in Table 12 and Table 13. Tables 10 to Tables 13 show results when G's model is compared to BS, SV, SVSI and SVJ models on three dimensions: *i)* by the number of matched pairs that G is a closer absolute distance to the market price, *ii)* by the dollar value of this G's improvement, and *iii)* by the basis points (bp) that G's improvement implies for an option portfolio. These comparisons are categorized by both calendar year and by leverage.

First, consider Table 10 comparing G, BS, SV, SVSI and SVJ models for ITM options. The columns left to right represent the year, the present value of all ITM matched pairs for that year, the total number of the matched pairs that year, the number of those matched pairs where an alternative model price is closer to the market price in absolute distance, the number of matched pairs where the G model price is closer to the market price, the dollar value of the alternative model price improvement, the dollar value of G improvement, and the net basis point advantage of G's model for that year.

[24]Economic improvement (bp) herein is relative to the model not the market, and thus "beating the market" is not being tested. Furthermore, economic improvement is based on a portfolio of one of each option per day when the actual daily volume experienced by market makers (or dealers) is greater.

Table 11 presents the same information categorized by the D/E ratio instead of by year, where D/E ranges from 0-200%. The totals for each column and each row are also presented.

The total number of ITM matched pairs of options is presented in Table 10. Geske's model is closer to the market price than the Black-Scholes model for 340,208 of these ITM matched pairs and Black-Scholes is closer on 145,429 pairs. The G model is closer to the market price than the SV/SVSI model for 209,916/210,462 of these ITM matched pairs and SV/SVSI is closer on 41,037/40,492 pairs. The G model is closer to the market price than the SVJ model for 169,754 of these ITM matched pairs and SVJ is closer on 81,195 pairs. Notice that the total numbers are different for BS and for SV, SVSI and SVJ model prices. This is because the matched SV, SVSI and SVJ pairs are calculated from a set of options whose number is equal or greater than 9 because of the number of parameters to be estimated while the number of options to estimate BS model is equal or greater than 6. Thus the number of matched pairs of the BS model is larger the number of matched pairs of the SV, SVSI and SVJ models.

In the following we explain in more detail the computation of the dollar and basis point improvement. More specifically, dollar improvement for each model is measured by considering all those matched pairs where a specific model is closer to the market price than the alternative model in absolute distance measured in dollars. The basis point advantage of Geske's model is then computed by dividing the net dollar improvement for that year or leverage category by the total value of options in that category. For example, in Table 10, across the sample years 1996-2005 the Geske's compound option model has a total dollar improvement of $611,870.16 and Black-Scholes has a dollar improvement of $19,990.32. Thus, the net dollar improvement of Geske's model is $591,879.84, and that divided by the total value of each option in this ITM portfolio, $3,972,966.56, produces the 1490 net basis point improvement.

Table 10 shows that by G being closer to the market price than BS on 70% of the ITM option matched pairs results in a basis point (bp) net improvement of on average 1490 bp for ITM options in an one of each option portfolio of options. The bp improvement are 1153 for SV, 1044 for SVSI and 705 for SVJ models. These numbers are calculated by constructing a one of each option portfolio containing one option for each strike price and time to expiration for each day and finding the market value of that one of each option portfolio each day for all days in a year. The basis point and dollar value improvements would generally be much larger for professionals who do not hold a one of each option portfolio, but instead hold all options in multiple amounts based on each dealer's share of the daily volume. Each option at a specific strike price and time to expiration generally has a much larger volume of trading which professionals will capture.

In Table 11, while the percentage pricing error of G's improvement relative to BS is monotonic in leverage as demonstrated Table 4 and Table 6, basis point improvement need not be since this depends on the dollar value of the options.

Next, consider Table 12 comparing G, BS, SV, SVSI and SVJ models for OTM options. The columns left to right represent the year, the present value of all OTM matched pairs for that year, the total number of the matched pairs that year, the number of those matched pairs where an alternative model price is closer to the market price in absolute distance, the number of matched pairs where the G model price is closer to the market price, the dollar value of the alternative model price improvement, the dollar value of G improvement, and the net basis point advantage of G's model relative to all the other models for that year.

Table 13 presents the same information categorized by the D/E ratio instead of by year, where D/E ranges from 0-200%. The totals for each column and each row are also presented.

The total number of OTM matched pairs of options is shown in Table 12. Geske's model is closer to the market price than the Black-Scholes model for 540,870 (70%) of these OTM matched pairs and Black-Scholes is closer on 227,871 pairs. The G model is closer to the market price than the SV/SVSI model for 358,553/346,300 (90%) of these OTM matched pairs and SV/SVSI is closer on 41,156/53,411 pairs. The G model is closer to the market price than the SVJ model for 285,086 (70%) of these OTM matched pairs and SVJ is closer on 114,621 pairs.

The net dollar improvement of G's model is $1,257,142.33 - 43,403.70 = 1,213,738.63$, and that divided by the total value of each option in the OTM portfolios $1,240,907.82 produces a 9781 basis point improvement. Table 11 shows that by G being closer to the market price than BS in a basis point net improvement of on average 9781 bp for OTM options in a one of each option portfolio of options. The basis point improvement are 8504 for SV, 6573 for SVSI and 5470 for SVJ models.

In Table 13, while the percentage pricing error of G's improvement relative to BS is monotonic in leverage as demonstrated Table 4 and Table 6, basis point improvement need not be since this depends on the dollar value of the options.

In this section we have demonstrated the considerable economic improvement of G's model relative to the BS, SV, SVSI and SVJ models for pricing the individual stock options. We have shown that the data necessary to implement G model for valuing individual stock options are readily available. In the next section, we compare Geske to Pan's (2002) models.

6. Comparison with Pan (2002)

6.1. Pan Description and Structural Parameter Characteristics

Pan (2002) extends the models of Heston (1993) and Bates (2000) by estimating the volatility and jump risk premia imbedded in options. Pan (2002) examines the joint time series of the S&P 500 index and near-the-money short-dated option prices with an arbitrage-free model which prices all three risk factors, including the volatility risk and the jump risk. An important feature of the jump-risk premium considered in Pan's model as compared with BCC's model is that the jump-risk premium is allowed to depend on the market volatility: when the market is more volatile, the jump-risk premium is higher.

Under the physical measure P, the dynamics of (S,V,r,q) are of the form

$$dS_t = [r_t - q_t + \eta^s V_t + \lambda V_t(\mu - \mu^*)]S_t dt + \sqrt{V_t} S_t dW_t^{(1)} + dZ_t - \mu S_t \lambda V_t dt \quad (12)$$

$$dV_t = \kappa_v(\bar{v} - V_t)dt + \sigma_v \sqrt{V_t}(\rho dW_t^{(1)} + \sqrt{1-\rho^2}\, dW_t^{(2)}) \quad (13)$$

$$dr_t = \kappa_r(\bar{r} - r_t)dt + \sigma_r \sqrt{r_t}\, dW_t^{(r)} \quad (14)$$

$$dq_t = \kappa_q(\bar{q} - q_t)dt + \sigma_q \sqrt{q_t}\, dW_t^{(q)} \quad (15)$$

Under the risk-neutral measure Q, the dynamics of (S,V) under Q are of the form

$$dS_t^Q = [r_t - q_t]S_t dt + \sqrt{V_t} S_t dW_t^{(1)}(Q) + dZ_t^Q - \mu^* S_t \lambda V_t dt \quad (16)$$

$$dV_t^Q = \kappa_v(\bar{v} - V_t + \eta^v V_t)dt + \sigma_v \sqrt{V_t}(\rho dW_t^{(1)}(Q) + \sqrt{1-\rho^2}\, dW_t^{(2)}(Q)) \quad (17)$$

Under the risk-neutral measure, the option price is a function of the risk-neutral probabilities recovered from inverting the characteristic functions.

The notation is as the following: κ_v, κ_r and κ_q are the mean-reversion rates; $\bar{v}$, $\bar{r}$ and $\bar{q}$ are the constant long-run means; σ_v, σ_r and σ_q are the volatility coefficients; ρ is the correlation of the Brownian shocks to price S and volatility V; λ is the constant coefficient of the state-dependent stochastic jump intensity λV_t; μ is the mean jump size under the physical measure; η^s is the constant coefficient of the return risk premium; η^v is the constant coefficient of the volatility risk premium; μ^* is the mean jump size of the jump amplitudes U^S under the risk-neutral measure; σ_J is the variance of the jump amplitudes U^S under the risk-neutral measure; r is a stochastic interest-rate process; $W = [W^{(1)}, W^{(2)}]^T$ is an adapted standard Brownian motion in P; $W(Q) = [W^{(1)}(Q), W^{(2)}(Q)]^T$ is an adapted standard Brownian motion in Q; Z is a pure-jump process in P; $Z(Q)$ is a pure-jump process in Q; $W^{(r)}$ and $W^{(q)}$ are independent adapted standard Brownian motions in P, independent also of W and Z.

The no-risk premia SV0 model is obtained by setting $\lambda = 0$ and $\eta_v = 0$. The volatility-risk premia model SV is obtained by setting $\lambda = 0$. The jump-risk premia SVJ0 model is obtained by setting $\eta_v = 0$. SVJ denotes the volatility and jump risk premia model.

Using Pan (2002) 's notation, under the risk neutral probability measure Q, the jump arrival intensity is $\{ \lambda V_t : t \geq 0 \}$ for some non-negative constant λ and the jump amplitudes U_i^S is normally distributed with Q -mean μ_J^* and Q -variance σ_J^2. Conditional on a jump event, the risk-neutral mean relative jump size is $\mu^* = E^Q(exp(U^S) - 1) = exp(\mu_J^* + \sigma_J^2/2) - 1$. By allowing the risk-neutral mean relative jump size μ^* to be different from its data generating counterpart μ, Pan accommodates a premium for jump-size uncertainty. All jump risk premia will be artificially absorbed by the jump-size risk premium coefficient $\mu - \mu^*$. The time-t expected excess stock return compensating for the jump-size uncertainty is $\lambda V_t(\mu - \mu^*)$. The linear specification λV of jump-arrival intensity is to allow for a state-dependent jump-risk premium; when the market is more volatile, the jump-risk premium implicit in option prices becomes higher.

Because options are non-linear functions of the state variables (S,V), the joint dynamics of the market observables S_n and C_n are complicated. In order to take advantage of the analytical tractability of the state variables (S,V), Pan proposed an ``implied-state" generalized method of moments (IS-GMM) approach. For any given set of model parameters θ, a proxy V_n^θ for the unobserved volatility V_n can be obtained by inverting $C_n = S_n f(V_n^\theta, \theta)$. Given the parameter-dependent V_n^θ, according to Duffie, Pan and Singleton (2000), the affine structure of (lnS, V) provides us a closed-form solution for the joint conditional moment-generating function, from which we can calculate the joint conditional moments of the stock return and volatility up to any order. For example, in Pan (2002) [25], she uses seven moments: the first four conditional moments of return, the first two conditional moments of volatility and the first cross moments of return and volatility. These conditional moments are used to build moment conditions. In this paper, for each firm, we first imply the volatility by inverting $C_n = S_n f(V_n^\theta, \theta)$, then we construct the seven moment conditions as performed by Pan (2002) and use the standard GMM estimation procedure afterwards to estimate the parameters. Each firm has a unique set of parameters.

[25] For detailed information on how to implement the IS-GMM, please refer to Pan (2002) .

Following Pan (2002), for each day of each firm, we first sort the options by time to expiration τ_n. Among all available options, we select those with a time to expiration that is larger than 15 calender days and as close as possible to 30 calendar days.[26] From the pool of options with the chosen time to expiration, we select all options with a strike price K nearest to the stock price S of this firm on this day. If a day has multiple calls selected, then one of these calls will be chosen at random. The combined time series $\{S_n, C_n\}$ is synchronized. The sample mean of τ_n is 34 days, with a sample standard deviation of 14 days. The sample median of τ_n is 32 days. The sample mean of the strike-to-spot price $\frac{K}{S}$ ratio is 1.014, with a sample standard deviation of 0.08134. The sample median of $\frac{K}{S}$ is 1.010.

Given the selected near-the-money and short-dated options, for all four models, we adopt Pan's IS-GMM method and perform joint estimations of the actual and risk-neutral dynamics using the time series $\{S_n, C_n\}$ of the individual stock options. The estimation results are reported in Table 14. Similar to BCC's Table 7, the mean reversion rate κ_v is significant across all models, the constant long-run mean $\bar{v}$ is not significant except for SVJ0 and the volatility coefficient σ_v is significant. The correlation coefficient ρ is significant and it is almost the same as BCC's estimate, which is around -0.60. η^s is the constant coefficient of the return risk premium and η^v is the constant coefficient of the volatility risk premium. η^s is only significant for the no-risk premia model SV0. η^v is not significant for SV or SVJ models. And also similar to BCC, except the jump intensity

[26] If the closest time to expiration is longer than 90 days, then it is not included in the sample.

coefficient λ, the mean and the variance of the jump sizes are not significant. The similarity of both sets of parameters shows the limitation in the current jump process assumption for stock returns of individual stocks.

6.2 Pricing Error Analysis of G vs. Pan's SV0, SV, SVJ0, and SVJ

Given these estimated parameters and the implied daily volatility for each firm, we further solve for the model prices of SV0, SV, SVJ0 and SVJ models in Pan's paper. we compared these prices with the those computed using Geske's model(from the ATM calibration as in Section 4) to find about Geske's improvements over Pan's SV0, SV, SVJ0 and SVJ model prices with respect to moneyness and time to expiration. For both the absolute/relative pricing errors for all models of in-the-money and out-of-the-money individual stock options, G is significantly superior to the SV0, SV, SVJ0 and SVJ models.

6.3. Graphs of Errors with respect to Time to Expiration

Figure 10/ 11 presents the absolute/relative pricing errors for all models of in-the-money individual stock options. The average is across all strike prices for the same time to expiration. Here G is shown to be superior to Pan's SV0, SV, SVJ0 and SVJ models. For the absolute pricing errors, G is always less than \$0.50, SVJ is from \$0.50 to \$1.00, SV0 and SV is from \$0.50 to \$2.00 and SVJ0 is from more than \$1.30 to \$1.70. For the relative pricing errors, G is always less than 0.05, SVJ is from 0.10 to 0.30, SV0 and SV is from 0.06 to 0.30 and SVJ0 is from more than 0.18 to as high as 0.58.

Figure 12/ 13 presents the absolute/relative pricing errors for all models of out-of-the-money individual stock options. The average is across all strike prices for the same time to expiration. G is again shown to be superior to Pan's SV0, SV, SVJ0 and SVJ models. For the absolute pricing errors, G is always less than \$0.50, SV0 and SV is from

$0.60 to $4.00, SVJ is from $1.80 to $4.50 and SVJ0 is from more than $2.80 to as high as $4.80. For the relative pricing errors, G is always less than 0.25 (25%), SV0 and SV is from 0.25 to 0.85, SVJ0 and SVJ is from 1.00 to 2.00.

