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In the modern advertising agency selection contest, each participating agency specifies not only its proposed creative campaign but also the budget required to purchase the agreed-on media. The advertiser selects the agency that offers the best combination of creative quality and media cost, similar to conducting a score auction. To participate in the contest, each agency needs to incur an up-front bid-preparation cost to cover the development of a customized creative campaign. Agency industry literature has called for the advertiser to fully reimburse such costs to all agencies that enter the contest. The authors analyze the optimal stipend policy of an advertiser facing agencies with asymmetric bid-preparation costs, such that the incumbent agency faces a lower bid-preparation cost than a competitor agency entering the contest. The authors show that reimbursing bid-preparation costs in full is never optimal, nor is reimbursing any part of the incumbent's bid-preparation cost. However, a stipend that is strictly lower than the competitor's bid-preparation cost can benefit the advertiser under certain conditions. The authors provide a sufficient condition (in terms of the distribution of agency values to the advertiser) for such a new-business stipend to benefit the advertiser.

Keywords: advertising agencies, contests, score auctions

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The Modern Advertising Agency Selection Contest: A Case for Stipends to New Participants

Advertising is one of the most important and expensive marketing activities in which any firm engages. In 2015, advertisers were projected to spend $592 billion on advertising worldwide, an increase of 6% over 2014 (eMarketer 2014). The United States is the dominant advertising market—spending by U.S. firms accounts for approximately one-third of the worldwide total. The majority of U.S. advertisers hire full-service advertising agencies to both develop and deliver their communication strategies (Horsky 2006). To select an agency, advertisers periodically hold a contest among several candidate agencies. In this article, we ask whether and when the advertiser looking to hire a full-service agency should offer stipends to the contest participants.

The advertising agency selection contest departs from many other procurement situations in that the participants incur a high cost in preparing each bid. Each agency needs to customize its product to the advertiser's specific business: to participate in a contest, an agency has to assign a dedicated team to develop its pitch, conduct marketing research specific to the advertiser's campaign goals, design several alternative creative approaches, and perform preliminary copy testing. We ask whether the advertiser should offer stipends to help defray these bid-preparation costs and thus encourage more agencies to participate in the contest. A participation stipend is different from the winner's compensation because it is awarded regardless of whether the agency wins the contract. The advertising industry press

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continuously debates the stipend question. Naturally, on the one hand, the various agency associations argue that agencies should always be compensated in full for their up-front work. For example, the Institute for Canadian Advertising strongly protested the Bell Canada contest in which the advertiser did not cover agencies' full bid-preparation costs (Brendan 1998). Gardner (1996, p. 33) expresses this position eloquently: "If you insist that the finalist agencies demonstrate their creative abilities on your product, thinking like professionals and working like professionals, then you should treat them like professionals. Pay them. You wouldn't do less for your lawyers, bankers or accountants." On the other hand, advertisers may perceive such compensation as an added and unnecessary cost. In practice, recent surveys have indicated that approximately half of major advertisers offer a stipend to cover some of the agencies' bid-preparation costs (American Association of Advertising Agencies [AAAA] 2007; Parekh 2009). Moreover, about 30% of agencies say they will pitch only if provided with an up-front fee (Borgwardt 2010).

The following specific managerial questions emerge from the industry debate: Why are stipends offered only in some of today's contests, and how can an advertising manager decide whether to offer such stipends in his or her particular contest? Why do real-world stipends tend to cover only a portion of the bid-preparation cost, and how should managers determine the best possible stipend level to offer? Why are stipends, when offered, often offered only to new participants and not to incumbent agencies? Should the advertising firm ever offer a stipend to all agencies?

Historically, industry practice fixed the compensation of the winning agency to 15% of the list price of media billings. As a result of the fixed compensation, the contest traditionally focused on selecting the agency with the best creative idea (Gross 1972). Currently, the media-buying aspect of the service is highly competitive, and only 5% of U.S. firms continue to compensate by a percentage of media billings based on the list price (Association of National Advertisers 2013). Instead, a modern full-service agency contest solicits each agency's bid of a media price in addition to its creative idea, recognizing that different agencies face different media costs owing to economies of scale and scope (Silk and Berndt 1993). The advertiser combines the creative and financial aspects of each pitch into an overall evaluation of each agency, often using a scorecard to keep track of the different aspects of those pitches (Argent 2014; Buccino 2009; Medcalf 2006; TheDrum.com 2010). The agency whose combination of creative quality and media price delivers the highest profit to the advertiser wins the contract. The contemporary advertising contest has thus evolved to resemble a score auction—a mechanism often used in other procurement settings to facilitate competition among suppliers with different costs and qualities (Beil and Wein 2003; Che 1993).

The lack of a theoretical or practical industry consensus regarding stipends calls for a careful analysis within a game-theoretic model that captures the essence of the contest and the entry game among invited agencies that precedes the contest. We propose to capture the essence of the modern advertising contest by a score auction with asymmetric bid-preparation costs and up-front participation stipends. We do not claim that real-world contests exactly follow the rules of a score auction as, for example, a government procurement contest would. The advertiser does not actually announce the bidding and scoring rules and can engage the contestants in additional price negotiations after the initial pitch. We thus use the score auction as a parsimonious model of both contract allocation and the price paid to the winning agency. Next, we provide an overview of our modeling assumptions.

To model the entry into the contest, we consider two kinds of agencies common within the advertising contest scene: (1) the incumbent agency, which is familiar with the advertiser and its industry and thus faces lower bid-preparation costs, and (2) a competing agency trying to win the advertiser's business.[1] In addition to the difference in bid-preparation costs, the agencies also differ in their ability to increase the advertiser's profit, which we call the "value" of an agency to the advertiser. The value of an agency arises from a combination of its expected creative quality and its media costs, the latter of which is private information of each agency. The model we propose begins with the advertiser publicly announcing the stipends available to each agency on entry into the contest, with the stipend to the competitor usually called a "new-business stipend" in the industry. The agencies then consider each other's incentives within an entry game and enter when their equilibrium expected surplus from participation exceeds the part of their bid-preparation cost that the stipend does not defray. In the final stage, the agencies that decided to enter bid in a score auction. Specifically, each agency reveals its creative quality to the advertiser during the pitch and submits a media-price bid, allowing the score auction to rank all contestants in terms of their profitability to the advertiser.

Two technical questions underlie all of the previously stated managerial questions: When is the higher advertiser profit that results from more contest participants worth the increased up-front cost of providing participation stipends? And how should these stipends depend on the agencies' bid-preparation costs and on the distribution of agencies' values to the advertiser? We find that the asymmetry in bid-preparation costs between the incumbent and the competitor is necessary for stipends to benefit the advertiser: when the agencies face the same bid-preparation cost (e.g., when they are both bidding for new business and the incumbent does not participate either because it was terminated by the advertiser or because it resigned the account to service a competitor), the advertiser should offer no stipends. When one agency's bid-preparation cost is lower than that of the other agency (we call the lower-cost agency an "incumbent"; see footnote 1), we obtain a general characterization of the optimal stipend scheme: First, we show that the incumbent should not receive a stipend, but a new-business stipend strictly lower than the competitor's bid-preparation cost can benefit the advertiser under certain

[1]"Incumbent" can be considered a mere label of the agency that faces a lower bid-preparation cost for another reason. To focus our analysis on the implication of asymmetry in bid-preparation costs, we assume that the two agencies are otherwise symmetric (i.e., their potential values to the advertiser are drawn from the same distribution). We thus abstract away from other advantages an actual incumbent might have.

conditions. Second, we provide a sufficient condition (in terms of the distribution of agency values) for a new-business stipend to benefit the advertiser.

To characterize when new-business stipends benefit the advertiser, and to illustrate how one can apply our sufficient condition, we then consider several tractable distributional families of values to advertiser in the population of agencies. We find that our sufficient condition is satisfied by a wide range of distributions, but we also find distributions that do not support any new-business stipends even when the incumbent has a lower bid-preparation cost than the competitor. Specifically, the advertiser should not offer any stipends when the population of agencies contains relatively many weak agencies that can deliver only a small profit to the advertiser. We also explore how our results would generalize under alternative sets of assumptions and find that they are robust to (1) adding more competitors, (2) changing the amount of information each agency has about the quality of its creative idea before entering the contest, and (3) situations in which the advertiser uses a reserve price in addition to stipends.

RELATED LITERATURE

This article contributes to the small amount of quantitative marketing literature concerning advertising agencies (i.e., Gross 1972; Horsky 2006; Silk and Berndt 1993; Villas-Boas 1994). Within this literature, the most related study is by Gross (1972), who examines the selection of the best creative campaign while keeping constant the media budget and the prize to the contest winner. At the time of Gross's writing, the advertising environment was different in that remunerations of agencies were standardized and the pitch did not include the media budget (for a detailed discussion, see the previous section). He correctly identifies the importance of evaluating several creative campaigns and screening them accurately, but he does not consider the stipend as a strategic variable. This article endogenizes the stipend decision and extends the analysis to the modern contests that solicit each agency's bid of a media price in addition to a demonstration of its creative quality. Subsequently, we also extend our analysis "back in time" and briefly consider the incentives for stipends in twentieth-century quality-only contests with a fixed prize for the winner. We find that new-business stipends can also be optimal in such a model, but stipends to the lower-cost "incumbent" agencies cannot be categorically ruled out.

Horsky (2006) is the first to address the bidimensionality of the modern advertising pitch. She documents the emergence of competition in the media-buying component and the appearance of the specialized media shops, which have the advantage of being media market makers because they are not bound by the industry practice of not working with rival advertisers. Unlike Horsky (2006), we restrict our work to the competition among the still-dominant full-service agencies and the process by which a specific one is chosen.

Although the agency selection process we study is usually called a "contest," we do not analyze a typical contest as conceptualized in the contest theory literature, beginning with Tullock (1980). Instead, we argue that the modern advertising agency selection "contest" resembles a scoring auction. Nevertheless, even in classic Tullock contests, the principal finds it optimal to level the playing field using "handicapping" policies that sometimes let a weaker player win with an inferior performance. We also find that the principal ("advertiser" in our nomenclature) wants to level the playing field by subsidizing the bidder with a higher participation cost. Therefore, leveling the playing field by somehow favoring or helping the weaker player seems to be a general idea, and we contribute by characterizing how and when participation stipends can level the playing field in procurement auctions with endogenous entry and asymmetric bid-preparation costs. We discuss our contribution to the procurement auction literature next, especially as it relates to leveling the playing field.

Starting with McAfee and McMillan (1989), the procurement-auction literature shows why a buyer facing asymmetric bidders may benefit from leveling the playing field using price-preference subsidies. Specifically, McAfee and McMillan analyzed government procurement price-preference policies that subsidize bids of weaker (higher-cost) bidders in first-price sealed-bid auctions and demonstrated that the buyer (i.e., the government) may benefit from such policies relative to when the contract is simply awarded to the lowest bidder. The reason costly price-preference policies can reduce the buyer's procurement cost is that they put pressure on the stronger bidders to lower their bids, and the resulting increase in competition can outweigh the inefficiency of not assigning the contract to the lowest-cost supplier. Flambard and Perrigne (2006) apply the theory to snow-removal contracts in Montreal, demonstrating that real-world asymmetries can be sufficiently strong to make price-preference policies profitable for the buyer. Branco (2002) shows that protection of the weaker bidders may provide an incentive for them to adopt more efficient technologies, which will eventually lower their costs (in the long run). Krasnokutskaya and Seim (2011) extend the theory of price-preference policies in first-price sealed-bid procurement auctions to the more realistic situation of endogenous entry. Following the endogenous-entry paradigm of Levin and Smith (1994), Krasnokutskaya and Seim's bidders need to incur a cost to learn their cost types and participate in the auction. No closed-form entry or bidding policies exist in the resulting model, so Krasnokutskaya and Seim use numerical methods to estimate equilibria under the model parameters calibrated on California highway procurement. They conclude that endogenous entry plays an important role in determining the optimal price-preference policy and its potential benefit to the buyer.

Analogous to the price-preference literature analyzing an existing yet controversial institution for leveling an asymmetric playing field in government procurement, we analyze an existing and controversial field-leveling practice in advertising agency selection contests: namely, participation stipends. The most closely related model is that by Gal, Landsberger, and Nemirovski (2007), who consider stipends in a different setting without an incumbent and without the auctioneer having any knowledge about the realized asymmetry in bid-preparation costs before the game. Like Gal, Landsberger, and Nemirovski, we study the second-price sealed-bid auction. The key benefit of this pricing rule is that, unlike Krasnokutskaya and Seim (2011), we can analyze a model with endogenous entry in closed form. In contrast to Gal, Landsberger, and Nemirovski, we allow the advertiser to know the extent of the entry-cost asymmetry between the incumbent and the competitor. Our results differ from theirs in that we find distributions for which entry subsidies are not optimal.

Like Gal, Landsberger, and Nemirovski (2007), and in contrast to Krasnokutskaya and Seim (2011), we endogenize entry by following Samuelson's (1985) framework to capture the idea of the agencies' costly bid preparation. He shows that when all bidders face the same bid-preparation cost and know their valuations of the auctioned object before they make their entry decisions, the entry game involves a selection of higher-value bidders. The endogenous distribution of participating bidders in turn influences the reserve price the auctioneer should use, and the auctioneer may want to limit the number of invitees. In contrast to Samuelson's key assumption, we allow the bid-preparation costs to differ across the bidders (capturing an important advantage of incumbency) and consider participation stipends. We find that this asymmetry is necessary for stipends to benefit the advertiser.

We also consider (in an extension of our main model) the possibility that agencies do not know their value to the advertiser when they enter the contest, and they need to incur a cost to learn it. This alternative assumption about endogenous entry is analogous to Levin and Smith's (1994) model used by Krasnokutskaya and Seim (2011). We extend Levin and Smith's model to asymmetric entry costs and confirm that an asymmetry in bid-preparation costs is necessary for stipends to benefit the advertiser, and the stipend should not reimburse any agency in full. In another extension, we allow the advertiser to charge a strategic reserve price, and we show that a new-business stipend continues to benefit the advertiser, with the benefit increasing in the amount of bid-preparation cost asymmetry.

MODEL: AN AUCTION WITH ASYMMETRIC BID-PREPARATION COSTS

To describe our model, we introduce the players (advertiser and agencies), define the rules of the auction-based contest with potential participation stipends, and discuss some of the key simplifying assumptions that make our analysis tractable. Table 1 summarizes the notation we use in our main model. Next, we describe the different actors in our model and their motivations.

Table 1
NOTATION

Symbol	*Definition*
i	Index of agencies; i = 0 represents the incumbent and i = 1 represents the competitor
x_i	The value of agency i to the advertiser (gain from trade between i and the advertiser)
k_i	Agency i's bid-preparation cost
V	The maximum value x_i in the population of agencies
F	Distribution of value x_i in the population of agencies
r_i	The stipend the advertiser offers to agency i
L_i	The entry threshold value such that agency i with $x_i \geq L_i$ enters
Π	The advertiser's expected profit
S_i	Agency i's expected surplus from participating in the contest
R	The reserve price (in Extension 3)
α	The mass point at zero (in Extension 5)
H	The distribution of advertising qualities (in the section "A Model of The Past")
P	The fixed contest prize (in the section "A Model of The Past")

Advertiser

A firm ("the advertiser" hereinafter) is soliciting a contract with a full-service advertising agency to develop and deliver a fixed amount of advertising exposures (e.g., measured in Gross Rating Points). When the advertiser does not hire an agency, it receives an outside-option payoff, which we normalize to zero.

Agencies

Two agencies qualified to bid on the contract are indexed by i = 0, 1: the incumbent agency i = 0 currently serves the advertiser, and the competitor agency i = 1 is interested in competing for the advertiser's business.[2] Each agency has its own creative quality q_i and media cost c_i. The quality q_i corresponds to the advertiser's profit lift from using agency i's creative in the entire campaign, whereas the cost c_i is agency i's cost of delivering the creative to consumers through the appropriate media (including any costs of servicing the advertiser, such as producing the final polished advertising copy).

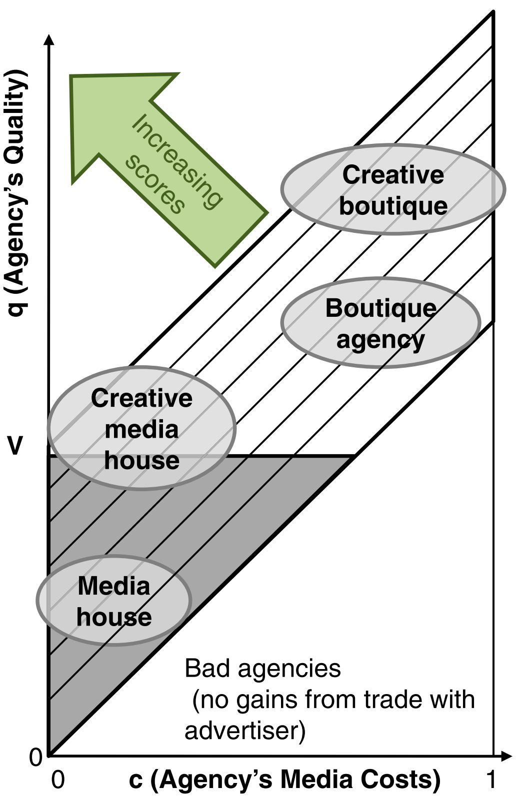
We assume that both q_i and c_i are private information of the agencies at the beginning of the game,[3] contest entrants' q_i is revealed during the pitch, and c_i remains private information throughout the game. At the beginning of the game, the advertiser only knows that $x_i \equiv q_i - c_i$ is distributed i.i.d. according to some distribution F(x) in the population of agencies. F has full support on the [0,V] interval. We consider only agencies with $q_i \geq c_i$ because agencies with $q_i < c_i$ cannot outperform the advertiser's outside-option profit (normalized to zero). Including them would introduce the possibility of no trade, but it would not change our qualitative results.

We call $x_i \equiv q_i - c_i$ the value of agency i to the advertiser. More precisely, x_i is the expected increase in profits (relative to the outside option) that the advertiser would make if agency i serviced the advertiser and delivered the contractual amount of advertising at its media cost c_i. Most of our results depend only on the distribution of x_i, but its decomposition into two dimensions (q, c) captures several diverse agency types that might be competing for the contract: a small creative boutique agency with little media clout is captured as (high c, high q), whereas a pure media house with a lot of media clout but little creative ability is captured as (low c, low q). For an illustration of these representative agencies, see Figure 1. Note that although they are different in terms of qualities and costs, the creative boutique can be quite similar to the media house in terms of value to the advertiser.

[2]Focusing on a single competitor greatly simplifies the analysis while exposing intuition. In the "Extensions" section, we generalize a special tractable case of the model to an arbitrary number of competitors.

[3]To learn its quality, each agency analyzes its initial creative ideas, available artists, and the nature of its match with the advertiser. This assumption results in a selection of higher-value agencies during the entry stage, but this selection is not critical to most of our results. In the "Extensions" section, we consider agencies that know only as much as the advertiser about their values before the contest, and we show that most of our results continue to hold.

Figure 1
DISTRIBUTION OF ADVERTISING AGENCIES IN (q,c) SPACE



Notes: The diagonally hatched parallelogram is the support P of joint distribution G. Uniform distribution on P yields the Uniform[0,V] distribution of scores $x = q - c$. Uniform distribution on the shaded triangle yields "decreasing-triangle" distribution $F(x) = 1 - (V - x)^2/V^2$.

Figure 1 also illustrates how the value distribution F may arise from an underlying bivariate distribution G of agency types (q_i, c_i): without loss of generality, we normalize the highest possible c_i to 1, so $0 \le c_i \le 1$, and allow an upper-limit V on the added value x_i that any agency can provide. As discussed previously, we consider only agencies with $q_i \ge c_i$. Therefore, we focus on G with support on a parallelogram defined by $\mathbf{P} \equiv \{(c_i, q_i)|c_i \in [0,1] \text{ and } q_i \in [c_i, c_i + V]\}$. The bivariate distribution G then implies a univariate $F(x) = \int_{(q,c)\in \mathbf{P}} \mathbf{1}(q - c \le x)dG(q,c)$ with support [0,V], where $\mathbf{1}$ is the indicator function.

Several of our example F distributions arise naturally from a simple bivariate G: a uniform G distribution with full support on $\mathbf{P}$ implies a uniform F on [0,V]. A uniform G distribution on $\mathbf{P}$ with an upper bound V on quality such that $q \le V \le 1$ implies a decreasing triangle distribution $F(x) = 1 - [(V - x)^2/V^2]$ with support on [0,V].

In addition to its own value x_i, each agency faces a fixed bid-preparation cost k_i of preparing a professional pitch specific to the advertiser's products. The activities that k_i covers include development and testing of creative ideas before the pitch as well as building and maintaining a relationship with the advertiser. Unlike the media and servicing cost c_i, which is incurred only by the winning agency, all agencies entering the contest need to spend their k_i to participate. The incumbent agency is already familiar with the advertiser and the industry, so its bid-preparation cost is lower than that of the competitor: $0 \le k_0 \le k_1$. It is natural to assume that everyone knows which agency is the incumbent and which is the competitor. Moreover, everyone also knows the approximate magnitude of these bid-preparation costs because agency executives discuss bid-preparation costs publicly (Medcalf 2006), and the costs mostly arise from easily estimated personnel hours needed to prepare a competitive pitch.[4] To simplify analysis, we let the bid-preparation costs be common knowledge in the beginning of the game. We believe our key results would continue to hold qualitatively even if the advertiser's belief also included some uncertainty around each k_i.

A key difference between the cost of participation k and the cost of production c is that k is verifiable. In the advertising example, the cost of creative development can be supported with invoices for materials and hours of employee time spent, whereas the cost of subsequent media buying remains private and inherently unverifiable.

Contest Rules and Timing of the Game

The contest game proceeds in three stages (for the timeline, see Figure 2): First, the advertiser announces the stipend r_i payable to agency i on its entry into the contest. In the second stage, the agencies get their private signals about the potential value x_i that they can deliver to the advertiser, and they play a simultaneous-move entry game to decide whether to sink the k_i and enter the contest. At this stage, each agency also submits receipts to substantiate the costs it incurred in preparing its bid, k_i.[5] In the final stage, the entrants bid for the contract in an auction that determines the winner and the contract price the advertiser pays for the winner's services. We discuss the score-auction mechanism next.