6.3. Graphs of Errors with respect to Moneyness

Figure 14/ 15 presents the absolute/relative pricing errors for all models of in-the-money individual stock options. The average is across all strike prices for the same moneyness. Here G is shown to be superior to the SV0, SV, SVJ0 and SVJ models. For both absolute/relative pricing errors, G is closest to the market price and is far below the rest, indicating the best fit.

Figure 16/ 17 presents the absolute/relative pricing errors for all models of out-of-the-money individual stock options. The average is across all strike prices for the same moneyness. G is again shown to be superior to the SV0, SV, SVJ0 and SVJ models. Similar to in-the-money options, but in even more prominent ways, for both absolute/relative pricing errors, G is closest to the market price and is far below the rest, indicating the best fit.

6.4. Summary

In this section, we have demonstrated that Geske's G model is also superior to Pan (2002)'s SV0, SV, SVJ0 and SVJ models. we again show that existing market leverage is both statistically and economically important to pricing the individual stock options. Therefore it is paramount to separate the economic effects of stochastic leverage and its induced stochastic volatility from any other assumed stochastic effects. Leverage is always present in the market and leverage has now been shown to be important to pricing individual stock options. Thus, if leverage is not properly treated prior to modeling other assumed stochastic effects, then the estimated parameters will be inaccurate because of the omitted variable.

7. Conclusions

In this paper, we present the first empirical evidence that Geske's compound option model can be used to *imply the market value* of the firm and thus the firm's debt. We show that this can be accomplished simply and parsimoniously with contemporaneous, liquid market prices for the equity and an options on the equity. We demonstrate with a very large sample (ten years with over 11,500 firms and over 2.5 million options) that Geske's model prices individual stock options better than the Black-Scholes (1973), Bakshi, Cao, and Chen (1997), or Pan (2002) models. Geske's model takes the theory of option pricing deeper into the theory of the firm by incorporating the effects of leverage consistent with Modigliani and Miller. Geske's model imbeds a stochastic process for the stock which characterizes how debt causes the individual stock risk to change stochastically and inversely with the equity price level. This paper demonstrates that this improvement is both statistically and economically significant for all strikes and all times to expiration. This paper also shows, as expected, that the improvements are greater the longer the time to expiration of the option, and the greater the market leverage in each firm. Finally, we show that while G's model is more parsimonious than the other competing option models which omit leverage, but incorporate many more parameters for stochastic processes for volatility, interest rates and jumps. We also have implemented G using a method similar to BCC by finding the volatility in each term structure bucket that minimizes the sum of squared errors of out of sample option prices from the previous day to demonstrate that G's performance is independent of using only contemporaneous ATM options. G's performance is better for both in and out-of-sample pricing, and avoids the criticisms of Ericsson and Reneby (2005). However, we would be remiss in not noting that after including leverage there is still room for improvement, and perhaps by also incorporating jumps or stochastic volatility at the firm level would result in an even better model.

References

1. Bakshi, G., C. Cao, and Z. Chen, 1997, Empirical performance of alternative option pricing models, Journal of Finance 52, 2003–2049.

2. Bates, D. S., 2000, Post-'87 crash fears in the S&P 500 futures options market, Journal of Econometrics 94, 181–238.

3. Black, E., 1976, Studies of stock price volatility changes, Proceedings of the 1976 Meetings of the Business and Economics Section.

4. Black, E., and M. Scholes, 1973, The pricing of options and corporate liabilities, Journal of Political Economy 81, 637–659.

5. Christie, A. A., "The stochastic behavior of common stock variances: Value, leverage and interest rate effects," Journal of Financial Economics, 1982, 10, 407-432.

6. Duan, J. C., "Maximum Likelihood Estimation Using Price Data of the Derivative Contract," Mathematical Finance, 4, 155-167.

7. Ericsson, J., and Reneby, J., "Estimating Structural Bond Pricing Models," The Journal of Business, 2005, 78, 2, 707-735.

8. Eraker, B., M. Johannes, and N. Polson, 2003, The Impact of Jumps in Volatility and Returns, Journal of Finance 58, 1269–1300.

9. Geske, Robert, 1979, The valuation of compound options, Journal of Financial Economics 7, 63–81.

10. Geske, Robert, and Yi Zhou, 2006, "A New Methodology For Measuring and Using the *Implied Market Value* of Aggregate Corporate Debt in Asset Pricing: Evidence from S&P 500 Index Put Option Prices.", UCLA Working Paper.

11. Geske, Robert, and Yi Zhou, 2007, "Predicting the Volatility of the S&P 500: Evidence from Index Options," UCLA Working Paper.

12. Guedes, J., and T. Opler, 1996, The Determinants of the Maturity of Corporate Debt Issues, Journal of Finance 51, 1809–1833.

13. Heston, S., 1993, A closed-form solution for options with stochastic volatility with applications to bond and currency options, Review of Financial Studies 6, 327–343.

14. Heston, S., and S. Nandi, 2000, A Closed-Form GARCH Option Valuation Model, Review of Financial Studies 13, 585–625.

15. Leland, H., Corporate debt value, bond covenants, and optimal capital structure, Journal of Finance, 1994, 49, 1213-1252.

16. Leland, H., and Toft, K., “Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads”, Journal of Finance, 51, 987-1019.

17. Liu, J., J. Pan, and T. Wang, 2005, An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks, Review of Financial Studies 18, 131–164.

18. Merton, R., 1973, Theory of Rational Option Pricing, Bell Journal of Economics 4, 141–183.

19. Pan, J., 2002, The jump-risk premia implicit in options: evidence from an integrated time series study, Journal of Financial Economics 63, 350.

20. Ross, S. A., 1976, Option and Efficiency, Quarterly Journal of Economics 90, 75–89.

21. Rubinstein, Mark, 1985, Nonparametric tests of alternative option pricing models using all reported trades and quotes on the 30 most active CBOE option classes from August 23, 1976 through August 31, 1978, Journal of Finance 40, 455–480.

22. Scott, L. O., 1987, Option pricing when the variance changes randomly: theory, estimation, and an application, Journal of Financial and Quantitative Analysis 22, 419–438.

23. Stein, E.M., and J.C. Stein, 1991, Stock price distributions with stochastic volatility: an analytic approach, Review of Financial Studies 4, 727–752.

24. Toft, K. and Prucyk, B.,“Options on Leveraged Equity: Theory and Empirical Tests”, 1997, Journal of Finance, 52, 3, 1151-1180.

25. Wiggins, J.B., 1987, Option values under stochastic volatility: theory and empirical estimates, Journal of Financial Economics 19, 351–377.

Figure 1: The Pricing Errors of Geske (G) and Black Scholes (BS) Model Prices. Black-Scholes model underprices most in the money call options (low K) and overprices most out of the money call options (high K) on the individual stock. ITM individual stock call options (low K) are shown to be under valued and OTM individual stock call options (high K) are shown to be over valued by the Black-Scholes model relative to the market prices. Geske's compound option model produces option values that are greater (less) than the Black-Scholes's values for in (out of) the money European individual stock call options, and could potentially eliminate the known Black-Scholes bias.

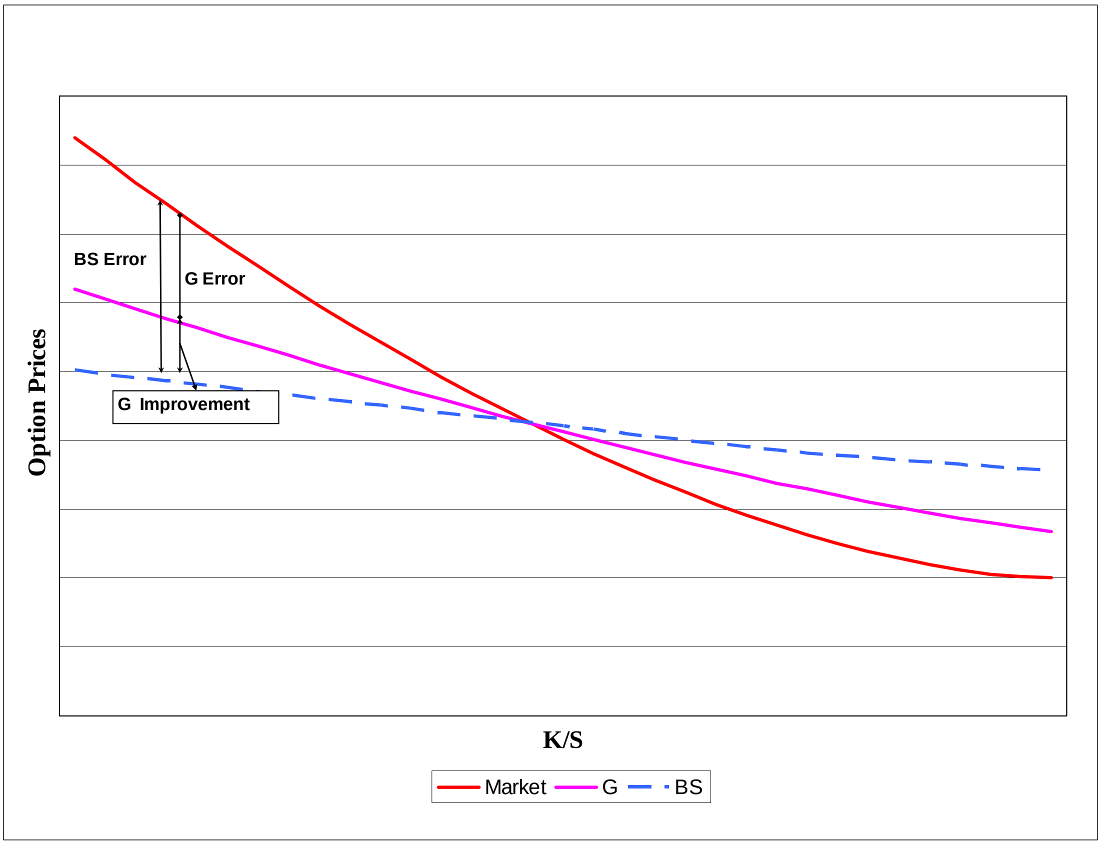

Figure 2: The Absolute Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call ITM vs. Time to Expiration.

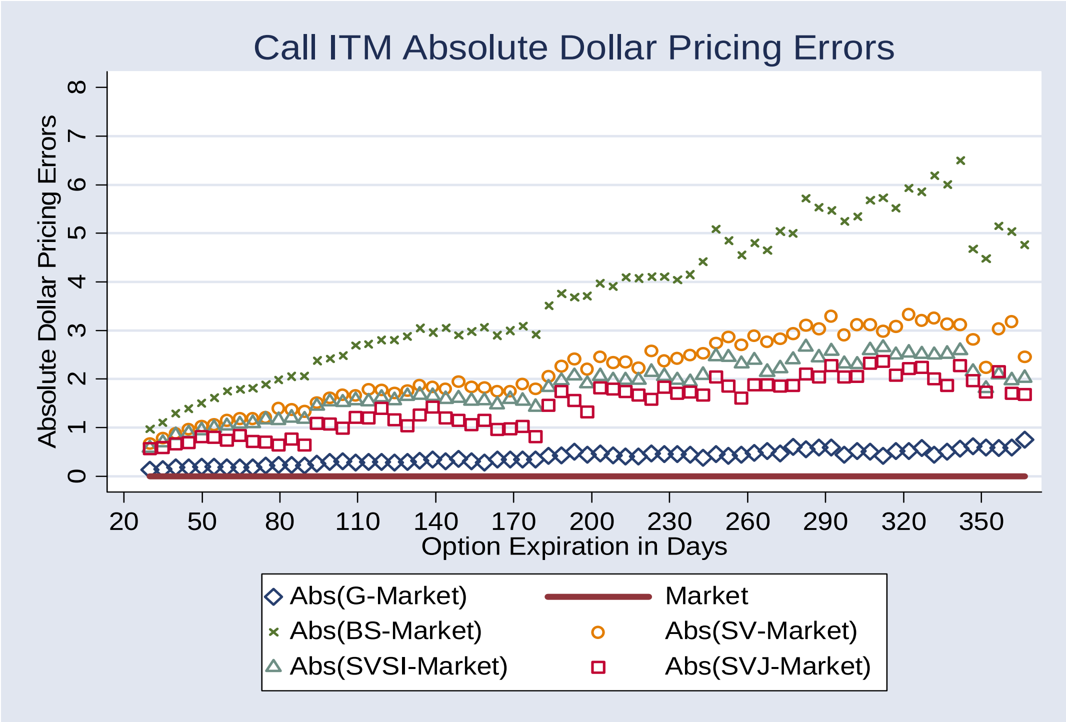

Figure 3: The Relative Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call ITM vs. Time to Expiration.

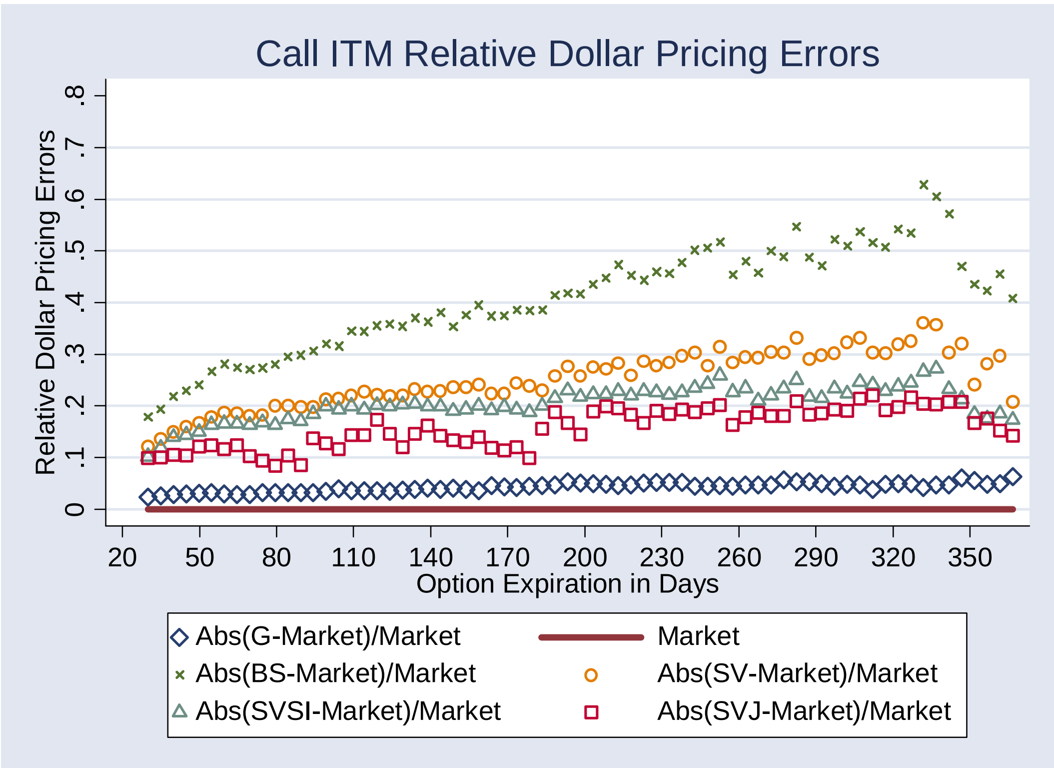

Figure 4: The Absolute Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ

Model Prices of Call OTM vs. Time to Expiration.