In the contest stage, the advertiser runs a second-score sealed-bid auction (Che 1993) with a reserve price of zero[6] to both allocate the contract and determine the price. In the auction, each agency pitches its creative ideas, credibly revealing its quality q_i, and bids an amount of money bid_i it is willing to accept for servicing the contract and delivering the resulting advertisements. The advertiser ranks bidders in terms of their proposed profit lift $q_i - bid_i$, awards the contract to the agency with the highest profit lift, and pays the winning agency the difference between its proposed lift and either the reserve price (if only one agency enters the contest) or the lift of the runner-up agency (when both enter). This auction is known to give each agency the incentive to bid its true cost c_i, so the auction is isomorphic to one in which agencies bid their values x_i, and the highest-value bidder wins and receives the difference between the two values as its compensation. The weakly dominant

[4]For a sense of magnitude, we note that according to the chief executive officer of M&C Saatchi, the cost of developing the average pitch is approximately \$100,000 internally (mainly in terms of time cost) and \$30,000 externally (Medcalf 2006).

[5]The submission of receipts to substantiate ongoing labor and other costs is common practice in the advertising agency industry and limits the potential for a moral hazard problem whereby agencies enter unprepared merely to collect the stipend. Reputation concerns are another reason such behavior should not happen.

[6]For a version of the model with a strategic reserve price, see the "Extensions" section.

Figure 2
TIMELINE OF THE GAME

strategy to bid value in a second-score auction with exogenous qualities (Che 1993; Vickrey 1961) shows that both the entry equilibrium and the auction price depend only on the agencies' values x_i, not on their underlying combinations of quality and media cost. It also follows that when only one agency enters the contest, it captures its entire value to the advertiser.

DISCUSSION OF ASSUMPTIONS

Having outlined our model, we next discuss its key assumptions. Our most important simplifying assumption is that the bid-preparation costs are known, but the advertiser cannot infer or influence the resulting quality. This assumption acknowledges that the creative quality-production process is idiosyncratic and noisy in the advertising setting. An unpredictable element of luck exists, coupled with difficult-to-predict agency-advertiser match shocks. For example, a creative boutique agency may be outspent by its rivals but still occasionally (i.e., not systematically) produce higher-quality creative (as was the case in the "Got Milk?" campaign by Goodby, Silverstein & Partners). We therefore abstract away from another potential reason for offering stipends: to improve advertising quality. Within our model, the stipends merely encourage participation in the contest, and they do not affect quality at the margin.

Mathematically speaking, suppose an agency i can invest any amount k into quality improvement and other costs related to participation in the contest, such that its resulting quality $q_i(k)$ increases in k with diminishing returns. Only the agency knows its quality-production function, which is idiosyncratic to its match with each particular advertiser. Such a quality-setting agency solves $k_i^* = \underset{k}{\operatorname{argmax}}\{S_i[q_i(k)] - k\}$ in the beginning of the game and enters whenever its expected surplus S_i exceeds the participation costs net of the stipend $S_i[q_i(k_i^*)] - k_i^* + r_i > 0$. Crucially, note that the optimal investment k_i^* and its associated quality $q_i(k_i^*)$ are unrelated to the stipend amount r_i, because the fixed stipend does not influence quality production at the margin. In our model, we effectively assume that $k_i \equiv k_i^*$ is common information (everyone knows how much the two agencies tend to spend on new pitches), each agency has a good sense of its idiosyncratic quality function $q_i(k)$, but the advertiser does not know these functions exactly and thus remains uncertain about q_i. We thus implicitly assume that qualities are exogenous in the sense of Engelbrecht-Wiggans, Haruvy, and Katok (2007).

Another key simplifying assumption is the second-price rule in our auction. We rely on the second-price auction to approximate the payoff in the real-world contest with closed-form expressions, but we do not claim that the real-world contest follows the rules of the second-score auction exactly. The auction rules are not as firmly codified as those in classic industrial procurement auctions, and some back-and-forth price negotiation often occurs between the advertiser and the agencies after the pitches are made. Another modeling idea borrowed from government contracting would be to assume a first-score sealed-bid auction whereby the agencies propose profit lifts, and the winning agency is paid its own bid as compensation for servicing the contract and delivering the advertising through media. Unfortunately, a closed-form analysis of this auction is not tractable given the Samuelson (1985) model of endogenous entry: the asymmetry in participation costs leads to an asymmetry in the distributions of entrants' values to the advertiser. Analyzing asymmetric first-price auctions is notoriously difficult (Maskin and Riley 2000) in that the equilibrium bidding strategies are often intractable (Kaplan and Zamir 2012). Because our goal is to merely approximate payoffs in a somewhat informal and iterative real-world contest, we chose the second-score auction for its tractability.[7]

We have reason to believe that our second-score pricing rule can actually be more realistic in the advertising agency selection contest than the first-price rule. Consider a postpitch renegotiation stage in which the agencies compete only on media costs: with the qualities revealed during the initial pitch, the advertiser can declare a temporary winner and invite the losing agency to drop its media-cost bid until the loser's score matches that of the temporary winner. From our discussions with advertisers, we understand that some postpitch price negotiations and adjustments do often occur. If such a back-and-forth negotiation over cost can be carried out quickly and costlessly, the resulting "ascending score" auction is revenue equivalent to a second-price sealed-bid auction because the highest-value agency wins and the second-highest-value agency drops out of the media-price bidding at its true media cost.

ENTRY GAME, ADVERTISER PROFITS, AND OPTIMAL NEW-BUSINESS STIPENDS

We show that the difference in agencies' bid-preparation costs makes them use different entry thresholds in the entry game, resulting in an asymmetry between bidders. Despite this asymmetry, analyzing the subsequent second-value auction is easy because each agency has a weakly dominant strategy to bid its value (Vickrey 1961). When only one agency enters the contest, the second-value auction awards it the contract for the reserve price of the advertiser's outside option. When both agencies enter, the weakly dominant bidding strategies imply that the auction awards the higher-value bidder the contract for the "price" of the lower value. In other words, the winner delivers its advertising as pitched, and the advertiser compensates the agency with the difference between the values. We now proceed to the entry stage by backward induction.

[7]First- and second-price rules in a sealed-bid auction are well known to be revenue equivalent when the values of all bidders are drawn from the same distribution (Che 1993; Vickrey 1961). Our setting involves an asymmetry between the value distribution of entering incumbents and that of the entering competitors, and so the advertiser profits depend on the pricing rule, and their relative order of profitability is ambiguous (Maskin and Riley 2000).

Agency i enters the contest when its expected surplus from bidding in the auction (denoted by S_i) exceeds its bid-preparation cost k_i minus its stipend r_i. Suppose the opponent agency –i uses a "threshold strategy" of entering if and only if $x_{-i} \geq L_{-i}$, where L_{-i} is the opponent threshold type indifferent between entering or not. The expected surplus of agency i satisfies the following:

$$S_i(x_i) = x_i F(L_{-i}) + \mathbf{1}(x_i > L_{-i}) \int_{L_{-i}}^{x_i} (x_i - x_{-i}) dF(x_{-i}). \tag{1}$$

The first part of S_i arises when the opponent does not enter; therefore, agency i pockets its value as surplus. The second part of S_i captures the competitive payoff in the auction when the opponent does enter and when x_i exceeds the opponent's entry threshold. Agency i enters when $k_i - r_i < S_i(x_i)$. Because S_i increases in x_i, agency i also uses a threshold entry strategy.

Because the opponent's entry threshold L_{-i} influences the surplus of the focal agency i (and vice versa), the agencies are engaged in an entry game. Consider the two threshold types L_i and suppose that the incumbent faces a lower bid-preparation cost even net of the stipend; that is, $k_0 - r_0 < k_1 - r_1$. Then the incumbent enters more often ($L_1 \geq L_0$) and the thresholds must satisfy

$$\begin{aligned} k_1 - r_1 &= L_1 F(L_0) + \int_{L_0}^{L_1} (L_1 - x_0) dF(x_0) \\ k_0 - r_0 &= L_0 F(L_1), \end{aligned} \tag{2}$$

where the ordering of the cutoffs ($L_1 \geq L_0$) implies that the threshold incumbent L_0 can win only when the competitor does not enter. By contrast, the threshold competitor L_1 can also win over weak incumbent entrants. The following lemma describes sufficient conditions for a pure-strategy entry equilibrium to exist (for a proof, see the Appendix):

Lemma 1: When $k_1 - r_1 \leq V - E(x)$, the entry game has a unique pure-strategy equilibrium with a pair of thresholds $V \geq L_1 \geq L_0 > 0$.

No general closed-form solution of Equation 2 exists, but we can exploit the structure of the entry system to obtain a general characterization of the advertiser's optimal stipend strategy. To analyze the advertiser's problem, we now derive its expected profit Π.

Figure 3 shows how the profit Π depends on the realized values of the two agencies: we can use Equation 2 to express Π entirely in terms of the two entry thresholds by substituting for r_i:

$$\begin{aligned} &\Pi(L_1, L_0) \\ &= \underbrace{\Pr(x_0, x_1 > L_1)}_{\Pr\text{ (both enter and incumbent is above } L_1)} \; \underbrace{E(\min(x_0, x_1) \mid x_0, x_1 > L_1)}_{\text{symmetric auction profit}} \\ &+ \underbrace{\Pr(L_0 < x_0 < L_1 < x_1)}_{\Pr\text{ (both enter and incumbent is below } L_1)} \; \underbrace{E(x_0 \mid L_0 < x_0 < L_1)}_{\text{incumbent loses and sets the price}} \\ &- \underbrace{[1 - F(L_1)]}_{\Pr\text{ (competitor enters)}} \underbrace{\left[k_1 - L_1 F(L_0) - \int_{L_0}^{L_1} (L_1 - x_0) dF(x_0)\right]}_{\text{competitor's stipend } r_1} \\ &- \underbrace{[1 - F(L_0)]}_{\Pr\text{ (incumbent enters)}} \; \underbrace{[k_0 - L_0 F(L_1)]}_{\text{incumbent's stipend } r_0}, \end{aligned} \tag{3}$$

where the first two terms correspond to the shaded regions in Figure 3, and the last two terms collect the expected stipend payments to the competitor and the incumbent, respectively.

The profit parametrization in Equation 3 transforms the advertiser's profit-maximization problem into a two-dimensional screening problem whereby a higher stipend r_i corresponds to a lower entry threshold L_i. Note that whereas a stipend that covers the full bid-preparation cost ($r_i = k_i$) corresponds to $L_i = 0$, no stipend ($r_i = 0$) corresponds to some positive entry threshold $L_i > 0$.

Several terms cancel each other out in the profit function. Most notably, the second profit term in Equation 3 generated by a competitor entering against a "weak" incumbent (i.e., $x_0 < L_1$) is effectively returned to the competitor as part of its stipend (for a detailed derivation, see the Web Appendix):

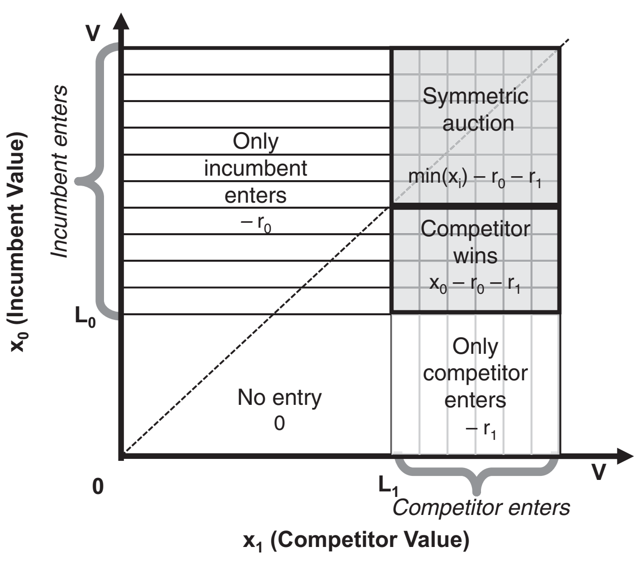
$$\begin{aligned} \Pi(L_1, L_0) = &\int_{L_1}^{V} z2f(z)[1 - F(z)]dz \\ &- [1 - F(L_1)][k_1 - L_1 F(L_1)] \\ &- [1 - F(L_0)][k_0 - L_0 F(L_1)]. \end{aligned} \tag{4}$$

The first term in Equation 4 is the net added profit from the auction competition, the second term is the expected payment to the competitor net of the increase in profits when the incumbent enters but is weak (below L_1), and the third term is the expected payment to the incumbent. Casting the stipend-optimization problem as a screening problem yields our main result:

P_1: For any continuous value distribution F on [0,V] and any bid-preparation costs $0 \leq k_0 \leq k_1$, no positive stipend for the incumbent agency $r_0 > 0$ can benefit the advertiser. A positive stipend for the competitor agency $r_1 > 0$ can benefit the advertiser only when $r_1 < k_1$ and $k_0 < k_1$. The optimal stipend for the competitor agency r_1 is positive for every $0 < k_1 < V - E(x)$ when $F(L_1)[1 - F(L_1)] - f(L_1)\int_0^{L_1} 1 - F(x)dx < 0$ for every L_1 in [0,V].

Figure 3

ADVERTISER PROFIT AS A FUNCTION OF AGENCY VALUES



Notes: The formulae show the advertiser profit. The horizontally (vertically) hatched region shows when the incumbent (competitor) enters. The shaded regions show the situations in which the auction delivers positive profit to the advertiser.

P_1 shows that stipends can benefit the advertiser only when an incumbent agency is present—that is, when one agency has a strictly lower bid-preparation cost. Moreover, the advertiser should only offer a stipend to the competitor and ensure that the stipend does not reimburse the competitor's bid-preparation cost in full. The sufficient condition shows that even offering the competitor a small compensation for its cost disadvantage is not always beneficial. Instead, the advertiser must consider whether the benefit of attracting a marginal competitor exceeds the marginal increase of the stipend.

The incumbent should not receive a stipend, because Equation 4 is increasing in L_0: a reduction in L_0 is clearly costly to the advertiser because it involves paying more incumbents more money, but the advertiser receives no benefit, because any increase in profit from the auction is paid to the competitor in the form of an increased stipend. In other words, increasing the "competitor wins" region of Figure 3 by lowering L_0 results in no net increase in profits after the competitor receives its stipend. Therefore, reducing L_0 through an increase in r_0 from zero is pointless.

To gain intuition into why a bid-preparation-cost asymmetry is necessary for a stipend to be optimal, suppose $k_0 = k_1$ and consider the marginal entrant $x_i = L_i$. The entry game in Equation 2 becomes symmetric with a common threshold $k(1 - r) = LF(L)$, so the marginal entrant pockets its entire value whenever it wins; therefore, it is unnecessary to subsidize its presence conditional on it winning. When it loses, its presence raises the auction price to L, but losing implies that the other agency has also entered and received its stipend. To make both agencies enter if $x_i > L$, the reimbursement policy must compensate them for any expected effects of their mutual competition, so the advertiser cannot benefit solely from a threshold agency losing the auction. Another intuition for the necessity of a cost asymmetry follows from Samuelson's (1985) result that the advertiser would actually want to charge a reserve price above its opportunity cost in the $k_0 = k_1$ case. With $k_0 = k_1$, a reserve price is isomorphic to a negative stipend because both instruments simply manipulate the common entry threshold.

The second part of P_1 shows that competitor (aka "new-business") stipends can be optimal when the two agencies face different bid-preparation costs, but only for some value distributions F, and only when the stipends are smaller than the competitor's bid-preparation cost k_1. To see why reimbursements in full are not optimal for the advertiser, consider the marginal entrant eliminated by a small increase in L_1 from $L_1 = 0$ (i.e., a small reduction of the stipend from $r_1 = k_1$). The entrant eliminated on the margin near $L_1 = 0$ has no gains from trade with the advertiser, so it loses the auction almost surely and does not put any competitive pressure on the incumbent. Naturally, subsidizing its entry cannot benefit the advertiser.

To gain intuition for the sufficient condition, consider the first partial derivative of the profit Equation 4 with $r_0 = 0$:

$$\frac{\partial \Pi(L_0, L_1 | r_0 = 0)}{\partial L_1} = F(L_1)[1 - F(L_1)] + (k_1 - L_1)f(L_1). \tag{5}$$

Note that because the "weak incumbent" term cancels out in Equation 3, offering no stipend to the incumbent makes the advertiser's profit independent of the incumbent's cost k_0. In other words, the profit depends only on the marginal competitor L_1 and its bid-preparation cost k_1. When $L_1 > 0$, the entrant eliminated on the margin by a small increase in L_1 can potentially win the subsequent auction against the incumbent, so whether its entry should be subsidized is not obvious from Equation 5. To provide a sufficient condition for a subsidy, consider the L_1 corresponding to no reimbursement ($r_1 = 0$): if the profit is decreasing at that L_1, some partial reimbursement (i.e., a reduction in L_1) is more profitable than no reimbursement at all. The sufficient condition of P_1 gives this derivative in terms of F alone.

P_1 UNDER SPECIFIC DISTRIBUTIONAL ASSUMPTIONS: EXAMPLES

We next turn to several specific distributional families to illustrate P_1, provide tractable examples of positive optimal competitor stipends, and give examples of distributions that do not support any competitor stipends even when $k_0 = 0$. Figure 4 illustrates all distributions considered in our examples. We invite the reader to glance at Figure 4 and guess which distributions support positive competitor stipends before examining our analysis of the examples. Another worthwhile question to ask is which distributions suggest larger stipends and which suggest smaller ones (keeping k_i fixed). We did not have strong intuitions prior to discovering P_1, and one of the main contributions of this research is to develop such intuition.

Examples of Distributions That Support Positive New-Business Stipends

Example 1: When F is uniform on [0,V] (i.e., $F(x) = x/V$ and $0 \le k_0 < k_1 < V$), the advertiser should offer no stipend to the incumbent agency and a stipend of $r_1 = (k_1^2 - k_0^2)/2k_1$ to the competitor.

We work out the uniform example in the main text because of its simplicity. The sufficient condition of P_1 reduces to $-L_1^2/2V^2 < 0$, so the advertiser should offer a new-business stipend at least for $k_1 \le V/2$. The entry equations become

$$2V(k_1 - r_1) = L_0^2 + L_1^2, \text{ and} \tag{2u}$$

$$V(k_0 - r_0) = L_0 L_1.$$

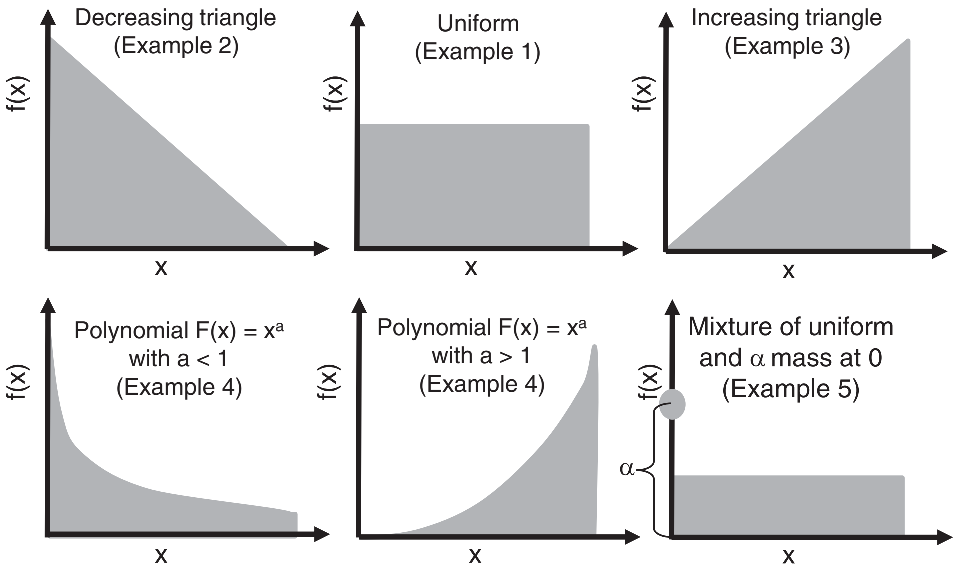
The advertiser profit function with no incumbent stipend ($r_0 = 0$) simplifies to a cubic:

$$3V^2\Pi(L_1, L_0 | r_0 = 0) = (V - L_1)(L_1^2 + V^2 + L_1 V - 3k_1 V), \tag{6}$$

where the lack of dependence on L_0 holds for any F, as Equation 4 shows. Finally, Equation 5 implies that $[\partial \Pi(L_0, L_1 | r_0 = 0)]/\partial L_1 = (k_1 V - L_1^2)/V^2$. Therefore, the first-order condition (FOC) suggests $L_1^2 = k_1 V$ as the optimal threshold, and it characterizes a maximum because the profit function is concave. The sufficient condition of P_1 is therefore loose in this particular example of F because the optimal L_1 is within the support of F for all $k_1 \le V$, and not only for $k_1 \le V/2$.[8]

[8]When $k_1 > V$, even the highest-value competitor $x_1 = V$ does not bring enough to the table to cover the bid-preparation cost, so no amount of competitive entry is beneficial to the advertiser. The optimal $L_1 = V$ can then be implemented by any stipend small enough that $r_1 \le V/2$.

Figure 4
SPECIFIC AGENCY-VALUE DISTRIBUTIONS ANALYZED TO ILLUSTRATE P_1



Notes: Each graph shows a probability density function. The graphs are merely illustrations and not necessarily drawn to comparable scale.

To derive the optimal stipend for any $k_1 \leq V$, we plug the optimal L_1 into the second equation in Equation 2u with $r_0 = 0$, solve for the incumbent threshold, and solve for r_1 using the first equation in Equation 2u. As one would expect from P_1, the stipend is less than k_1 (indeed, less than half of k_1), increasing in the difference between the two bid-preparation costs, and vanishes when the costs become equal. Notably, the stipend is also independent of V—a simplification specific to the uniform F.

Comparing the optimal stipend policy with a no-stipend status quo ($r_i = 0$) is interesting and results in $L_1^2 = k_1 V + V\sqrt{k_1^2 - k_0^2}$. Therefore, relative to the optimal stipend policy, the status quo involves less entry by the competitor and more entry by the incumbent.

Example 2: When F is the decreasing triangle distribution on $[0,V]$—that is, $F(x) = 1 - [(V - x)^2/V^2]$ and $0 = k_0 < k_1 < V$—the advertiser should offer no stipend to the incumbent agency and a stipend of $r_1 = k_1/3$ to the competitor.