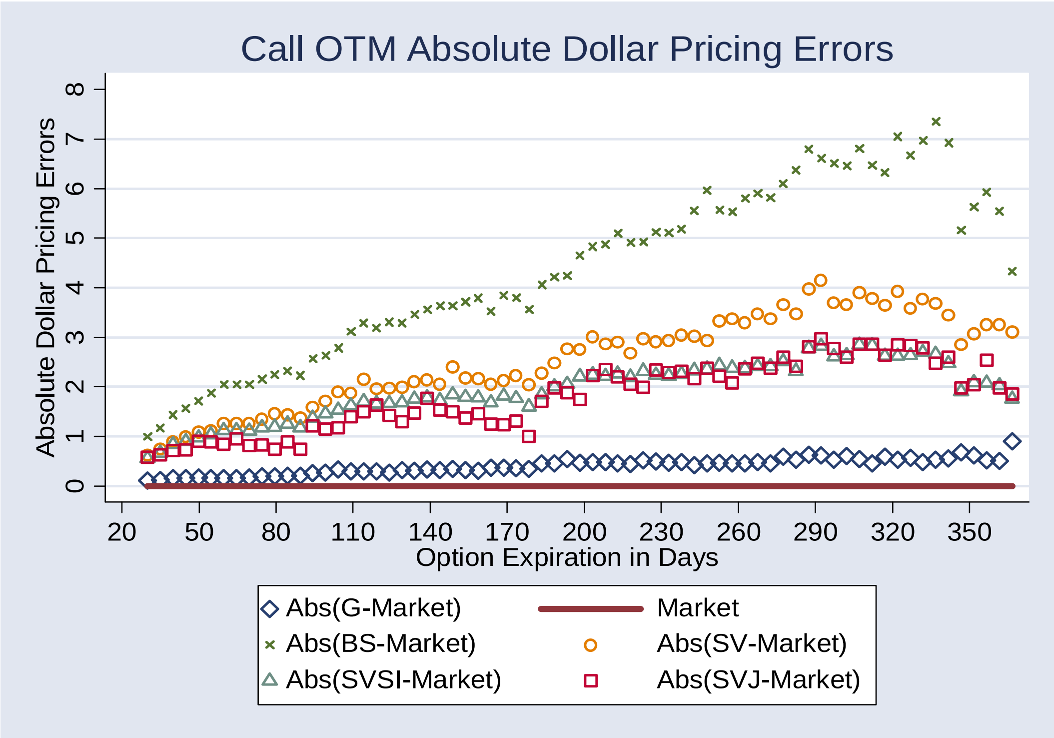

Figure 5: The Relative Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call OTM vs. Time to Expiration.

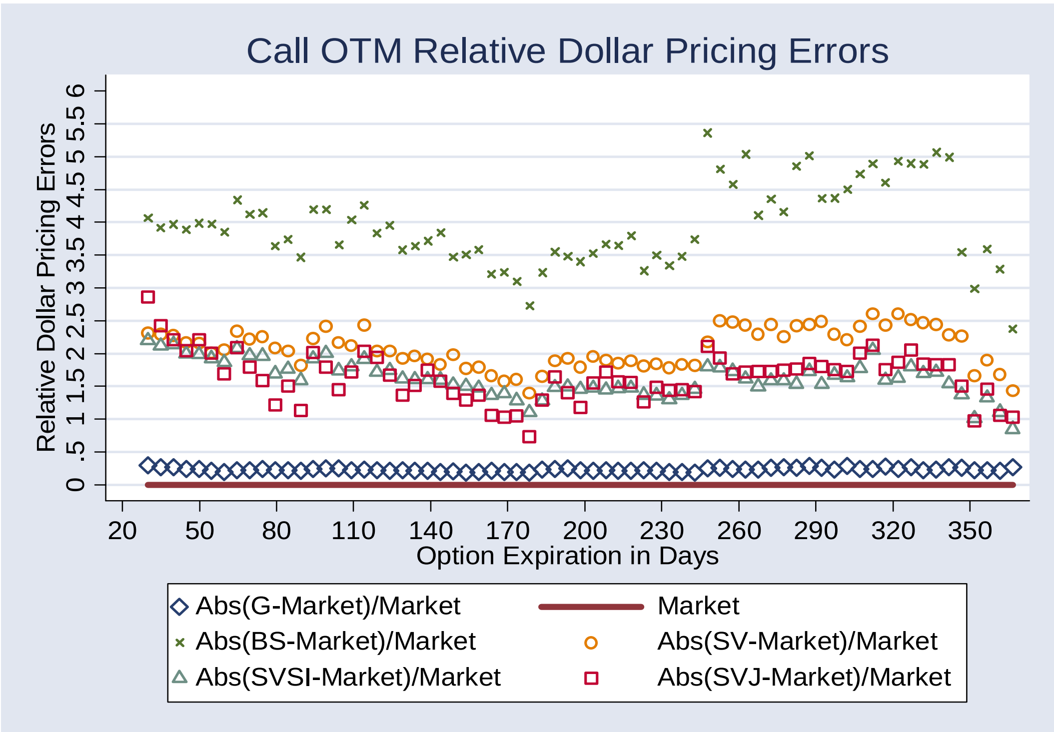

Figure 6: The Absolute Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call ITM vs. Moneyness.

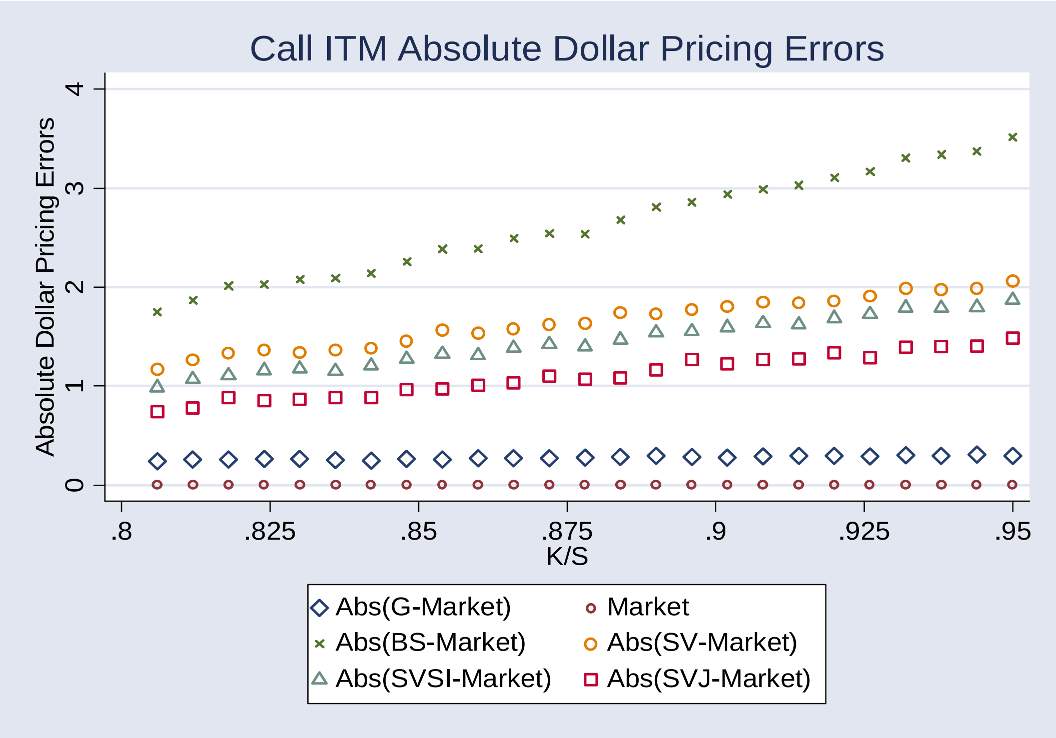

Figure 7: The Relative Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call ITM vs. Moneyness.

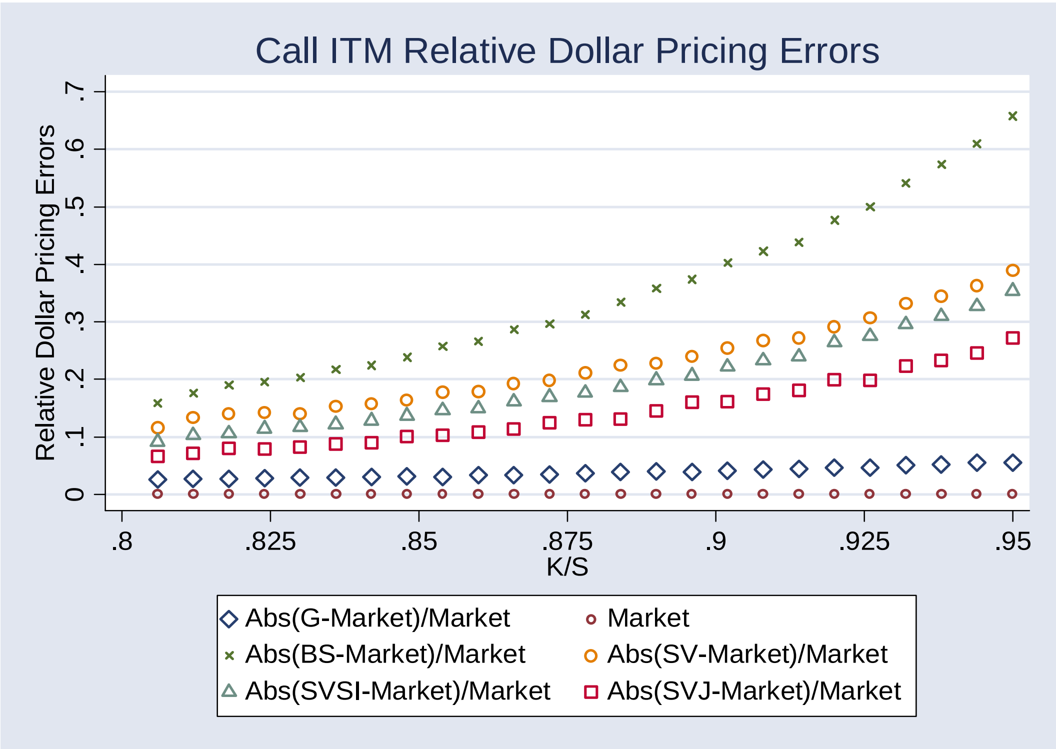

Figure 8: The Absolute Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call OTM vs. Moneyness.

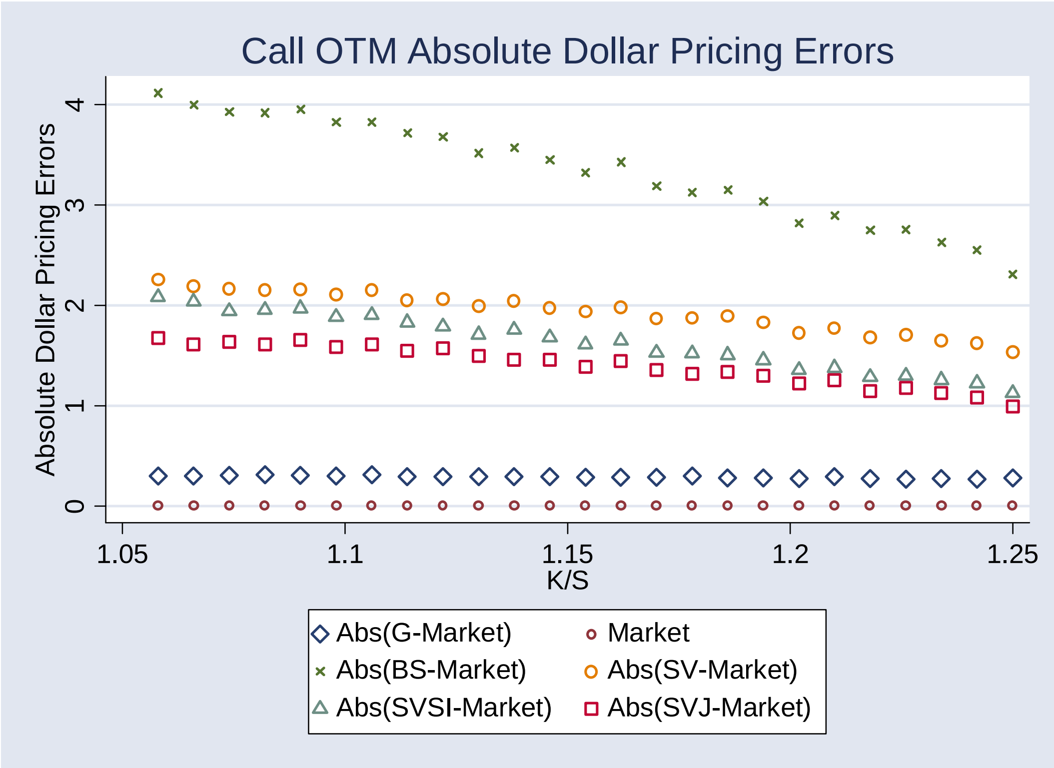

Figure 9: The Relative Pricing Errors of G, BS, and BCC's SV, SVSI and SVJ Model Prices of Call OTM vs. Moneyness.

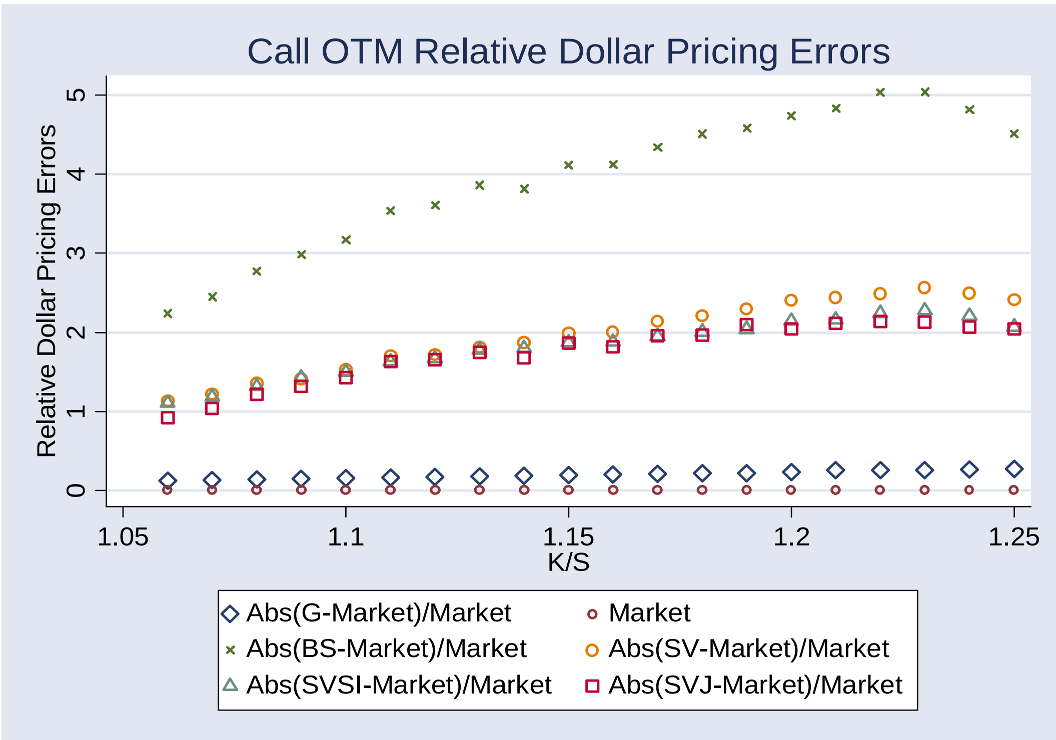

Figure 10: The Absolute Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call ITM vs. Time to Expiration.