The proof of Example 2 is analogous to that of Example 1, except the entry system does not have a simple solution tractable to us when $k_0 > 0$ beyond a bound $r_1 \leq k_1/3$. Setting $k_0 = 0$ ensures that the incumbent always enters and makes the optimal stipend simple.

Example 3: When F is the increasing triangle distribution on $[0,1]$—that is, $F(x) = x^2$ and $0 = k_0 < k_1 < V$—the advertiser should offer no stipend to the incumbent agency and a stipend of $r_1 = k/3 + z/9 - 1/3z$, where $z = \sqrt[3]{3(9k_1 + \sqrt{3 + 81k_1^2})}$. It can be shown that $r_1 > k_1/2$.

Example 3 demonstrates that simple distributional assumptions do not necessarily lead to simple optimal stipends, even when $k_0 = 0$. Comparing the three distributions in Examples 1–3, we conjecture an intuitive comparative static in the relative probability of more creative agencies: when relatively more high-value agencies (with x close to V) than low-value agencies (with x close to 0) are present, the optimal reimbursement proportion increases.

Examples of Distributions That Do Not Support Positive Competitor Stipends

Our next example enables us to begin describing the kinds of value distributions that satisfy P_1's sufficient condition for positive competitor stipends. We consider the polynomial family $F(x) = x^a$ for $a > 0$ with support on the unit interval. A large subset of the polynomial family does not admit any stipends.

Example 4: When $F(x) = x^a$, $a < 1$, P_1's sufficient condition for a positive competitor stipend does not hold for small-enough k_1. When $0 = k_0 < k_1 < (1 - a)(1 - a^2)^{1/a}$, no $r_1 > 0$ benefits the advertiser.

The first part of the example shows that the sufficient condition can fail, and the second part focuses on a tractable special case in which all possible stipends can be ruled out. To understand what is special about $a < 1$, note that the restriction singles out the distributions that put a lot of probability mass near zero. To gain intuition into why mass near zero makes competitor stipends unprofitable, note that a lot of mass at the bottom of the support encourages competitive entry by increasing the chance that a weak incumbent is present. When the marginal competitor is itself weak (i.e., when k_1 is small), subsidizing its entry cannot be profitable: either it wins and pockets almost its entire contribution to the social surplus or it loses and puts little pressure on a high-value incumbent. Thus, despite the asymmetry in bid-preparation costs ($k_0 < k_1$), the intuition behind Example 4 is analogous to the reason that stipends are not profitable under general F when $k_0 = k_1$.

In confirmation of this intuition, it can be shown that the sufficient condition of P_1 fails for small-enough k_1 when F is a mixture of a mass point at 0 and a continuous distribution on [0,V], regardless of the shape of F on the rest of its support (for a detailed proof, please contact the authors). Our last example provides a closed-form illustration, including the possibility that no stipends benefit the advertiser:

Example 5: When F is a mixture of a Uniform[0,1] and an $\alpha > 0$ mass-point at zero, then for $0 = k_0 < k_1$, the optimal competitor stipend is $r_1 = \max[0, (k_1 - \alpha)/2]$.

Example 5 returns full circle to the Uniform distribution of Example 1, in which $k_0 = 0$ implied that the competitor's stipend should be half of its bid-preparation cost. Mixing in a mass of "bad" agencies at the low boundary of the support clearly reduces the stipend and sometimes even cancels out its profitability to the advertiser. As discussed previously, the k_1 costs that do not support positive stipends are the lower ones, because a low k_1 implies a low-value marginal competitor.

These last two examples illustrate that new-business stipends should not be universal. One explanation for the difference in new-business-stipend popularity between the United States and Europe could be the difference in the underlying distributions of agency values to the advertiser: the U.S. agency landscape may be more competitive, with many marginal agencies as in Examples 4 (with a < 1) and 5.

EXTENSIONS: TIMING OF ENTRY, MULTIPLE COMPETITORS, AND RESERVE PRICES

Extension 1: Agencies Do Not Know Their Values to Advertiser Before Entry

In our main model, we assume that each agency knows its expected value to the advertiser in the beginning of the game. This "private information" assumption is plausible if each agency can predict the quality of its pitched ad before developing it, perhaps based on the available artists or other stable characteristics of the agency, such as a match between it and the advertiser. Alternatively, the agencies might be "shooting in the dark" at the time of entry, not learning the value of their work until after the pitch. In our first extension, we consider this alternative situation and show that our qualitative results continue to hold.

Suppose that agency i = 0, 1 faces a publicly known bid-preparation cost k_i (as in the main model) but does not know its value x_i at the time of entry. Instead, both agencies know only that their values will be drawn i.i.d. from some distribution F at the time of the contest. For example, this assumption captures the possibility that the ad quality is in the eye of the beholder (the advertiser). When $k_0 = k_1$, the model reduces to Levin and Smith (1994), who find a symmetric mixed-strategy equilibrium whereby each agency enters with the same probability ρ that makes the agency indifferent between entering and not entering. Suppose $0 < k_0 < k_1$, and let agency i enter with probability ρ_i. Then agency i enters when

$$(7)\qquad k_i - r_i \le (1 - \rho_{-i})E(x_i) + \rho_{-i}E_{x_i}\left[\int_0^{x_i}(x_i - x_{-i})dF(x_{-i})\right],$$

where the left-hand side of the inequality is the cost of entry, and the right-hand side is the expected benefit of entry. The first term on the right-hand side corresponds to i capturing its entire value in the event of –i not entering, and the second term corresponds to the expected payoff from the auction between two entrants. Contrast Equation 7 with Equation 1 and note the additional expectation operator on the right-hand side in the former—a consequence of the additional uncertainty about the agency's own x.

We focus on the tractable uniform case F = Uniform[0,V] and begin our solution of the modified contest game with an analysis of the entry stage. The uniform assumption implies $E(x_i) = V/2 > E_{x_i}[\int_0^{x_i}(x_i - x_{-i})dF(x_{-i})] = V/6$. When the two bid-preparation costs net of stipends are similar such that $(k_i - r_i) \in [V/6, V/2]$, each agency would prefer that the other not enter the contest, and multiple equilibria result: a mixed-strategy equilibrium in which both agencies are indifferent between entering and not entering $\{\rho_0, \rho_1\} = \{3/2 - 3(k_1 - r_1)/V, 3/2 - 3(k_0 - r_0)/V\}$ and two pure-strategy "pre-emption" equilibria $\{\rho_0, \rho_1\} = \{0, 1\}$, $\{\rho_0, \rho_1\} = \{1, 0\}$. We assume that the agencies play the mixed-strategy equilibrium because it approaches a symmetric equilibrium as $k_0 - r_0 \to k_1 - r_1$. When $k_0 - r_0 \le V/6 < k_1 - r_1 \le V/2$, the incumbent agency always enters and the competitor does not: $\{\rho_0, \rho_1\} = \{1, 0\}$. Finally, when $k_i - r_i \le V/6$, both agencies always enter $\{\rho_0, \rho_1\} = \{1, 1\}$. The following proposition gives the optimal policy:

P_2: When F is Uniform[0,V] and the agencies do not know their value to the advertiser at the entry stage, the advertiser should offer no stipend to the incumbent agency. The competitor agency should receive a positive stipend $r_1 = k_1 - V/6$ only when $0 \le k_0 \le V/6 < k_1 \le V/2$.

P_2 shows that the selection of higher-value agencies at the entry stage is not necessary for the qualitative results of P_1 to hold. Instead, the critical piece is indeed pricing pressure on an incumbent whenever the incumbent has a low-enough bid-preparation cost to enter regardless of competition, but the competitor's cost is high enough that the competitor would stay out of the contest without a stipend. Also echoing the result of P_1, the optimal competitor stipend does not fully cover its bid-preparation cost, and equal bid-preparation costs make stipends suboptimal.

Extension 2: More Than One Competitor

In our second extension, we relax the assumption of a single competitor. For simplicity, suppose that $k_0 = 0$ and let N competitors face the same bid-preparation cost $k > 0$. We need this equal-cost assumption to make the entry game tractable, relying on the results of Samuelson (1985) to guarantee a simple pure-strategy threshold equilibrium. For tractability, assume that the competitors' values x_i are drawn i.i.d. from F uniform on [0,V].

One would expect that with more competitors, the advertiser would have to become stingier with the new-business stipends: although each entrant collects r, the more bidders that are already participating, the smaller the incremental decrease in the auction price from adding one more bidder. This intuition is incomplete, because the presence of additional competitors also makes entry less likely by reducing the expected surplus given entry. Thus, the entry threshold rises and the agencies that decide to enter drive the profit up faster than the same number of exogenous entrants would. Notably, these effects cancel each other out, and $r = k/2$ remains the optimal reimbursement policy regardless of the number of potential competitors, as in Example 1 with a single competitor and $k_0 = 0$.

P_3: When F is uniform and $N \geq 2$ potential competitors exist with participation cost k in addition to the incumbent with no participation cost ($k_0 = 0$), the optimal new-business stipend is the same as when only one competitor exists—namely, $r = k/2$. The expected advertiser profit increases in N.

A key simplifying aspect of P_3 is the lack of dependence of the optimal reimbursement proportion on the number of potential competitors. Figure 5 illustrates how profits increase in N. The fact that profits are increasing in N differs from the case of optimal reserve prices: the received intuition from auctions with bid-preparation costs is that increasing N can reduce the auctioneer's profit by reducing entry. For example, Samuelson (1985, p. 53) concludes that "expected procurement cost need not decline with increases in the number of potential bidders."

Extension 3: Strategic Reserve Price

So far, we have assumed that the advertiser cannot use reserve price above its outside option. This assumption is realistic for settings in which the advertiser does not have enough commitment to reject positive (i.e., above outside option) offers. Without such commitment, the advertiser will be tempted to drop the reserve price and reauction the contract in case all current bids are below its reserve. When such reauctioning is instantaneous, the result is an instance of the Coase conjecture—the advertiser cannot credibly use a reserve above its outside option (McAfee and Vincent 1997).

Suppose instead that the advertiser can announce a public reserve $R > 0$ and commit not to reauction the contract when no bids exceed R. To investigate the impact of a reserve price on optimal reimbursements and expected costs, we focus on the single-competitor case. In the Web Appendix, we show in full generality that the main qualitative conclusions of P_1 continue to hold: incumbent stipends are never optimal, and offering a stipend to the competitor agency can be profitable for the advertiser.