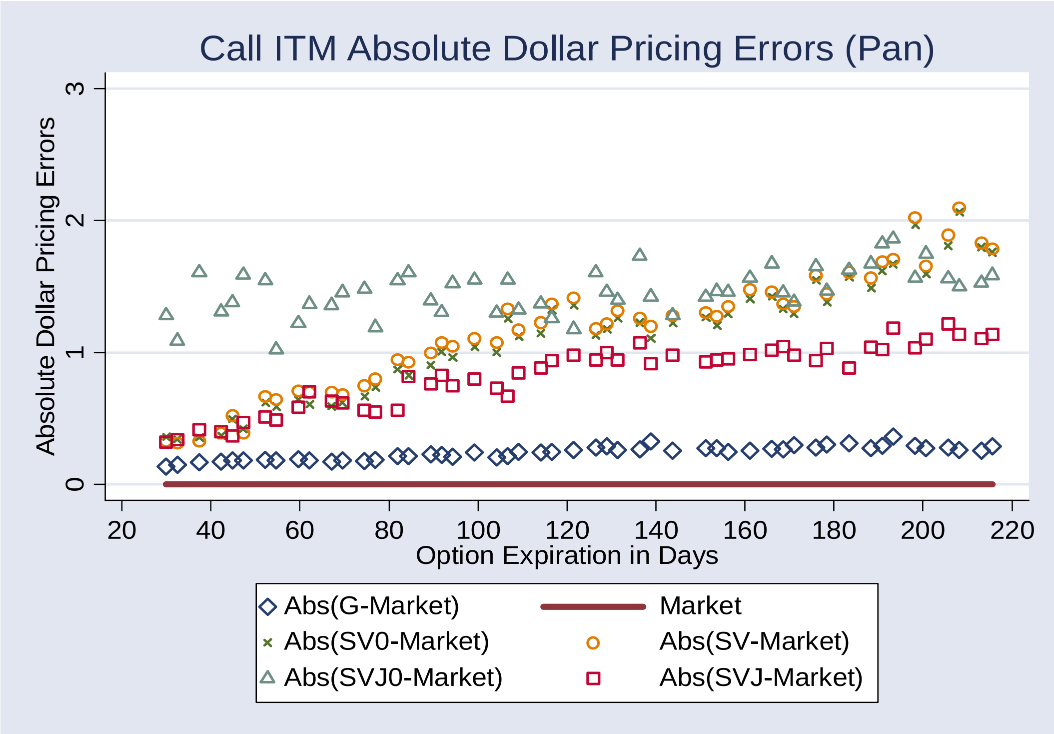

Figure 11: The Relative Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call ITM vs. Time to Expiration.

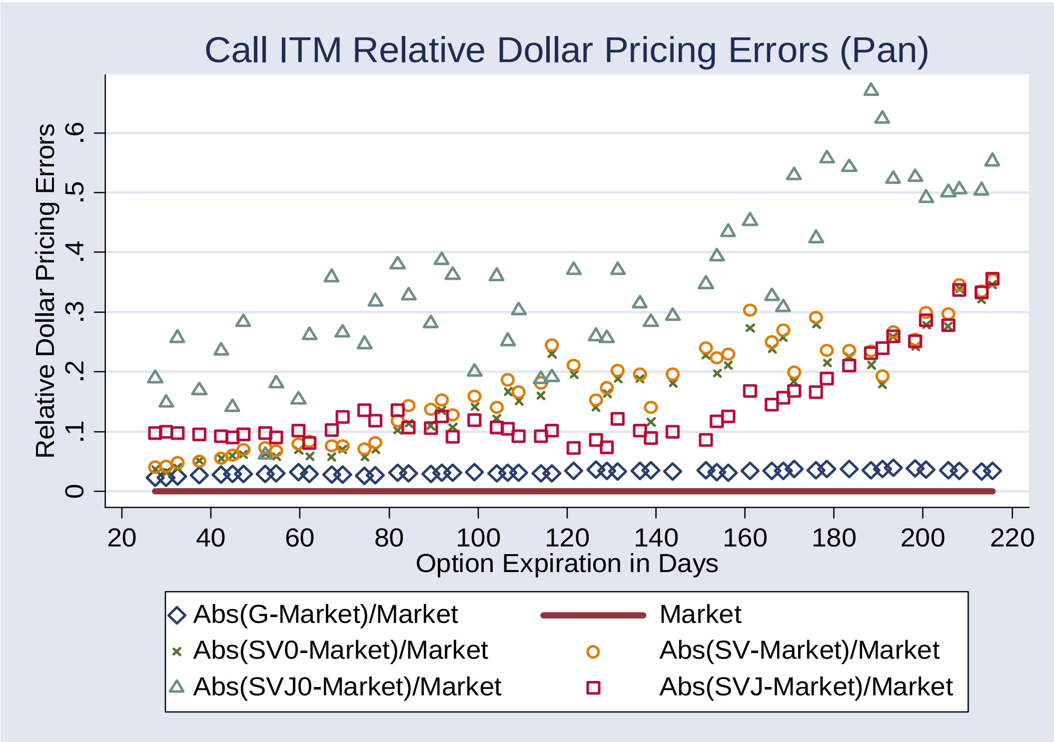

Figure 12: The Absolute Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call OTM vs. Time to Expiration.

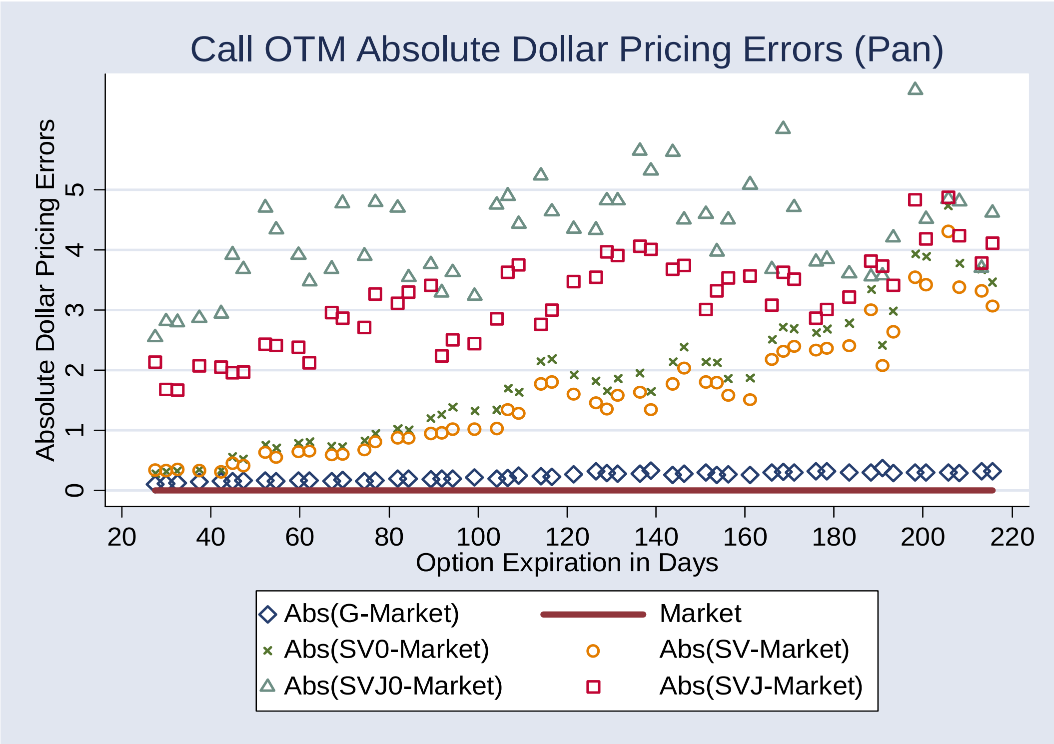

Figure 13: The Relative Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call OTM vs. Time to Expiration.

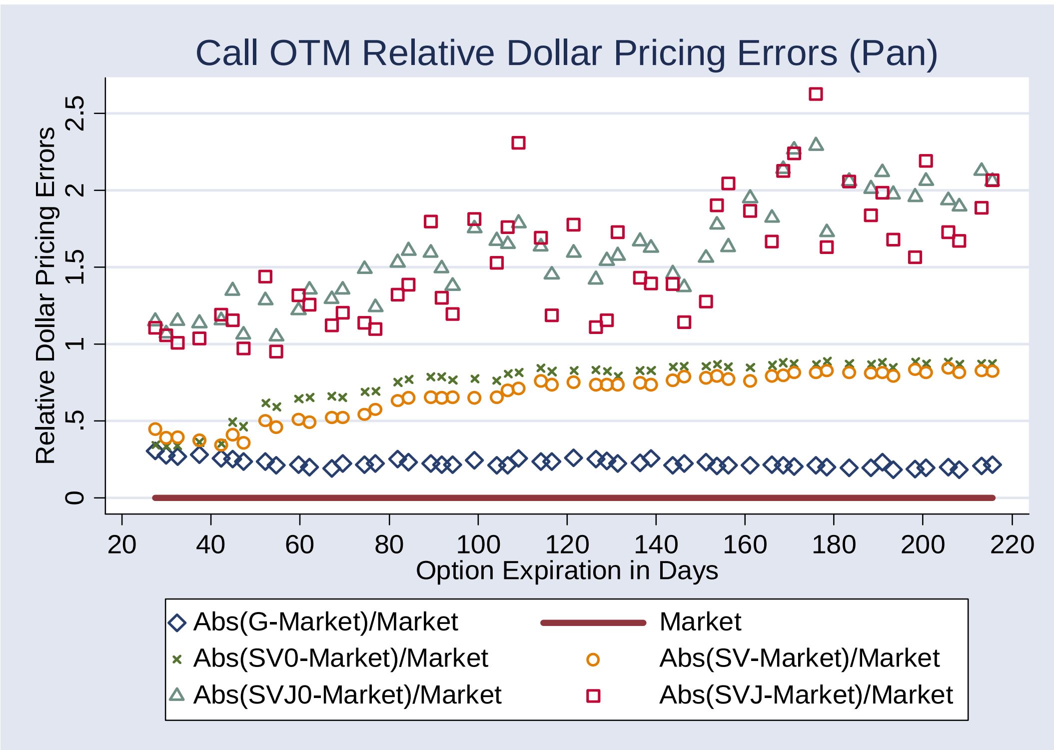

Figure 14: The Absolute Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call ITM vs. Moneyness.

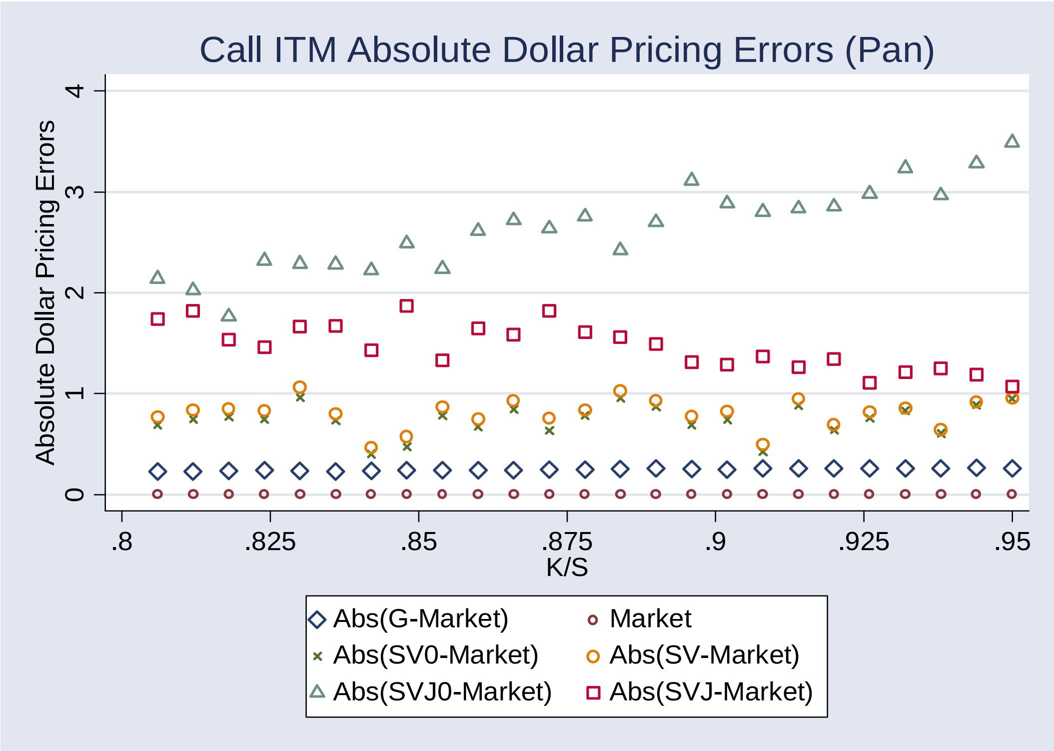

Figure 15: The Relative Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call ITM vs. Moneyness.