A general characterization of the optimal reserve-stipend strategy $\{R^*, r_1^*\}$ is not tractable for an arbitrary F, but we show that when F is Uniform[0,1], the optimal reserve price is $R^* = 1/2 - (k_0\sqrt{2})/d$ and the optimal competitor stipend is $r_1^* = k_1/2 - k_0(4k_0 + d\sqrt{2})/2d^2$, where

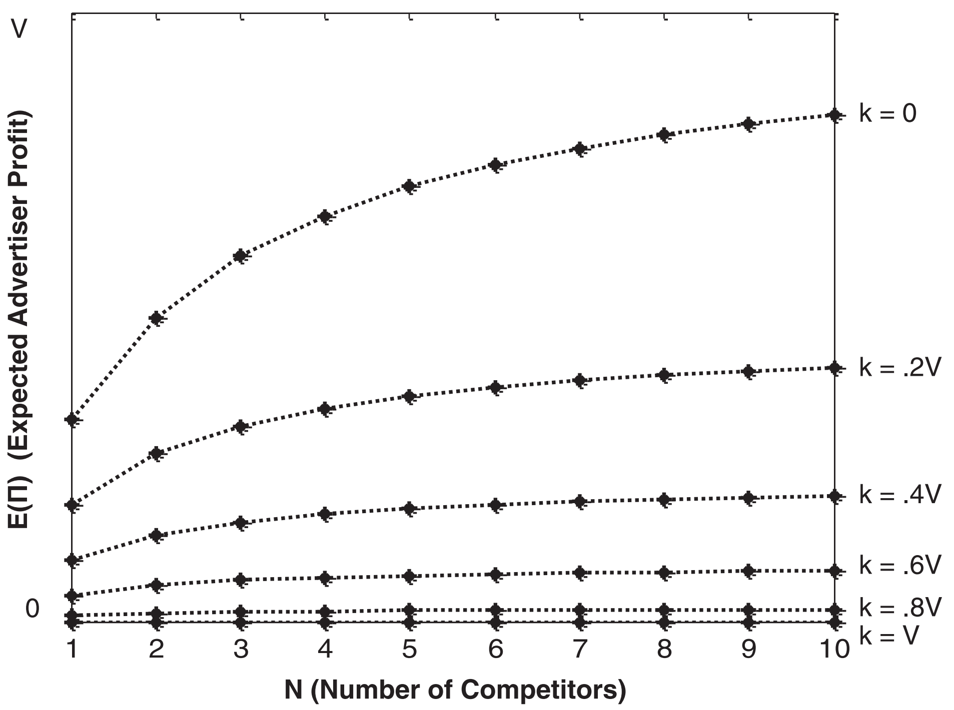
$$d = \sqrt{1 + 4k_1 + \sqrt{(1 + 4k_1)^2 - (4k_0)^2}}.$$

For any $k_0 > 0$, both the optimal reserve and the optimal competitor stipend increase with k_1 (for a proof, see the Web Appendix).

When F is Uniform[0,1] and $k_0 = 0$, the formulas simplify dramatically, and we obtain the optimal reserve $R^* = 1/2$ familiar from textbooks, along with the now-familiar expression for the optimal incumbent's stipend $r_1^* = k_1/2$ (same as in Example 1). This special case (F is Uniform [0,1] and $k_0 = 0$) also makes it tractable to solve for the optimal reserve in the absence of stipends. We find that as k_1 rises, the optimal reserve price drops but remains above 2/5. Importantly, the advertiser is better off with a new-business stipend and its corresponding optimal reserve than

Figure 5

INCREASED COMPETITION INCREASES ADVERTISER PROFITS



Notes: Each line represents a different level of k. The N does not count the incumbent.

with an optimal reserve alone, and the profit difference increases as k_1 (and, thus, the difference between k_1 and k_0) increases (for an illustration, see Figure 6). Therefore, a new-business stipend is a step in the right direction from a mechanism that is optimal without asymmetries in bid-preparation costs to a mechanism that is optimal under that asymmetry. Figure 6 illustrates the advertiser profits with and without a strategic reserve as well as with and without the optimal competitor stipend.

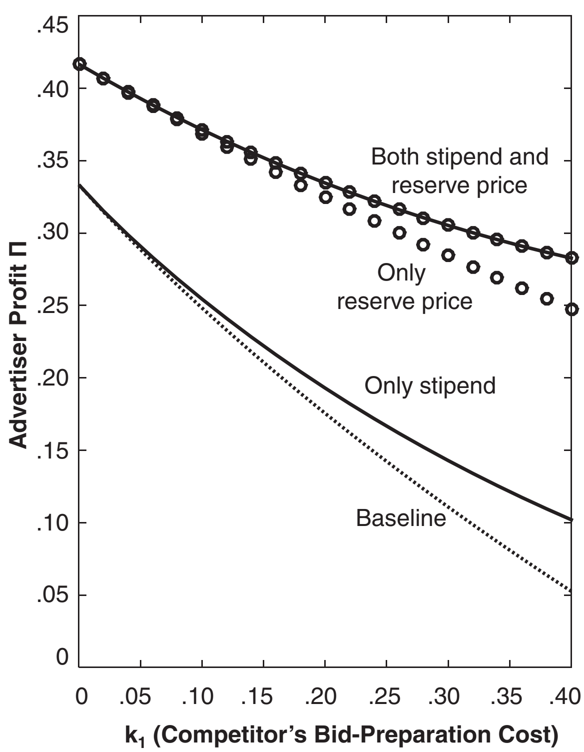
A MODEL OF THE PAST: A QUALITY CONTEST WITH A FIXED PRIZE

It is useful to contrast the modern auction-based contest with the traditional twentieth-century contest whereby agencies competed for the best creative quality and the winner received a fixed prize (traditionally 15% of the media billings). Consider two agencies $i = 0, 1$ and assume that each agency has a different, privately known creative quality q_i and a publicly known bid-preparation cost k_i, such that $0 \le k_0 \le k_1$. Let qualities be distributed i.i.d. according to a distribution H on [0,V], and suppose that the agency with the higher quality wins the contest and receives a fixed prize P.

The entry game is different from the auction contest because the winner's payoff does not depend on the quality of the loser. As a result, the incumbent always enters, but

Figure 6

ADVERTISER PROFIT WITH RESERVE PRICES AND/OR COMPETITOR STIPENDS



Notes: The baseline uses neither the reserve price nor stipends. The solid line indicates the presence of the optimal competitor stipend. The circles indicate the presence of the optimal reserve price. F Uniform[0,1] and k_0 = 0 throughout.

the competitor stays out of the contest when its quality is low (see the proof of P_4). P_4 provides a sufficient condition for the advertiser to use a new-business stipend to encourage more competitor entry:

> P_4: For any continuous-quality distribution H[0,V] and any bid-preparation costs $0 \le k_0 \le k_1$, let L_1 satisfy $k_1 = PH(L_1)$. A positive stipend for the competitor agency $r_1 > 0$ benefits the advertiser when $P[1 - H(L_1)] < \int_0^{L_1} H(x)dx$.

The sufficient condition in P_4 is derived analogously to that in P_1, but P_4 is weaker than P_1 because positive incumbent stipends cannot be categorically ruled out: when the bid-preparation costs of both agencies are high enough and similar enough, the advertiser can benefit from offering *both* agencies a stipend. Nevertheless, incumbent stipends can be ruled out when k_0 is sufficiently smaller than k_1 such that the optimal r_1 is $k_0 \le k_1 - r_1$. Then the incumbent always enters, and offering it a stipend cannot benefit the advertiser. A uniform-distribution example illustrates both possibilities:

> Example 7: When H is Uniform[0,1] and $0 \le k_0 < -2P^2 + \sqrt{P^4 + 2P^3} < k_1 < P$, the advertiser maximizes its profit by offering no stipend to the incumbent and offering the following positive stipend to the competitor: $r_1 = k_1 + 2P^2 - P\sqrt{2(k_1 + P + 2P^2)}$. When $k_0 = k_1$ and $2\left(\sqrt{P^3 + P^4} - P^2\right) < k_i < P$, positive stipends to both agencies benefit the advertiser.

For details of Example 7, see the Appendix. The contrast with Example 1 is striking: the seemingly simpler contest with a fixed prize involves a much more complicated new-business stipend scheme that depends on the prize. Notably, r_1 is not even monotonic in P. Also in contrast to Example 1, which rules out stipends under $k_0 = k_1$, Example 7 gives a sufficient condition for the advertiser to offer both agencies a stipend under symmetric bid-preparation costs.

DISCUSSION

Advertisers commonly hold contests to select an advertising agency, and the level of contest activity seems to have increased recently (Finneran 2009; Parekh 2010). The compensation method of advertising agencies has been in flux since the demise of the standard compensation contract based on 15% of the media billings. This article focuses on the observation that the advertising agency selection contest has become similar to a procurement score auction in which the pitch involves not only the creative idea but also a proposed price for buying the media. One of the consequences of the modern contest is increased competition, whereby the compensation for an occasional contest victory does not provide enough profit to cover up-front bid-preparation costs for all the contests in which the agency participates (Rice 2006). To cover such costs, advertising agency associations worldwide recommend reimbursing the losing agencies for contest-preparation expenses (Brendan 1998; Gardner 1996), but the advertisers predictably resist this idea because it seems like an added cost. Two recent surveys have indicated that approximately half of today's advertising contests involve some form of stipend to help defray the costs (AAAA 2007; Parekh 2009). The industry has not reached a consensus, as advertising practitioners continue debating the pros and cons of reimbursements as well as the details of optimal policies.

We analyzed the score-auction model with endogenous entry and asymmetric bid-preparation costs to account for the difference between the incumbent agency that currently serves the advertiser and a competitor agency that is bidding against the incumbent for new business. Our analysis indicates that the advertisers are right to resist demands for reimbursements in full and right to resist offering stipends to the incumbent agency. However, we find that offering a stipend to agencies competing with the incumbent for new business can increase the advertiser's overall profit. The optimality of a new-business stipend depends on (1) the asymmetry in bid-preparation costs and (2) the distribution of value-to-advertiser in the population of agencies. First, when the incumbent and the competitor face the same bid-preparation cost, the advertiser should not offer any stipends. Or, more commonly, when the incumbent does not participate in the contest and only new-to-the-account agencies compete, no stipends should be offered. Therefore, the business reason for providing reimbursements is not necessarily to increase the competition in general but to increase pricing pressure on the incumbent, which enters the contest more often. Second, we provide and analyze a simple sufficient condition on the distribution of agency values to the advertiser for new-business stipends to benefit the advertiser. We find that new-business stipends benefit the advertiser only when the population distribution of agencies is not too concentrated near the bottom of its support.

The statistic that approximately half the real-world contests offer no reimbursement may thus be partially explained by contests in which the incumbent agency does not participate for whatever reason. Furthermore, it can be explained by some contests that attract agencies with values to advertiser concentrated at the bottom of their support.

Regarding the details of the optimal new-business-stipend policy, we provide a set of assumptions (uniform distribution of agency profitability to the advertiser) under which the optimal reimbursement policy is simple—a fixed proportion of the creative development costs regardless of the costs' magnitude, the number of potential competitors, or the presence of a strategic reserve. However, we also illustrate that seemingly simple assumptions (e.g., a triangle distribution) can also imply fairly complex relationships between the magnitude of the bid-preparation costs and the optimal new-business stipend policy.

The entry game into our auction-driven score contest involves a systematic selection of higher-valued agencies because the lower-valued agencies are less likely to win and are thus less likely to cover their bid-preparation costs. We find that this selection at the entry stage is not necessary for the aforementioned qualitative results: even when the agencies do not know their values at entry time, the advertiser should not offer a stipend to the incumbent, and new-business stipends can benefit the advertiser only when the incumbent faces a lower bid-preparation cost than the competitor. The critical force for the optimality of new-business stipends is thus the pricing pressure on an incumbent when the incumbent has a lower bid-preparation cost than the competitor.

Many of our qualitative results extend to contests based solely on quality that award the winner a fixed prize. Such contests were often used in advertising agency selection during the twentieth century, with the prize being 15% of the list price of the media billings. We provide a sufficient condition for new-business stipends in that context as well. One result that does not extend is the suboptimality of incumbent stipends: we describe a situation in which both agencies should receive a stipend.

In our modeling, we have assumed that the incumbent agency has an advantage in lower bid-preparation costs but has no cost advantage in executing the creative and media buying tasks after the contest. In other words, in our setup, the incumbent and the competitor are asymmetric only in the bid-preparation costs *before* the auction and ex ante symmetric in parameters active *during* the auction. We did not formally examine the generalization to incumbent advantages at both stages of the auction, but we speculate that the case for new-business stipends would strengthen under such assumptions. The literature we reviewed (e.g., Branco 2002; McAfee and McMillan 1989) found that the auctioneer should give an advantage to the "weaker" bidder during the auction by allowing that bidder to win the auction even if it quoted a higher price. Given these results, it would stand to reason that in our scenario, if the incumbent also continued to have an advantage during the auction (and not just during entry), the case for giving a stipend to the new participants would strengthen to induce them to take part in this contest that is already biased against them for an additional reason. We conjecture that the same reasoning would likely apply if the advertiser's familiarity with the incumbent makes it more uncertain about the profit lift expected from the new entrant.

Our research is the first to highlight the role of an incumbent in a score-auction contest environment. Any client firm wishing to hire a service provider in a context in which incumbents might exist could apply our model, for example, in choosing a new outside accounting/auditing or legal office or an outside consulting firm. However, for our reimbursement strategy to apply, differences in bid-preparation costs as well as precontest quality differences among bidders need to exist. The advertising contest is also theoretically analogous to a contest among architects to design an extension or renovate an existing city museum and then supervise its construction. A more appealing design will please residents and increase attendance, donations, and tourism to the city. The most creative architect might not necessarily be an efficient construction supervisor. In the architect-selection context, our results imply that the contest organizers should not offer a reimbursement of design costs unless they have a clear "incumbent" that will participate in the contest no matter what (e.g., the architect who designed the original building, or one that did a preliminary study and already offered a possible design).

APPENDIX: PROOFS OF PROPOSITIONS AND DETAILS OF EXAMPLES

Proof of Lemma 1

Integration by parts yields the following right-hand side of the first equation in Equation 2: $L_0F(L_0) + \int_{L_0}^{L_1} F(z)dz$. Plugging in $L_0 = (k_0 - r_0)/F(L_1)$, which solves the second equation, yields the first equation in terms of only L_1:

$$k_1 - r_1 = \frac{k_0 - r_0}{F(L_1)} F\left(\frac{k_0 - r_0}{F(L_1)}\right) + \int_{\frac{k_0 - r_0}{F(L_1)}}^{L_1} F(z)dz.$$

The right-hand side of the equation is obviously continuous and increasing in L_1 large enough that $L_1F(L_1) \geq k_0 - r_0$, so the intermediate value theorem implies that a unique $L_1 \geq L_0$ exists that satisfies the equation as long as $k_1 - r_1 \leq \int_0^V F(z)dz = V - E(x)$. Because the solution thus involves a $L_0 = (k_0 - r_0)/[F(L_1)] \leq L_1 \leq V$, the condition is sufficient to guarantee a pair of thresholds (L_0, L_1) is between 0 and V. It remains to be checked whether $L_0 \leq V$ (i.e., that $(k_0 - r_0)/[F(L_1)] \leq V$). *QED Lemma 1*

Proof of P_1

In the profit Equation 4, substitute for L_0 using the second equation in Equation 2:

$$\Pi(L_1, r_0) = \int_{L_1}^{V} z2f(z)[1 - F(z)]dz - [1 - F(L_1)][k_1 - L_1F(L_1)] - r_0\left[1 - F\left(\frac{k_0 - r_0}{F(L_1)}\right)\right].$$

Note that for every $L_0 < L_1$ such that $r_0 \geq 0$, the profit is decreasing in r_0:

$$\frac{\partial\Pi(L_1, r_0)}{\partial r_0} = -f\left(\frac{k_0 - r_0}{F(L_1)}\right)\frac{r_0}{F(L_1)} - \left[1 - F\left(\frac{k_0 - r_0}{F(L_1)}\right)\right] < 0.$$

Fix any $r_0 > 0$ and let $\Pi^*(r_0)$ be the optimal profit achieved by manipulating L_1 in $\Pi(L_1, r_0)$ given the fixed r_0. By the envelope theorem, $\Pi^*(r_0)$ is decreasing in r_0: $d\Pi^*/dr_0 = \partial\Pi/\partial r_0|_{(r_0, L_1^*)} < 0$. Therefore, no $r_0 > 0$ can benefit the advertiser more than $r_0 = 0$.

When $k_0 = k_1$ and $r_i = 0$, the entry thresholds are the same: $L_0 = L_1$. Offering a stipend to the competitor agency would result in $L_0 > L_1$, and the competitor agency would play the role of agency 0 in the profit function Equation 4. This argument shows that the advertiser would be better off reducing the competitor stipend back to zero. When $k_0 = k_1$, the advertiser does not benefit from offering a stipend to both agencies either: let $L \equiv L_0 = L_1$ in Equation 4 and note that

$$d\Pi(L)/dL = \underbrace{2f(L)}_{\text{prob marginal entrant}}\underbrace{[k - LF(L)]}_{\text{stipend}} + \underbrace{2[1 - F(L)]}_{\text{number of stipends paid}}F(L) > 0.$$

The second term combines the marginal decrease in profit of Lf(L) per entrant and the marginal savings in the per capita stipend amount of Lf(L) + F(L). Because the latter exceeds the former, the advertiser always benefits from raising L.

Next, consider the $0 \leq k_0 < k_1$ case and let $r_0 = 0$. The suboptimality of reimbursing the competitor in full follows from Equation 5 with $L_1 = 0$ $\partial\Pi(L_0, L_1|r_0 = 0)/\partial L_1|_{L_1=0} = k_1f(0) > 0$. The sufficient condition for the optimality of a positive stipend ensures that $\partial\Pi(L_0, L_1|r_0 = 0)/\partial L_1|_{L_1=\overline{L}_1} < 0$, where $\overline{L}_1$ is the entry threshold corresponding to $r_1 = 0$: $k_1 \equiv \int_0^{\overline{L}_1}[\overline{L}_1 - x]dF(x) = \int_0^{\overline{L}_1}F(x)dx$, where the second equality follows from integration by parts. Expressing the cost k_1 in terms of the $\overline{L}_1$ threshold in Equation 5 yields the condition. Setting $\overline{L}_1 = V$ yields the upper bound on k_1 that admits an entry threshold interior to the support of F. *QED P_1.*

Details of Example 2

The sufficient condition of P_1 simplifies to $L^a(1 - a^2 - L_1^a)/(1 + a) < 0$, which is violated for $a < 1$ and for small-enough L_1 such that $L_1^a < 1 - a^2$. Recall that the condition is the derivative at L_1 corresponding to $r_1 = 0$, so the entry Equation 2 satisfies $k_1 = (aL_0^{a+1} + L_1^{a+1})/(1 + a), k_0 = L_0L_1^a$. Setting $k_0 = 0$ makes the entry game tractable with $k_1 = L_1^{a+1}/(a + 1)$; therefore, $L_1^a < 1 - a^2 \Leftrightarrow k_1 = L_1^{a+1}/(a + 1) < (1 - a)(1 - a^2)^{\frac{1}{a}}$. The fact that the condition is violated shows that the profit function is increasing in L_1 at $L_{max} = [(a + 1)k_1]^{1/(a+1)}$, corresponding to $r_1 = 0$. It remains to be shown that the profit function is also increasing for all L between 0 ($r_1 = k_1$) and L_{max} ($r_1 = 0$). The derivative of the profit function is $\partial\Pi/\partial L_1 = (ak_1 + L_1 - aL_1 - L_1^{1+a})/L_1^{1-a} > 0 \Leftrightarrow ak_1 + L_1(1 - a - L_1^a) > 0$. Expressing k_1 in terms of L_{max} results in $ak_1 + L_1(1 - a - L_1^a) = (aL_{max}^{a+1})/(a + 1) + L_1(1 - a - L_1^a) > 0 \Leftrightarrow a(L_{max}^{a+1} - L_1^{a+1}) + L_1(1 - a^2 - L_1^a) > 0$, which holds because $L_1 < L_{max}$ and $1 - a^2 - L_1^a > 0$. Therefore, the profit is indeed increasing on the entire [0, L_{max}] range. *QED Example 2.*

Details of Example 5

With $k_0 = 0$, the incumbent always enters, and the marginal competitor entrant satisfies: $2(k_1 - r_1) = 2\alpha L_1 + (1 - \alpha)L_1^2$. The basic structure of advertiser profit remains as in Equation 4 with $F(x) = \alpha + (1 - \alpha)x$ and $f(x) = (1 - \alpha)$ for any $x > 0$. The profit with $r_0 = 0$ simplifies to $\Pi(L_1, L_0|r_0 = 0) = (1 - \alpha)^2\Pi(L_1, L_0|r_0 = 0, \alpha = 0) - \alpha(1 - \alpha)(1 - L_1)(k_1 - L_1)$, where the second term captures the possibility that the competitor enters with $x_1 > L_1$ and the incumbent either is weak or does not enter. With the help of Equation 5, the derivative can be rearranged as

$$\frac{\partial\Pi(L_1, L_0|r_0 = 0)}{\partial L_1} \propto \frac{\partial\Pi(L_1, L_0|r_0 = 0, \alpha = 0)}{\partial L_1} + \alpha(1 - L_1)^2,$$

where the derivative with $\alpha = 0$ is $k_1 - L_1^2$, as shown in the derivation of Example 1. Therefore, the FOC is $k_1 + \alpha = 2\alpha L_1 + (1 - \alpha)L_1^2$. The second-order condition implies that the profit function $\Pi(L_1, L_0|r_0 = 0)$ is concave in L_1: $\partial^2\Pi(L_1, L_0|r_0 = 0)/\partial L_1^2 \propto -2L_1 - 2\alpha(1 - L_1) < 0$. Therefore, the FOC characterizes the maximum, and $\partial\Pi(L_1, L_0|r_0 = 0)/\partial L_1 < 0$ characterizes all L_1 smaller than the solution to the FOC. The right-hand side of the entry equation is the same as the right-hand side of the FOC, so it is immediate that both equations hold at $r_1 = (k_1 - \alpha)/2$. When $r_1 < 0$, the advertiser would want to charge a participation fee instead of awarding a stipend. Equivalently, the positive root of the FOC is too large to support a stipend. From concavity of the profit function in L_1, the best nonnegative stipend to use in this situation is thus $r_1 = 0$. *QED Example 5.*

Proof of P_2

The advertiser makes a positive profit only when both agencies enter but pays the stipend to each entrant independently:

$$\Pi(\rho_0, \rho_1) = \rho_0\rho_1\underbrace{\left(\frac{V}{3}\right)}_{E[\min(x_i)]} - \rho_0\underbrace{\left[k_0 - \frac{(3 - 2\rho_1)V}{6}\right]}_{r_0} - \rho_1\underbrace{\left[k_1 - \frac{(3 - 2\rho_0)V}{6}\right]}_{r_1}.$$

First, suppose that both bid-preparation costs exceed the competitive contest payoff $k_i \in [V/6, V/2]$ and consider a stipend policy that results in $(k_i - r_i) \in [V/6, V/2]$ and the associated mixed-strategy equilibrium $\{\rho_0, \rho_1\} = \{3/2 - [3(k_1 - r_1)/V], 3/2 - [3(k_0 - r_0)/V]\}$. Expressing the profit function in terms of the stipends makes it clear the advertiser cannot benefit from positive stipends: $\Pi(r_0, r_1) =$

$3/4[V + (4k_0k_1/V) - 2(k_0 + k_1)] - 3r_0r_1/V$. Therefore, the advertiser maximizes its within-equilibrium payoff by setting stipends to zero (setting only one stipend to zero is revenue neutral). Alternatively, the advertiser can set $k_i - r_i = V/6$ to induce guaranteed entry by both agencies and obtain a profit of $2V/3 - k_0 - k_1$, which is lower than $\Pi(0, 0)$ as long as $k_i > V/6$. Finally, the advertiser cannot benefit from setting only one $k_i - r_i = V/6$ to induce guaranteed entry by agency i, because the best response to $\rho_i = 1$ is $\rho_{-i} = 0$ as long as $k_i \in [V/6, V/2]$, and the advertiser only makes a positive profit when both agencies enter. Therefore, the advertiser should not offer stipends when $k_i \in [V/6, V/2]$.