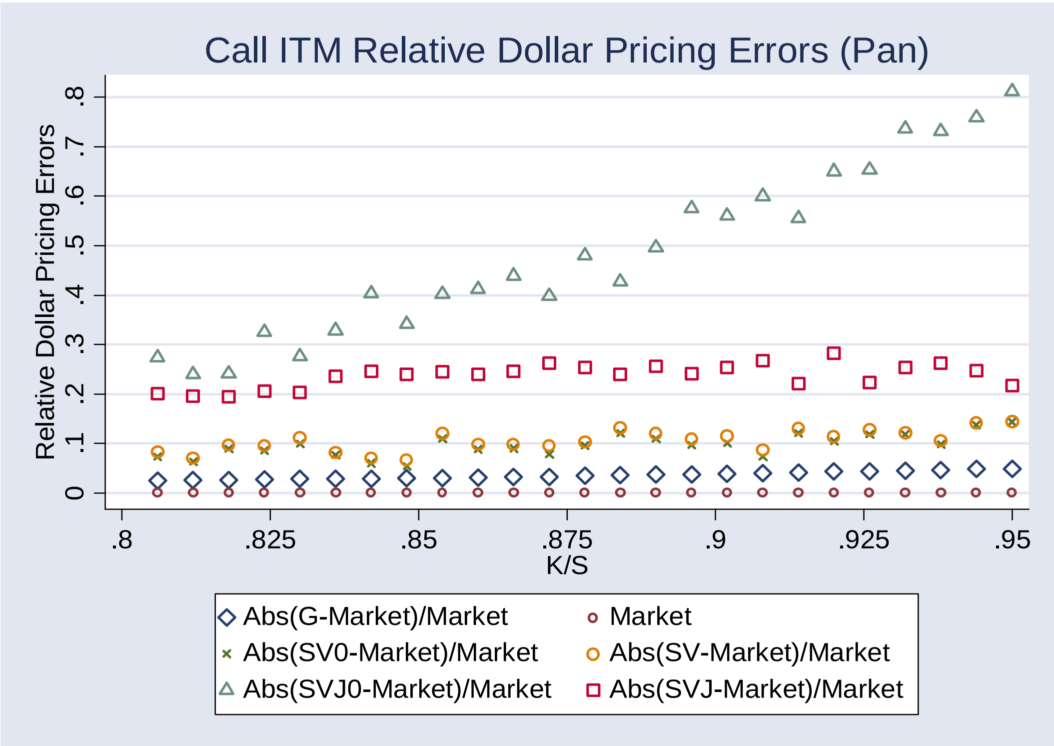

Figure 16: The Absolute Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call OTM vs. Moneyness.

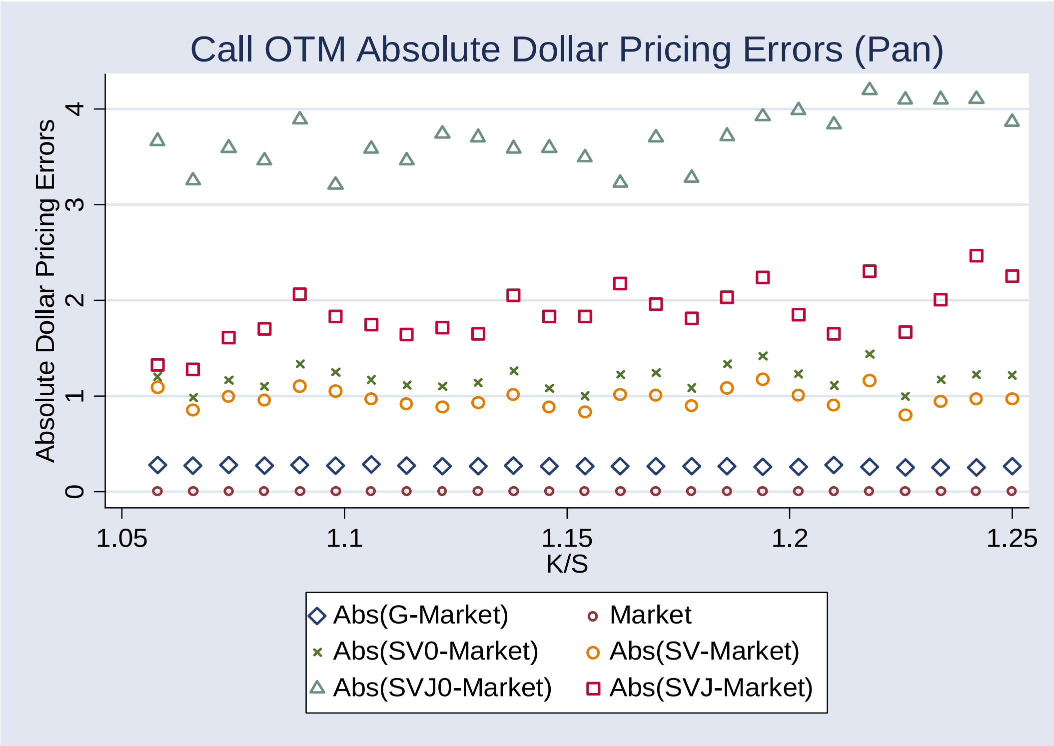

Figure 17: The Relative Pricing Errors of G and Pan's SV0, SV, SVJ0 and SVJ Model Prices of Call OTM vs. Moneyness.

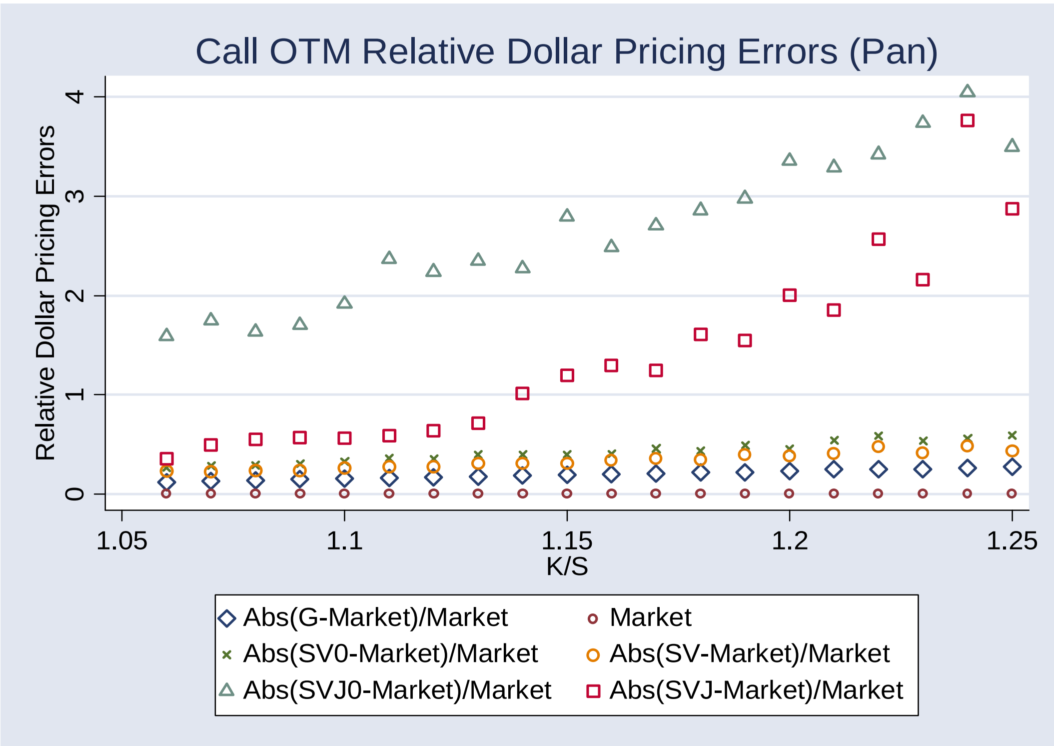

Table 1. Sample Properties of Individual Stock Options.

The reported numbers are respectively the average bid-ask mid-point price, the average trading volume and the total number of options, for all categories partitioned by moneyness and term of expiration. The sample period extends from January 4, 1996 through December 30, 2005 for a total of 3,487,894 calls. denotes the spot individual stock price and is the exercise price. ITM, ATM and OTM denote in-the-money, at-the-money and out-of-the money options, respectively.

Moneyness		**Days-to-Expiration**					
	K/S	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**Subtotal**
ITM	**[0.4--0.75)**	$17.11	$16.36	$16.68	$16.11	$18.33	**$16.96**
		39.07	32.94	31.39	30.61	25.72	**31.28**
		23,227	19,013	35,686	34,512	34,896	**147,334**
ITM	**[0.75--0.85)**	$8.82	$9.10	$9.58	$10.08	$11.45	**$9.79**
		66.32	49.77	46.94	40.01	31.44	**47.32**
		51,675	36,424	53,061	47,899	44,140	**233,199**
ITM	**[0.85--0.95)**	$5.00	$5.64	$6.23	$7.10	$8.35	**$6.30**
		127.31	82.03	71.77	53.71	38.20	**80.00**
		148,720	97,161	110,430	97,337	90,114	**543,762**
ATM	**[0.95--1.05]**	$2.10	$2.77	$3.43	$4.41	$5.56	**$3.45**
		253.01	157.62	124.47	95.45	54.94	**150.61**
		272,856	189,277	180,000	166,865	160,515	**969,513**
OTM	**(1.05--1.15]**	$0.90	$1.37	$1.83	$2.59	$3.48	**$2.02**
		214.53	147.23	125.37	104.18	61.97	**132.47**
		183,237	151,037	160,659	164,088	162,928	**821,949**
OTM	**(1.15--1.25]**	$0.52	$0.87	$1.21	$1.79	$2.47	**$1.48**
		137.3	109.12	94	84.3	56.05	**92.28**
		67,369	58,096	81,249	90,112	94,760	**391,586**
OTM	**(1.25--2.50]**	$0.25	$0.45	$0.66	$1.08	$1.56	**$0.94**
		91.24	81.15	70.47	67.43	53.82	**68.89**
		48,214	41,948	87,638	97,511	105,240	**380,551**
Subtotal	**[0.40--2.50]**	**$3.00**	**$3.36**	**$4.04**	**$4.52**	**$5.42**	**$4.06**
		182.65	**121.8**	**95.8**	**79.33**	**51.43**	**107.92**
		795,298	**592,956**	**708,723**	**698,324**	**692,593**	**3,487,894**

Table 2: Annual Distributions of Individual Stock Options.

The reported numbers are the total number of options, for all categories partitioned by moneyness K/S and days to expiration for each year. ITM, ATM and OTM denote In, At, and Out of-the money options.

Moneyness		Year	Days-to-Expiration						Year	Days-to-Expiration					
	K/S		21-40	41-60	61-110	111-170	171-365	Subtotal		21-40	41-60	61-110	111-170	171-365	Subtotal
ITM	[0.40--0.75)	1996	1,633	1,479	2,670	2,735	2,506	11,023	2001	1,698	1,294	1,944	1,946	2,325	9,207
ITM	[0.75--0.85)		4,387	3,158	4,657	4,302	3,709	20,213		3,888	2,488	3,147	2,832	2,918	15,273
ITM	[0.85--0.95)		13,629	8,944	10,873	9,365	8,214	51,025		11,122	6,427	6,448	5,644	5,644	35,285
ATM	[0.95--1.05]		27,021	18,726	18,918	17,085	15,456	97,206		21,986	14,240	12,442	11,487	10,925	71,080
OTM	(1.05--1.15]		15,972	13,572	15,587	15,894	15,082	76,107		17,623	14,316	13,977	13,814	13,398	73,128
OTM	(1.15--1.25]		5,230	4,520	6,862	7,188	7,088	30,888		7,226	6,384	8,683	9,414	9,910	41,617
OTM	(1.25--2.50]		3,487	2,795	6,902	6,540	6,564	26,288		5,980	5,448	10,733	12,364	14,473	48,998
Subtotal			71,359	53,194	66,469	63,109	58,619	312,750		69,523	50,597	57,374	57,501	59,593	294,588
ITM	[0.40--0.75)	1997	1,682	1,550	3,558	3,052	2,475	12,317	2002	1,572	1,306	2,012	1,845	2,107	8,842
ITM	[0.75--0.85)		4,586	3,677	5,834	4,974	3,668	22,739		3,723	2,480	3,020	2,685	2,589	14,497
ITM	[0.85--0.95)		14,970	10,259	12,218	10,361	8,439	56,247		11,527	6,957	7,208	6,302	6,038	38,032
ATM	[0.95--1.05]		28,541	19,982	19,209	17,052	14,793	99,577		23,013	15,018	13,784	12,831	12,542	77,188
OTM	(1.05--1.15]		17,296	14,095	15,589	15,087	12,770	74,837		16,815	13,507	14,147	14,612	14,510	73,591
OTM	(1.15--1.25]		5,892	4,691	6,783	6,876	6,143	30,385		6,509	5,743	8,223	9,266	10,152	39,893
OTM	(1.25--2.50]		3,901	2,955	6,411	6,676	5,608	25,551		5,226	4,613	10,484	11,738	13,167	45,228
Subtotal			76,868	57,209	69,602	64,078	53,896	321,653		68,385	49,624	58,878	59,279	61,105	297,271
ITM	[0.40--0.75)	1998	2,025	1,506	3,170	3,080	3,045	12,826	2003	2,423	1,906	3,578	3,430	3,257	14,594
ITM	[0.75--0.85)		4,835	3,337	5,269	4,736	4,224	22,401		4,911	3,412	5,148	4,570	4,280	22,321
ITM	[0.85--0.95)		15,166	9,569	11,107	9,771	8,599	54,212		14,116	9,070	10,678	9,376	8,477	51,717
ATM	[0.95--1.05]		27,451	18,856	17,231	15,890	14,304	93,732		25,780	18,044	17,950	16,638	16,301	94,713
OTM	(1.05--1.15]		18,550	14,779	15,347	15,017	13,105	76,798		15,305	13,116	14,723	16,009	17,329	76,482
OTM	(1.15--1.25]		6,219	5,310	7,807	7,987	6,564	33,887		4,739	4,479	6,356	7,941	9,767	33,282
OTM	(1.25--2.50]		3,955	3,769	8,753	8,761	6,917	32,155		2,683	2,462	5,195	6,686	9,804	26,830
Subtotal			78,201	57,126	68,684	65,242	56,758	326,011		69,957	52,489	63,628	64,650	69,215	319,939
ITM	[0.40--0.75)	1999	3,833	2,761	5,900	5,421	4,898	22,813	2004	2,488	2,213	3,635	3,733	4,154	16,223
ITM	[0.75--0.85)		6,987	4,486	6,745	6,030	5,187	29,435		5,994	4,414	6,338	5,698	5,818	28,262
ITM	[0.85--0.95)		17,144	10,613	11,692	10,204	9,143	58,796		18,112	12,896	14,789	13,109	12,826	71,732
ATM	[0.95--1.05]		28,587	18,780	16,886	15,337	14,354	93,944		33,764	25,408	24,791	23,455	23,546	130,964
OTM	(1.05--1.15]		21,265	16,147	15,817	15,339	13,983	82,551		19,467	18,004	20,295	21,932	24,267	103,965
OTM	(1.15--1.25]		8,150	6,532	8,565	9,222	8,279	40,748		6,474	5,983	8,570	10,488	13,145	44,660
OTM	(1.25--2.50]		5,000	4,016	8,204	8,823	8,306	34,349		4,607	4,271	8,439	10,815	12,991	41,123
Subtotal			90,966	63,335	73,809	70,376	64,150	362,636		90,906	73,189	86,857	89,230	96,747	436,929
ITM	[0.40--0.75)	2000	3,507	2,774	4,931	4,718	5,272	21,202	2005	2,366	2,224	4,288	4,552	4,857	18,287
ITM	[0.75--0.85)		6,450	4,160	5,558	4,986	4,820	25,974		5,914	4,812	7,345	7,086	6,927	32,084
ITM	[0.85--0.95)		14,154	8,800	8,996	8,144	7,694	47,788		18,780	13,626	16,421	15,061	15,040	78,928
ATM	[0.95--1.05]		23,529	15,510	13,518	12,532	12,277	77,366		33,184	24,713	25,271	24,558	26,017	133,743
OTM	(1.05--1.15]		20,082	15,107	13,987	13,556	12,467	75,199		20,862	18,394	21,190	22,828	26,017	109,291
OTM	(1.15--1.25]		9,757	7,949	9,307	9,853	9,088	45,954		7,173	6,505	10,093	11,877	14,624	50,272
OTM	(1.25--2.50]		9,021	7,582	13,288	14,098	13,384	57,373		4,354	4,037	9,229	11,010	14,026	42,656
Subtotal			86,500	61,882	69,585	67,887	65,002	350,856		92,633	74,311	93,837	96,972	107,508	465,261
							ITM	[0.40--0.75)	ALL	23,227	19,013	35,686	34,512	34,896	147,334
							ITM	[0.75--0.85)		51,675	36,424	53,061	47,899	44,140	233,199
							ITM	[0.85--0.95)		148,720	97,161	110,430	97,337	90,114	543,762
							ATM	[0.95--1.05]		272,856	189,277	180,000	166,865	160,515	969,513
							OTM	(1.05--1.15]		183,237	151,037	160,659	164,088	162,928	821,949
							OTM	(1.15--1.25]		67,369	58,096	81,249	90,112	94,760	391,586
							OTM	(1.25--2.50]		48,214	41,948	87,638	97,511	105,240	380,551
							Subtotal			795,298	592,956	708,723	698,324	692,593	3,487,894

Table 3: Call ITM Pricing Errors by Calendar Year.

PANEL A: Total Number Of Options						
			Option Expiration (in Days)			
YEAR	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
1996	19,639	13,574	18,186	16,387	14,413	**82,199**
1997	21,233	15,478	21,593	18,368	14,558	**91,230**
1998	22,001	14,396	19,509	17,547	15,844	**89,297**
1999	27,959	17,846	24,296	21,617	19,148	**110,866**
2000	24,103	15,718	19,467	17,816	17,738	**94,842**
2001	16,703	10,207	11,528	10,412	10,869	**59,719**
2002	16,788	10,722	12,216	10,798	10,708	**61,232**
2003	21,442	14,379	19,391	17,363	16,001	**88,576**
2004	26,586	19,513	24,753	22,521	22,774	**116,147**
2005	27,055	20,660	28,045	26,686	26,799	**129,245**
TOTAL	**223,509**	**152,493**	**198,984**	**179,515**	**168,852**	**923,353**

PANEL B: Pricing Error Improvement						
			Option Expiration (in Days)			
YEAR	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
1996	15%	30%	51%	76%	100%	**51%**
1997	22%	33%	57%	92%	100%	**58%**
1998	16%	24%	29%	33%	51%	**30%**
1999	19%	28%	34%	41%	50%	**35%**
2000	20%	25%	33%	43%	55%	**36%**
2001	13%	13%	19%	24%	37%	**22%**
2002	10%	10%	11%	16%	25%	**15%**
2003	6%	9%	15%	20%	26%	**16%**
2004	8%	20%	24%	32%	44%	**27%**
2005	14%	27%	33%	46%	57%	**38%**
TOTAL	**14%**	**21%**	**27%**	**36%**	**47%**	**30%**

PANEL C: Rank Sum Test p Value						
			Option Expiration (in Days)			
YEAR	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
1996	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2005	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table 4: Call ITM Pricing Errors by Leverage.

PANEL A: Total Number Of Options						
	Option Expiration (in Days)					
D/E	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
(0.00-0.10]	42,604	29,413	36,324	33,531	32,107	**173,979**
(0.10-0.20]	39,326	27,133	33,904	31,550	28,659	**160,572**
(0.20-0.30]	30,827	21,262	27,406	24,666	23,892	**128,053**
(0.30-0.60]	54,149	36,672	51,041	44,069	42,208	**228,139**
(0.60-1.00]	31,314	21,030	28,226	25,226	23,444	**129,240**
(1.00-1.50]	17,263	11,603	15,162	14,083	12,881	**70,992**
(1.50-2.00]	8,026	5,380	6,921	6,390	5,661	**32,378**
TOTAL	**223,509**	**152,493**	**198,984**	**179,515**	**168,852**	**923,353**

PANEL B: Pricing Error Improvement						
	Option Expiration (in Days)					
D/E	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
(0.00-0.10]	4%	7%	9%	13%	18%	**11%**
(0.10-0.20]	9%	15%	21%	27%	35%	**22%**
(0.20-0.30]	12%	16%	27%	35%	52%	**28%**
(0.30-0.60]	19%	28%	37%	51%	72%	**42%**
(0.60-1.00]	23%	31%	38%	53%	66%	**43%**
(1.00-1.50]	30%	45%	56%	67%	93%	**59%**
(1.50-2.00]	36%	51%	70%	77%	96%	**64%**
TOTAL	**14%**	**21%**	**27%**	**36%**	**47%**	**30%**

PANEL C: Rank Sum Test p Value						
	Option Expiration (in Days)					
D/E	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
(0.00-0.10]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.10-0.20]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.20-0.30]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.30-0.60]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.60-1.00]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(1.00-1.50]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(1.50-2.00]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table 5: Call OTM Pricing Errors by Calendar Year.

PANEL A: Total Number Of Options						
Option Expiration (in Days)						
YEAR	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
1996	24,689	20,887	29,351	29,622	28,734	**133,283**
1997	27,089	21,741	28,783	28,639	24,521	**130,773**
1998	28,724	23,858	31,907	31,765	26,586	**142,840**
1999	34,415	26,695	32,586	33,384	30,568	**157,648**
2000	38,860	30,638	36,582	37,507	34,939	**178,526**
2001	30,829	26,148	33,393	35,592	37,781	**163,743**
2002	28,550	23,863	32,854	35,616	37,829	**158,712**
2003	22,727	20,057	26,274	30,636	36,900	**136,594**
2004	30,548	28,258	37,304	43,235	50,403	**189,748**
2005	32,389	28,936	40,512	45,715	54,667	**202,219**
TOTAL	**298,820**	**251,081**	**329,546**	**351,711**	**362,928**	**1,594,086**

PANEL B: Pricing Error Improvement						
Option Expiration (in Days)						
YEAR	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
1996	60%	87%	65%	100%	71%	**77%**
1997	75%	86%	97%	97%	91%	**90%**
1998	42%	64%	70%	66%	81%	**64%**
1999	83%	54%	66%	66%	81%	**71%**
2000	90%	53%	75%	80%	98%	**80%**
2001	48%	37%	39%	42%	52%	**44%**
2002	26%	25%	28%	31%	37%	**30%**
2003	36%	31%	32%	34%	38%	**35%**
2004	89%	42%	48%	44%	55%	**55%**
2005	89%	67%	68%	65%	72%	**72%**
TOTAL	**49%**	**48%**	**55%**	**55%**	**65%**	**59%**

PANEL C: Rank Sum Test p Value						
Option Expiration (in Days)						
YEAR	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
1996	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
1997	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
1998	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
1999	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2000	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2001	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2002	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2003	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2004	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
2005	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table 6: Call OTM Pricing Errors by Leverage.

PANEL A: Total Number Of Options						
			Option Expiration (in Days)			
D/E	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
(0.00-0.10]	50,291	40,911	44,782	47,783	47,619	**231,401**
(0.10-0.20]	52,491	44,018	53,393	57,570	59,084	**266,606**
(0.20-0.30]	38,976	32,660	43,367	46,347	49,459	**210,849**
(0.30-0.60]	74,469	63,909	88,884	92,458	96,955	**416,762**
(0.60-1.00]	44,040	37,109	53,653	58,130	59,828	**252,822**
(1.00-1.50]	25,338	21,291	29,753	32,488	33,164	**142,057**
(1.50-2.00]	13,215	11,183	15,714	16,935	16,531	**73,589**
TOTAL	**298,820**	**251,081**	**329,546**	**351,711**	**362,640**	**1,594,086**

PANEL B: Pricing Error Improvement						
			Option Expiration (in Days)			
D/E	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
(0.00-0.10]	20%	15%	18%	21%	24%	**20%**
(0.10-0.20]	56%	35%	37%	38%	45%	**41%**
(0.20-0.30]	87%	43%	53%	48%	62%	**55%**
(0.30-0.60]	61%	67%	67%	68%	81%	**70%**
(0.60-1.00]	44%	70%	77%	70%	87%	**72%**
(1.00-1.50]	26%	92%	91%	92%	96%	**81%**
(1.50-2.00]	24%	89%	91%	88%	73%	**83%**
TOTAL	**49%**	**48%**	**55%**	**55%**	**65%**	**59%**

PANEL C: Rank Sum Test p Value						
			Option Expiration (in Days)			
D/E	**21-40**	**41-60**	**61-110**	**111-170**	**171-365**	**TOTAL**
(0.00-0.10]	0.0005	0.0003	0.0000	0.0000	0.0006	**0.0000**
(0.10-0.20]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.20-0.30]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.30-0.60]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(0.60-1.00]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(1.00-1.50]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
(1.50-2.00]	0.0000	0.0000	0.0000	0.0000	0.0000	**0.0000**
TOTAL	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

Table 7: Implied Parameters and In-Sample Fit.

The structural parameters of a given model are estimated daily by minimizing the sum of squared pricing errors between the market price and the model price for each option. The first line is the sample average of the estimated parameters; the second line is the standard errors in parentheses. Following Bakshi, Cao, and Chen (1997), the structural parameters' definitions are as the following: κ_v, θ_v/κ_v, and σ_v (κ_R, θ_R/κ_R, and σ_R) are respectively the speed of adjustment, the long-run mean, and the variation coefficient of the diffusion volatility $V(t)$ (the spot interest rate $R(t)$). The parameter ρ represents the correlation between volatility and spot return. The parameter μ_J represents the mean jump size, λ the frequency of the jumps per year, and σ_J the standard deviation of the logarithm of one plus the percentage jump size. V_J is the instantaneous variance of the jump component. BS, SV, SVSI, and SVJ, respectively, stand for the Black-Scholes, the stochastic-volatility model, the stochastic-volatility and stochastic-interest-rate model, and the stochastic-volatility model with random jumps. (See Appendix 2 for comparison)

Parameters	**BS**	**SV**	**SVSI**	**SVJ**
κ_v		1.67	1.64	1.67
		(0.75)	(0.55)	(0.28)
θ_v		0.09	0.06	0.05
		(0.14)	(0.08)	(0.05)
σ_v		0.51	0.48	0.41
		(0.27)	(0.22)	(0.12)
ρ		−0.66	−0.69	−0.68
		(0.20)	(0.16)	(0.11)
λ				0.77
				(0.48)
μ_J				−0.06
				(0.10)
σ_J				0.12
				(0.12)
V_J				0.14
				(0.15)
κ_R			0.59	
			(0.45)	
θ_R			0.02	
			(0.01)	
σ_R			0.55	
			(0.74)	
Implied Volatility (%)	54.65	52.21	51.92	49.06
	(0.20)	(0.18)	(0.14)	(0.10)

Table 8: Out-of-Sample Pricing Errors (we).

For a given model, we compute the price of each option using the previous day's implied parameters and implied stock volatility. The reported absolute pricing error is the sample average of the absolute error. G, BS, SV, SVSI, and SVJ, respectively, stand for the Geske, Black-Scholes, the stochastic-volatility model, the stochastic-volatility and stochastic-interest-rate model, and the stochastic-volatility model with random jumps.

Panel A: Absolute Pricing Error								
	Moneyness		Days to Expiration					
	K/S	Model	21-40	41-60	61-110	111-170	171-365	Subtotal
ITM	[0.4--0.75)	G	0.06	0.08	0.10	0.13	0.19	0.11
		BS	0.14	0.29	0.59	0.94	2.05	0.94
		SV	0.12	0.13	0.19	0.27	0.55	0.22
		SVSI	0.30	0.44	0.48	0.58	1.23	0.57
		SVJ	0.31	0.39	0.49	0.62	1.22	0.61
ITM	[0.75--0.85)	G	0.08	0.11	0.14	0.18	0.26	0.14
		BS	0.55	0.99	1.69	2.42	3.66	1.86
		SV	0.28	0.41	0.64	0.80	1.21	0.56
		SVSI	0.52	0.74	1.08	1.41	2.10	1.03
		SVJ	0.51	0.70	1.03	1.35	2.06	1.04
ITM	[0.85--0.95)	G	0.10	0.14	0.17	0.21	0.29	0.16
		BS	1.46	2.10	2.95	3.84	4.72	2.86
		SV	0.55	0.83	1.22	1.50	1.90	1.01
		SVSI	0.84	1.24	1.73	2.19	2.89	1.54
		SVJ	0.82	1.20	1.67	2.10	2.80	1.55
ATM	[0.95--1.05]	G	0.10	0.13	0.16	0.22	0.31	0.17
		BS	1.30	1.86	2.26	3.08	4.26	2.47
		SV	0.83	1.21	1.61	1.99	2.42	1.43
		SVSI	1.08	1.58	2.08	2.68	3.42	1.95
		SVJ	1.06	1.54	2.03	2.58	3.32	1.95
OTM	(1.05--1.15]	G	0.08	0.11	0.15	0.21	0.30	0.15
		BS	1.63	2.46	3.41	4.71	5.83	3.68
		SV	0.63	0.98	1.38	1.81	2.20	1.27
		SVSI	0.85	1.31	1.83	2.48	3.20	1.79
		SVJ	0.84	1.29	1.78	2.40	3.12	1.78
OTM	(1.15--1.25]	G	0.06	0.09	0.13	0.19	0.28	0.14
		BS	0.77	1.31	2.28	3.50	4.98	2.86
		SV	0.35	0.56	0.89	1.25	1.66	0.88
		SVSI	0.51	0.80	1.32	1.89	2.67	1.40
		SVJ	0.52	0.80	1.29	1.85	2.66	1.40
OTM	(1.25--2.50]	G	0.06	0.07	0.09	0.14	0.23	0.12
		BS	0.25	0.49	0.93	1.59	2.90	1.55
		SV	0.18	0.25	0.39	0.58	0.91	0.45
		SVSI	0.24	0.36	0.60	0.95	1.64	0.79
		SVJ	0.24	0.36	0.61	0.96	1.64	0.81
Subtotal	[0.40--2.50]	G	0.09	0.12	0.14	0.20	0.28	0.04
		BS	1.20	1.79	2.29	3.20	4.35	1.29
		SV	0.52	0.80	1.03	1.34	1.75	0.99
		SVSI	0.76	1.14	1.44	1.94	2.65	1.48
		SVJ	0.75	1.12	1.41	1.88	2.59	1.48

Table 9: Out-of-Sample Relative Pricing Errors (II).