Next, suppose that only the competitor's cost exceeds the competitive contest payoff: $0 \le k_0 \le V/6 < k_1 \le V/2$. Offering any stipend to the incumbent that always enters even with $r_0 = 0$ is obviously not beneficial. The minimum stipend that induces the competitor to enter is $r_1 = k_1 - V/6$, and the resulting profit exceeds the (zero) profit without competitor entry when

$$\Pi(\text{comp entry}) = \underbrace{\left(\frac{V}{3}\right)}_{E[\min(x_i)]} - \underbrace{\left(k_1 - \frac{V}{6}\right)}_{r_1} = \frac{V}{2} - k_1 > 0 = \Pi(\text{no comp entry}) \Leftrightarrow k_1 < \frac{V}{2}.$$

QED P_3.

Proof of P_3

A competitor with $x_i = L_1$ knows it can win only if all other competitors stay out of the auction *and* the incumbent's x_0 is below L_1. Therefore, the entry threshold satisfies the following:

$$\Pr(N - 1\ x_i\text{'s} < L_1)\Pr(x_0 < L_1)E(L_1 - x_0|x_0 < L_1) = \left(\frac{L_1}{V}\right)^{N-1}\frac{L_1^2}{2} = k - r.$$

Fix k for clarity and consider the expected price that the advertiser obtains. Let π_n be the advertiser's profit when $n \ge 2$ agencies (including both the incumbent and/or the competitors) exist with x_i above the L_1 cutoff. The uniform assumption yields π_n in a closed form: π_n is the expected second-highest draw from $n \ge 2$ draws that are x_i distributed i.i.d. on [L, V] according to the uniform distribution $\Pr(x < z) = (z - L)/(V - L)$. When n draws are distributed i.i.d. according to F(x), the density of the second-smallest order statistic is $n(n-1)f(x)[1 - F(x)]F^{n-2}(x)$. $F(z) = (z - L)/(V - L)$, which implies that

$$\pi_n = \int_L^V zn(n-1)\left(\frac{1}{V-L}\right)\left(\frac{V-z}{V-L}\right)\left(\frac{z-L}{V-L}\right)^{n-2}dz = \frac{2L + (n-1)V}{n+1}.$$

To keep notation compact, define π_1 as the expected profit with one competitor (by definition, above L_1) and the incumbent below L_1: $\pi_1 \equiv L_1/2$.

The advertiser averages over all possible numbers n = 0, 1, 2, . . . , N of entering competitors. Denoting the probability that a single entrant enters as $s \equiv (V - L)/V$, the expected profit is

$$\Pi_N(s) = \sum_{n=1}^{N}\binom{N}{n}s^n(1-s)^{N-n}[s\pi_{n+1} + (1-s)\pi_n - nr] = -rsN + \sum_{n=1}^{N}\binom{N}{n}s^n(1-s)^{N-n}[s\pi_{n+1} + (1-s)\pi_n],$$

where the second equality follows from $\sum_{n=1}^{N}\binom{N}{n}s^n(1-s)^{N-n}n = E[n|n \sim \text{Binomial}(s, N)] = sN$. Adding all the probabilities that a given π_n occurs yields

$$\sum_{n=1}^{N}\binom{N}{n}s^{n+1}(1-s)^{N-n}\pi_{n+1} + \sum_{n=1}^{N}\binom{N}{n}s^n(1-s)^{N-n+1}\pi_n = Ns(1-s)^N\pi_1 + s^{N+1}\pi_{N+1} + \sum_{m=2}^{N}\left[\binom{N}{m-1} + \binom{N}{m}\right]s^m(1-s)^{N-m+1}\pi_m.$$

Finally, reparametrizing in terms of the entry probability s uses the uniform assumption $V(1-s)^{N+1} = 2(k - r) \Rightarrow r = k - [V(1-s)^{N+1}]/2$. The profits π_m follow from $L = (1-s)V \Rightarrow \pi_1 = (1-s)V/2, \pi_m = V - 2s[V/(m+1)]$ for $m \ge 2$. The previous four transformations yield

$$\Pi_N(s) = sN\left[V(1-s)^{N+1} - k\right] + \left\{s^{N+1}\pi_{N+1} + \sum_{m=2}^{N}\left[\binom{N}{m-1} + \binom{N}{m}\right]s^m(1-s)^{N-m+1}\left[V - 2s\left(\frac{V}{m+1}\right)\right]\right\}.$$

Although the part of the equation in braces does not easily simplify further, its derivative in s does: $d/ds\{...\} = sNV(N+1)(1-s)^N$. Therefore, the first derivative of the profit in s takes a remarkably simple form: $d\Pi_N/ds = N[V(1-s)^{N+1} - k]$. Because $d^2E(\Pi)/ds^2 = -N(N+1)(V - v)(1-s)^N < 0$, the FOC characterizes the optimal entry probability: $dE(\Pi)/ds = 0 \Leftrightarrow V(1-s^*)^{N+1} = k \Leftrightarrow r^* = k/2$. At the optimal s, $\Pi_N(s^*) = [s^*(1-s^*)^{N+2}N(N-1)V]/(2+N)$, which can be shown to be increasing in N. *QED P_3.*

Proof of P_4

In the entry game, it is natural to look for an equilibrium in threshold strategies because a higher q_i implies a higher probability of winning the contest. Suppose the opponent $-i$ enters when $q_{-i} \ge L_{-i}$. The expected entry surplus of agency i is

$$S_i(q_i) = P\underbrace{H(L_{-i})}_{\Pr(-i\text{ not enter})} + P\underbrace{(1 - H(L_{-i}))}_{\Pr(-i\text{ enter})}\underbrace{\max\left\{0, \frac{H(q_i) - H(L_{-i})}{1 - H(L_{-i})}\right\}}_{\Pr(q_i > q_{-i}|-i\text{ enter})} = \begin{cases} q_i < L_{-i}: PH(L_{-i}) \\ q_i \ge L_{-i}: PH(q_i) \end{cases}.$$

Suppose $k_0 - r_0 \le k_1 - r_1$, and let $L_0 = 0$ (i.e., assume the incumbent enters regardless of its quality). In response, the competitor plays a threshold strategy that satisfies $k_1 - r_1 = PH(L_1)$. Closing the loop, always entering ($L_0 = 0$) is the incumbent's best response to such a competitor: either the q_0 is low ($q_0 < L_1$), in which case $S_0(q_0) = PH(L_1) = k_1 - r_1 > k_0 - r_0$, or q_0 is high, in which case $S_0(q_0) = PH(q_0) >$

$PH(L_1) = k_1 - r_1 > k_0 - r_0$. Therefore, $\{L_0 = 0, k_1 - r_1 = PH(L_1)\}$ is a Nash equilibrium as long as the stipends are such that $k_0 - r_0 \le k_1 - r_1$.

Next, suppose that $k_0 < k_1$ and consider stipends that maintain $k_0 - r_0 \le k_1 - r_1$. Express the advertiser profit in terms of L_1 as

$$\Pi(L_1) = \underbrace{H(L_1)\int_{L_1}^{V} z dH(z)}_{\substack{Pr(1\text{ enter}\,\&\,q_0 < L_1)\\ \times E(q_1|1\text{ enter})}} + \underbrace{2\int_{L_1}^{V} z[H(z) - H(L_1)]dH(z)}_{Pr(1\text{ enter}\,\&\,q_0 > L_1)\times E(\max q_i|1\text{ enter})}$$

$$+ \underbrace{H(L_1)\int_{0}^{V} z dH(z)}_{Pr(1\text{ not enter})\times E(q_0)} - \underbrace{[1 - H(L_1)]}_{Pr(1\text{ enter})}\underbrace{[k_1 - PH(L_1)]}_{r_1}$$

$$= 2\int_{L_1}^{V} zH(z)dH(z) + H(L_1)\int_{0}^{L_1} z dH(z) - [1 - H(L_1)] \times [k_1 - PH(L_1)].$$

The first derivative of the profit functions is $d\Pi/dL_1 = h(L_1)[k_1 + P - 2PH(L_1) - \int_0^{L_1}(L_1 - z)dH(z)]$. To find the $d\Pi/dL_1$ at $r_1 = 0$, substitute $k_1 = PH(L_1)$. Differentiation by parts yields that $d\Pi/dL_1|_{r_1=0} < 0$ iff $P[1 - H(L_1)] < \int_0^{L_1} H(x)dx$ at L_1 that satisfies $k_1 = PH(L_1)$. *QED P₄*.

Details of Example 7

Letting $H(L_1) = L_1$ simplifies the FOC to a quadratic: $0 = d\Pi/dL_1 = k_1 + P - 2PL_1 - L_1^2/2$. The second-order condition implies Π is concave. The positive root of the quadratic FOC equation is $L_1^* = -2P + \sqrt{2(k_1 + P + 2P^2)}$, implying the optimal stipend through the entry-threshold condition $k_1 - r_1 = PL_1$. Marginal analysis only makes sense when the highest-quality competitor enters even with $r_1 = 0$ (i.e., when $k_1 < P$). The stipend implied by L_1^* is positive when $L_1^* < k_1/P$ corresponding to $r_1 = 0$. Substitutingfor L_1^* yields $-2P^2 + \sqrt{P^4 + 2P^3} < k_1$. The stipend implied by L_1^* is small enough that $k_0 \le k_1 - r_1$ as long as $PL_1^* > k_0 \Leftrightarrow -2P^2 + \sqrt{P^4 + 2P^3} > k_0$. Thus, the first constraint. Next, consider the $k_0 = k_1 \equiv k$ case in which the advertiser must pay both agencies to lower the entry threshold L_1. The general profit function becomes $\Pi(L_1) = 2\int_{L_1}^{V} zH(z)dH(z) + H(L_1)\int_0^{L_1} zdH(z) - 2[1 - H(L_1)][k - PH(L_1)]$, and the first derivative under the uniform assumption $H(L_1) = L_1$ is $d\Pi/dL_1 = 2k + 2P - 4PL_1 - L_1^2/2$. Substituting the $k = PL_1$ yields the first derivative at $r_i = 0$: $d\Pi/dL_1|_{r_i=0} = -2k + 2P - k^2/2P^2$. Positive stipends to both agencies benefit the advertiser when $d\Pi/dL_1|_{r_i=0} < 0$, yielding the second inequality of the example. *QED Example 7*.

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