For a given model, we compute the price of each option using the previous day's implied parameters and implied stock volatility. The reported relative pricing error is the sample average of the model price minus market price, divided by the market price. G, BS, SV, SVSI, and SVJ, respectively, stand for the Geske, Black-Scholes, the stochastic-volatility model, the stochastic-volatility and stochastic-interest-rate model, and the stochastic-volatility model with random jumps.

Panel B: Relative Pricing Error								
	Moneyness		Days to Expiration					
	K/S	Model	21-40	41-60	61-110	111-170	171-365	Subtotal
ITM	[0.4--0.75)	G	-0.64%	-0.63%	-0.46%	-0.18%	0.41%	-0.22%
		BS	0.07%	1.39%	3.53%	6.22%	12.15%	5.53%
		SV	0.30%	0.48%	1.05%	1.65%	3.04%	1.04%
		SVSI	0.58%	0.88%	1.70%	2.48%	4.07%	1.73%
		SVJ	0.31%	0.09%	-0.22%	-0.47%	-0.38%	-0.23%
ITM	[0.75--0.85)	G	-0.77%	-0.94%	-0.60%	-0.12%	0.89%	-0.30%
		BS	5.16%	9.80%	16.09%	22.47%	31.65%	17.00%
		SV	3.52%	5.23%	7.55%	8.83%	11.38%	6.33%
		SVSI	4.56%	6.44%	8.97%	10.65%	13.02%	7.82%
		SVJ	2.06%	2.20%	1.74%	2.26%	3.11%	1.84%
ITM	[0.85--0.95)	G	-1.08%	-1.44%	-0.71%	0.06%	1.43%	-0.45%
		BS	26.84%	36.06%	45.22%	53.80%	61.77%	42.99%
		SV	11.27%	16.84%	21.68%	23.37%	25.49%	17.89%
		SVSI	12.98%	19.34%	24.01%	26.28%	27.96%	20.19%
		SVJ	2.54%	5.20%	4.26%	8.34%	9.68%	5.05%
ATM	[0.95--1.05]	G	0.17%	-0.94%	-0.10%	0.77%	2.19%	0.39%
		BS	77.98%	77.36%	70.51%	73.91%	81.48%	76.39%
		SV	52.67%	56.25%	58.40%	53.94%	52.73%	55.72%
		SVSI	58.22%	62.14%	62.48%	59.28%	55.95%	60.18%
		SVJ	13.74%	19.10%	13.15%	21.79%	22.63%	20.25%
OTM	(1.05--1.15]	G	1.07%	0.59%	1.35%	2.26%	3.26%	1.77%
		BS	359.06%	336.39%	308.52%	271.69%	237.24%	300.58%
		SV	165.10%	151.26%	139.96%	112.82%	95.34%	145.57%
		SVSI	201.04%	179.34%	155.46%	125.76%	102.04%	161.54%
		SVJ	37.05%	51.47%	31.47%	51.84%	47.96%	67.14%
OTM	(1.15--1.25]	G	-0.79%	-0.06%	0.99%	2.65%	3.45%	1.68%
		BS	452.65%	403.70%	420.29%	393.65%	371.78%	404.52%
		SV	203.33%	185.68%	197.32%	163.33%	137.15%	191.44%
		SVSI	249.48%	225.82%	234.66%	192.02%	149.96%	216.18%
		SVJ	87.75%	66.48%	61.31%	78.31%	79.83%	109.46%
OTM	((1.25--2.50]	G	-2.08%	-1.07%	-0.32%	1.43%	2.56%	0.95%
		BS	252.04%	331.82%	376.53%	355.86%	414.18%	362.14%
		SV	167.28%	156.64%	173.96%	154.83%	187.77%	175.16%
		SVSI	170.42%	146.00%	155.68%	147.90%	163.09%	161.57%
		SVJ	159.46%	98.97%	94.30%	86.14%	129.92%	128.53%
Subtotal	[0.40--2.50]	G	-0.12%	-0.69%	0.04%	1.03%	2.24%	0.38%
		BS	141.78%	154.31%	163.82%	165.37%	177.61%	154.91%
		SV	94.04%	95.31%	103.61%	92.65%	94.91%	96.85%
		SVSI	109.66%	108.10%	110.25%	101.40%	94.38%	104.70%
		SVJ	44.99%	42.55%	36.45%	44.65%	55.39%	50.17%

Table 10: In the Money Option Basis Point Improvement of G vs. BS, SV, SVSI and SVJ by Year.

The columns left to right represent the year, the present value of all matched pairs for that year, the total number of the matched pairs that year, the number of those matched pairs where an alternative model price is closer to the market price in absolute distance, the number of matched pairs where the G model price is closer to the market price, the dollar value of the alternative model price improvement, the dollar value of G improvement, and the net basis point advantage of G's model for that year.

YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	BS	G	BS	G	**BP**
1996	338,731.04	43,904	15,025	28,879	1,922.28	46,347.79	**1312**
1997	390,477.02	45,449	14,849	30,600	2,255.64	57,683.90	**1420**
1998	410,362.05	45,267	14,090	31,177	2,206.80	65,360.25	**1539**
1999	596,078.97	53,883	17,538	36,345	3,172.49	60,913.38	**969**
2000	463,607.30	40,662	14,177	26,485	2,824.13	35,936.64	**714**
2001	239,251.05	31,561	9,550	22,011	1,285.91	32,561.97	**1307**
2002	217,488.91	32,001	9,407	22,594	1,229.67	34,230.91	**1517**
2003	330,128.05	50,510	14,288	36,222	1,456.23	60,254.65	**1781**
2004	448,605.59	68,351	18,544	49,807	1,745.59	96,961.86	**2122**
2005	538,236.58	74,049	17,961	56,088	1,891.57	121,618.81	**2224**
TOTAL	**3,972,966.56**	**485,637**	**145,429**	**340,208**	**19,990.32**	**611,870.16**	**1490**

YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SV	G	SV	G	**BP**
1996	176,191.06	21,201	3,890	17,311	818.27	19,922.89	**1084**
1997	203,143.53	22,330	3,496	18,834	775.78	24,821.64	**1184**
1998	226,184.16	23,181	3,877	19,304	1,114.41	27,856.65	**1182**
1999	358,364.61	30,004	5,427	24,577	1,670.21	31,410.18	**830**
2000	290,954.73	22,619	5,033	17,586	2,106.77	20,164.53	**621**
2001	130,899.57	16,729	3,037	13,692	689.80	15,251.67	**1112**
2002	119,279.77	17,167	3,951	13,216	809.16	14,923.91	**1183**
2003	179,514.36	25,732	4,277	21,455	765.37	24,342.82	**1313**
2004	234,997.59	33,699	4,019	29,680	654.37	38,384.59	**1606**
2005	295,032.64	38,291	4,030	34,261	722.54	48,324.27	**1613**
TOTAL	**2,214,562.02**	**250,953**	**41,037**	**209,916**	**10,126.66**	**265,403.16**	**1153**

YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVSI	G	SVSI	G	**BP**
1996	176,191.06	21,201	3,801	17,400	828.76	18,435.70	**999**
1997	203,143.53	22,330	3,159	19,171	740.90	23,454.53	**1118**
1998	226,184.16	23,181	3,574	19,607	1,048.84	25,843.79	**1096**
1999	358,359.42	30,003	5,705	24,298	1,999.54	26,877.46	**694**
2000	290,954.73	22,619	5,385	17,234	2,411.71	16,456.51	**483**
2001	130,899.57	16,729	3,170	13,559	796.42	13,638.75	**981**
2002	119,279.77	17,167	4,027	13,140	818.59	13,283.30	**1045**
2003	179,514.36	25,732	4,123	21,609	774.09	22,456.05	**1208**
2004	235,000.94	33,700	3,812	29,888	706.17	36,684.86	**1531**
2005	295,034.19	38,292	3,736	34,556	719.34	44,840.69	**1495**
TOTAL	**2,214,561.73**	**250,954**	**40,492**	**210,462**	**10,844.36**	**241,971.65**	**1044**

YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVJ	G	SVJ	G	**BP**
1996	176,183.00	21,199	7,509	13,690	1,675.46	12,726.92	**627**
1997	203,143.53	22,330	7,603	14,727	1,862.56	16,009.22	**696**
1998	226,184.16	23,181	7,242	15,939	2,025.08	18,995.29	**750**
1999	358,364.61	30,004	10,398	19,606	3,222.69	19,176.95	**445**
2000	290,954.73	22,619	8,400	14,219	3,204.00	12,319.93	**313**
2001	130,896.89	16,728	5,674	11,054	1,198.92	10,067.44	**678**
2002	119,274.87	17,166	6,441	10,725	1,239.57	9,584.53	**700**
2003	179,511.36	25,731	8,516	17,215	1,469.75	16,301.17	**826**
2004	234,999.21	33,699	9,592	24,107	1,599.76	26,507.29	**1060**
2005	295,034.19	38,292	9,820	28,472	1,774.66	33,604.15	**1079**
TOTAL	**2,214,546.55**	**250,949**	**81,195**	**169,754**	**19,272.47**	**175,292.88**	**705**

Table 11: In the Money Option Basis Point Improvement of G vs. BS, SV, SVSI and SVJ by Leverage. The columns left to right represent the D/E ratio, the present value of all matched pairs for that D/E ratio, the total number of the matched pairs for that D/E ratio, the number of those matched pairs where an alternative model price is closer to the market price in absolute distance, the number of matched pairs where the G model price is closer to the market price, the dollar value of the alternative model price improvement, the dollar value of G improvement, and the net basis point advantage of G's model for that D/E ratio.

D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	BS	G	BS	G	BP
(0.00-0.10]	843,791.52	74,671	26,623	48,048	4,805.16	77,343.97	**860**
(0.10-0.20]	794,288.56	90,522	27,379	63,143	3,886.21	119,535.70	**1456**
(0.20-0.30]	582,667.38	71,915	20,729	51,186	2,561.26	108,186.78	**1813**
(0.30-0.60]	934,583.84	126,861	36,078	90,783	4,418.84	164,932.31	**1717**
(0.60-1.00]	459,269.37	69,741	19,440	50,301	2,247.09	84,286.08	**1786**
(1.00-1.50]	246,241.28	35,768	10,386	25,382	1,349.05	39,546.59	**1551**
(1.50-2.00]	112,124.61	16,159	4,794	11,365	722.72	18,038.72	**1544**
TOTAL	**3,972,966.56**	**485,637**	**145,429**	**340,208**	**19,990.32**	**611,870.16**	**1490**
D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SV	G	SV	G	BP
(0.00-0.10]	522,825.16	41,352	9,479	31,873	3,478.38	38,776.98	**675**
(0.10-0.20]	462,305.29	49,499	8,592	40,907	2,159.03	51,695.37	**1072**
(0.20-0.30]	315,026.26	36,646	5,045	31,601	1,068.28	45,155.55	**1399**
(0.30-0.60]	500,334.83	64,224	9,229	54,995	1,788.72	70,333.43	**1370**
(0.60-1.00]	237,689.25	34,914	4,914	30,000	819.90	35,807.21	**1472**
(1.00-1.50]	122,231.81	16,910	2,601	14,309	512.46	16,358.08	**1296**
(1.50-2.00]	54,149.42	7,408	1,177	6,231	299.90	7,276.54	**1288**
TOTAL	**2,214,562.02**	**250,953**	**41,037**	**209,916**	**10,126.66**	**265,403.16**	**1153**
D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVSI	G	SVSI	G	BP
(0.00-0.10]	522,825.16	41,352	10,222	31,130	4,004.71	32,267.46	**541**
(0.10-0.20]	462,305.29	49,499	8,521	40,978	2,319.21	46,892.72	**964**
(0.20-0.30]	315,029.61	36,647	4,871	31,776	1,040.33	42,014.09	**1301**
(0.30-0.60]	500,334.83	64,224	8,723	55,501	1,844.12	64,906.49	**1260**
(0.60-1.00]	237,685.61	34,914	4,626	30,288	837.72	33,425.69	**1371**
(1.00-1.50]	122,231.81	16,910	2,453	14,457	519.36	15,379.01	**1216**
(1.50-2.00]	54,149.42	7,408	1,076	6,332	278.90	7,086.20	**1257**
TOTAL	**2,214,561.73**	**250,954**	**40,492**	**210,462**	**10,844.36**	**241,971.65**	**1044**
D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVJ	G	SVJ	G	BP
(0.00-0.10]	522,817.93	41,350	15,343	26,007	5,095.49	23,931.86	**360**
(0.10-0.20]	462,297.39	49,497	16,497	33,000	4,122.48	34,619.04	**660**
(0.20-0.30]	315,029.61	36,647	10,438	26,209	2,239.02	31,409.57	**926**
(0.30-0.60]	500,334.83	64,224	19,964	44,260	4,040.63	46,281.83	**844**
(0.60-1.00]	237,685.56	34,913	10,831	24,082	1,960.75	23,305.66	**898**
(1.00-1.50]	122,231.81	16,910	5,581	11,329	1,171.22	10,806.14	**788**
(1.50-2.00]	54,149.42	7,408	2,541	4,867	642.89	4,938.77	**793**
TOTAL	**2,214,546.55**	**250,949**	**81,195**	**169,754**	**19,272.47**	**175,292.88**	**705**

Table 12: Out of the Money Option Basis Point Improvement of G vs. BS, SV, SVSI and SVJ by Year. The columns left to right represent the year, the present value of all matched pairs for that year, the total number of the matched pairs that year, the number of those matched pairs where an alternative model price is closer to the market price in absolute distance, the number of matched pairs where the G model price is closer to the market price, the dollar value of the alternative model price improvement, the dollar value of G improvement, and the net basis point advantage of G's model for that year.

YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	BS	G	BS	G	**BP**
1996	98,971.46	63,054	21,822	41,232	4,022.43	78,818.84	**7557**
1997	104,249.84	58,286	19,510	38,776	3,915.89	84,059.36	**7688**
1998	116,627.00	65,101	21,589	43,512	4,537.47	103,088.96	**8450**
1999	169,542.84	71,767	21,811	49,956	5,082.04	110,459.07	**6215**
2000	208,359.31	75,462	25,638	49,824	7,422.82	100,988.47	**4491**
2001	132,732.74	78,866	23,077	55,789	4,422.19	119,188.80	**8646**
2002	97,468.48	75,991	20,232	55,759	3,647.40	116,614.04	**11590**
2003	77,947.90	71,939	17,962	53,977	2,536.20	127,171.59	**15990**
2004	111,154.94	101,898	28,029	73,869	3,622.13	193,455.39	**17078**
2005	123,853.31	106,377	28,201	78,176	4,195.12	223,297.81	**17690**
TOTAL	**1,240,907.82**	**768,741**	**227,871**	**540,870**	**43,403.70**	**1,257,142.33**	**9781**
YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SV	G	SV	G	**BP**
1996	51,330.26	30,482	4,073	26,409	1,067.04	36,807.50	**6963**
1997	55,350.77	30,013	3,986	26,027	1,004.52	40,275.05	**7095**
1998	60,390.29	32,620	4,261	28,359	1,186.80	48,539.25	**7841**
1999	96,875.80	39,548	4,288	35,260	1,644.85	57,870.40	**5804**
2000	114,847.81	39,844	5,296	34,548	2,519.72	55,523.24	**4615**
2001	69,351.98	42,021	4,236	37,785	1,199.70	57,511.13	**8120**
2002	52,623.50	41,669	4,289	37,380	972.70	55,010.62	**10269**
2003	41,277.85	37,468	2,838	34,630	643.09	54,297.71	**12998**
2004	57,125.38	50,998	3,888	47,110	869.64	78,033.24	**13508**
2005	65,050.81	55,046	4,001	51,045	896.90	92,982.46	**14156**
TOTAL	**664,224.45**	**399,709**	**41,156**	**358,553**	**12,004.95**	**576,850.60**	**8504**
YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVSI	G	SVSI	G	**BP**
1996	51,330.26	30,482	5,534	24,948	1,457.59	27,771.45	**5126**
1997	55,350.77	30,013	5,303	24,710	1,340.26	30,836.87	**5329**
1998	60,390.32	32,621	5,397	27,224	1,506.15	37,514.76	**5963**
1999	96,875.80	39,548	6,104	33,444	2,408.57	43,367.37	**4228**
2000	114,844.02	39,844	7,491	32,353	3,410.52	37,917.69	**3005**
2001	69,351.98	42,021	5,605	36,416	1,689.68	44,599.34	**6187**
2002	52,623.72	41,670	4,940	36,730	1,209.90	44,268.83	**8182**
2003	41,277.85	37,468	3,167	34,301	740.78	45,789.70	**10914**
2004	57,125.38	50,998	4,545	46,453	1,039.64	65,914.20	**11357**
2005	65,050.81	55,046	5,325	49,721	1,157.94	74,592.13	**11289**
TOTAL	**664,220.91**	**399,711**	**53,411**	**346,300**	**15,961.04**	**452,572.34**	**6573**
YEAR	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVJ	G	SVJ	G	**BP**
1996	51,330.26	30,482	10,757	19,725	2,790.65	23,359.48	**4007**
1997	55,350.77	30,013	10,338	19,675	2,755.15	26,494.18	**4289**
1998	60,390.32	32,621	10,414	22,207	2,900.70	34,623.07	**5253**
1999	96,875.80	39,548	11,907	27,641	3,784.89	37,939.73	**3526**
2000	114,846.24	39,844	12,364	27,480	4,853.99	34,686.91	**2598**
2001	69,351.98	42,021	11,772	30,249	3,198.85	38,449.13	**5083**
2002	52,623.47	41,670	12,688	28,982	3,028.31	35,311.75	**6135**
2003	41,277.85	37,468	9,416	28,052	1,871.34	38,224.47	**8807**
2004	57,122.05	50,995	12,278	38,717	2,426.51	56,636.48	**9490**
2005	65,050.61	55,045	12,687	42,358	2,716.81	67,930.41	**10025**
TOTAL	**664,219.35**	**399,707**	**114,621**	**285,086**	**30,327.19**	**393,655.60**	**5470**

Table 13: Out of the Money Option Basis Point Improvement of G vs. BS, SV, SVSI and SVJ by Leverage. The columns left to right represent the D/E ratio, the present value of all matched pairs for that D/E ratio, the total number of the matched pairs for that D/E ratio, the number of those matched pairs where an alternative model price is closer to the market price in absolute distance, the number of matched pairs where the G model price is closer to the market price, the dollar value of the alternative model price improvement, the dollar value of G improvement, and the net basis point advantage of G's model for that D/E ratio.

D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	BS	G	BS	G	BP
(0.00-0.10]	244,864.80	94,124	33,002	61,122	9,264.33	114,277.68	**4289**
(0.10-0.20]	248,380.07	135,081	39,364	95,717	8,493.22	230,658.20	**8945**
(0.20-0.30]	171,881.89	107,515	30,692	76,823	5,549.48	216,696.30	**12284**
(0.30-0.60]	307,088.04	211,527	61,224	150,303	10,199.87	355,311.28	**11238**
(0.60-1.00]	151,090.87	125,908	35,329	90,579	5,221.39	203,333.54	**13112**
(1.00-1.50]	76,513.58	63,126	18,770	44,356	2,934.87	86,917.13	**10976**
(1.50-2.00]	41,088.57	31,460	9,490	21,970	1,740.54	49,948.20	**11733**
TOTAL	**1,240,907.82**	**768,741**	**227,871**	**540,870**	**43,403.70**	**1,257,142.33**	**9781**
D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SV	G	SV	G	BP
(0.00-0.10]	137,144.28	50,080	8,043	42,037	3,416.30	62,748.01	**4326**
(0.10-0.20]	139,899.55	74,312	7,905	66,407	2,600.75	108,783.91	**7590**
(0.20-0.30]	89,960.33	55,904	5,050	50,854	1,460.86	94,618.74	**10355**
(0.30-0.60]	162,351.41	108,753	9,829	98,924	2,463.04	161,447.65	**9793**
(0.60-1.00]	77,934.74	65,442	5,997	59,445	1,084.50	89,945.62	**11402**
(1.00-1.50]	37,545.37	30,531	2,968	27,563	672.10	38,143.77	**9980**
(1.50-2.00]	19,388.77	14,687	1,364	13,323	307.41	21,162.90	**10756**
TOTAL	**664,224.45**	**399,709**	**41,156**	**358,553**	**12,004.95**	**576,850.60**	**8504**
D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVSI	G	SVSI	G	BP
(0.00-0.10]	137,140.40	50,079	10,187	39,892	4,514.28	44,983.79	**2951**
(0.10-0.20]	139,899.55	74,312	10,123	64,189	3,484.15	83,382.74	**5711**
(0.20-0.30]	89,960.98	55,905	6,552	49,353	1,846.31	76,273.74	**8273**
(0.30-0.60]	162,350.96	108,753	12,948	95,805	3,311.38	127,073.61	**7623**
(0.60-1.00]	77,934.88	65,444	7,788	57,656	1,469.04	72,865.41	**9161**
(1.00-1.50]	37,545.37	30,531	3,941	26,590	881.71	30,442.37	**7873**
(1.50-2.00]	19,388.77	14,687	1,872	12,815	454.18	17,550.67	**8818**
TOTAL	**664,220.91**	**399,711**	**53,411**	**346,300**	**15,961.04**	**452,572.34**	**6573**
D/E	PV	NUMBER	NUMBER	NUMBER			
		TOTAL	SVJ	G	SVJ	G	BP
(0.00-0.10]	137,141.58	50,079	17,673	32,406	6,551.02	38,465.09	**2327**
(0.10-0.20]	139,899.55	74,312	21,607	52,705	6,499.77	75,581.55	**4938**
(0.20-0.30]	89,958.59	55,903	14,245	41,658	3,741.18	69,448.93	**7304**
(0.30-0.60]	162,350.61	108,751	30,307	78,444	7,198.69	110,761.15	**6379**
(0.60-1.00]	77,934.88	65,444	17,740	47,704	3,382.44	60,236.59	**7295**
(1.00-1.50]	37,545.37	30,531	9,032	21,499	1,959.41	24,845.90	**6096**
(1.50-2.00]	19,388.77	14,687	4,017	10,670	994.67	14,316.39	**6871**
TOTAL	**664,219.35**	**399,707**	**114,621**	**285,086**	**30,327.19**	**393,655.60**	**5470**

Table 14: Implied Parameters For Pan(2002)'s Models.

The structural parameters of a given model are estimated by firm by IS-GMM. The first line is the sample average of the estimated parameters; the second line is the standard errors in parentheses. Following Pan2002 Pan2002 , the structural parameters' definitions are as the following: κ_v is the mean-reversion rate, $\bar{v}$ is the constant long-run mean, σ_v is the volatility coefficient, ρ is the correlation of the Brownian shocks to price S and volatility V , λ is the constant coefficient of the state-dependent stochastic jump intensity λV_t , μ is the mean relative jump size under the physical measure, η^s is the constant coefficient of the return risk premium, η^v is the constant coefficient of the volatility risk premium, μ^* is the mean jump size of the jump amplitudes U^S under the risk-neutral measure and σ_J is the variance of the jump amplitudes U^S under the risk-neutral measure. SV0, SV, SVJ0, and SVJ, respectively, stand for the no risk premia model, the volatility-risk premia model, the jump-risk premia model and the volatility and jump risk premia model. For conciseness, the reported are the average of each parameter across all the firms.

Parameters	**SV0**	**SV**	**SVJ0**	**SVJ**
κ_v	16.31	24.40	18.22	12.08
	(5.92)	(4.57)	(8.12)	(8.61)
$\bar{v}$	0.02	0.01	0.01	0.01
	(0.02)	(0.01)	(0.01)	(0.01)
σ_v	0.63	0.67	0.56	0.57
	(0.31)	(0.39)	(0.19)	(0.16)
ρ	−0.64	−0.69	−0.59	−0.59
	(0.36)	(0.40)	(0.14)	(0.15)
η^S	3.71	1.02	−0.73	−0.46
	(0.93)	(1.61)	(3.14)	(3.33)
η^v		1.25		−0.51
		(2.91)		(3.69)
λ			10.33	11.13
			(3.89)	(3.81)
μ_J			−0.17	−6.36
			(13.78)	(15.24)
σ_J			3.76	4.13
			(3.23)	(3.09)
μ^*			−11.94	−9.10
			(12.31)	(10.44)
Implied Volatility (%)	56.11	50.71	45.45	35.47

APPENDIX I

In this appendix we discuss comparisons of BS, G, BCC and Pan. Where possible we tried to implement the different models with the same methodology. This was simple for BS and G because BS is a special case of G. In both BS and G we can *imply* the parameters directly from contemporaneous prices of the stock and at-the-money options on the stock. We also used the alternate volatility estimation methodology of finding the volatility that minimizes the sum of squared errors for pricing equity index options on any day. This comparison allows us to show that the G model dominates BCC and BS when the models are implemented with identical methodologies. Furthermore, we lag the volatility estimate by one day in order for the estimate to be out of sample, as in BCC. As mentioned above, this methodology is necessary to implement models such as BCC which assume many other stochastic complexities and require many more option prices in order to estimate their required parameters. Appendix II reproduces Table 3 from BCC (1997) and illustrates that BS (1973) does much worse without a volatility term structure primarily because BCC has 5 to 9 times as many parameters as BS which has only 1 parameter. We are able to effectively reproduce Table 3. In the comparisons with Pan we do not use the implied state generalized method of moments technique for the competing models.

APPENDIX II

The equation below from BCC, p. 210, # 7, describes the dynamics for all three BCC embedded models SV, SVSI, and SVJ subject to the relevant parameters and boundary condition for a put or call option.

$$\frac{1}{2} VS^2 \frac{\partial^2 C}{\partial S^2} + [R - \lambda \mu_J] S \frac{\partial C}{\partial S} + \rho \sigma_v VS \frac{\partial^2 C}{\partial S \partial V} + \frac{1}{2} \sigma_v^2 V \frac{\partial^2 C}{\partial V^2} + [\theta_v - \kappa_v V] \frac{\partial C}{\partial V}$$
$$+ \frac{1}{2} \sigma_R^2 R \frac{\partial^2 C}{\partial R^2} + [\theta_R - \kappa_R R] \frac{\partial C}{\partial R} - \frac{\partial C}{\partial \tau} - RC$$
$$+ \lambda E\{C(t, \tau; S(1 + J), R, V) - C(t, \tau; S, R, V)\} = 0.$$

The table below from BCC, p. 218, Table 3, shows a parameterization for all three BCC models.

Implied Parameters and In-Sample Fit

Each day in the sample, the structural parameters of a given model are estimated by minimizing the sum of squared pricing errors between the market price and the model-determined price for each option. The daily average of the estimated parameters is reported first, followed by its standard error in parentheses. The parameters in the groups under "All Options", "Short-Term Options", and "At-the-Money Options" are obtained by respectively using all the available options, only short-term options, and only ATM options in the day as input into the estimation. For each model, SSE in a given column group denotes the daily average sum of squared errors for all options after the All-Options-Based, Maturity-Based, or Moneyness-Based treatment. The structural parameters κ_v, θ_v/κ_v, and σ_v (κ_R, θ_R/κ_R, and σ_R) are respectively the speed of adjustment, the long-run mean, and the variation coefficient of the diffusion volatility $V(t)$ (the spot interest rate $R(t)$). The parameter μ_J represents the mean jump size, λ the frequency of the jumps per year, and σ_J the standard deviation of the logarithm of one plus the percentage jump size. V_J is the instantaneous variance of the jump component. BS, SV, SVSI, and SVJ, respectively, stand for the Black-Scholes, the stochastic-volatility model, the stochastic-volatility and stochastic-interest-rate model, and the stochastic-volatility model with random jumps.

	All Options				Short-Term Options				At-the-Money Options			
Parameters	BS	SV	SVSI	SVJ	BS	SV	SVSI	SVJ	BS	SV	SVSI	SVJ
κ_v		1.15	0.98	2.03		1.62	1.47	3.93		0.99	0.71	1.74
		(0.03)	(0.04)	(0.06)		(0.09)	(0.08)	(0.08)		(0.02)	(0.02)	(0.04)
θ_v		0.04	0.04	0.04		0.04	0.04	0.04		0.04	0.04	0.04
		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
σ_v		0.39	0.42	0.38		0.44	0.45	0.40		0.40	0.43	0.40
		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)		(0.00)	(0.00)	(0.00)
ρ		−0.64	−0.76	−0.57		−0.76	−0.80	−0.52		−0.70	−0.79	−0.58
		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)		(0.01)	(0.01)	(0.01)
λ				0.59				0.61				0.68
				(0.02)				(0.02)				(0.02)
μ_J				−0.05				−0.09				−0.04
				(0.00)				(0.00)				(0.00)
σ_J				0.07				0.14				0.06
				(0.00)				(0.00)				(0.00)
$\sqrt{V_J}$ (%)				6.15				12.30				6.65
				(0.22)				(0.17)				(0.21)
κ_R			0.58				0.40				0.69	
			(0.02)				(0.02)				(0.02)	
θ_R			0.02				0.02				0.02	
			(0.00)				(0.00)				(0.00)	
σ_R			0.03				0.03				0.03	
			(0.00)				(0.00)				(0.00)	
Implied	18.23	18.66	18.65	19.38	18.15	18.45	18.54	20.65	18.74	18.48	18.36	19.03
Volatility (%)	(0.14)	(0.14)	(0.15)	(0.16)	(0.14)	(0.14)	(0.14)	(0.15)	(0.14)	(0.14)	(0.15)	(0.16)
SSE	69.60	10.63	10.68	6.46	28.09	5.48	5.16	2.63	25.34	5.98	5.45	5.31