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Erratum to "Optimal Reverse-Pricing Mechanisms" by Martin Spann, Robert Zeithammer, and Gerald Häubl

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In our paper about optimal reverse pricing mechanisms [Spann M, Zeithammer R, Häubl G (2010) Optimal reverse-pricing mechanisms. *Marketing Sci.* **29**(6):1058–1070] (hereafter, ORPM), some of the mathematical derivations implicitly assume that the name-your-own-price seller interprets the outside-market posted price p differently than the buyers. This note shows that all of the qualitative results in ORPM continue to hold under the more natural assumption of common knowledge that p is the upper bound of wholesale cost. Interestingly, the proofs and algebraic expressions are often simpler than those in ORPM.

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Introduction: The Two Assumptions About the Meaning of p

Before going through the affected sections of the original paper, we expand on the nature of the original assumption implicit in the math of Spann et al. (2010) (hereafter, ORPM). In describing the model, ORPM assumes that the outside market price p is common knowledge in the beginning of the game, and p is an informative upper bound of the wholesale cost w. ORPM correctly shows how the buyers use p in optimizing their bids. However, in deriving the seller profit, the algebra in ORPM is set up as if the seller believed his wholesale cost w to be distributed Uniform$[0, 1]$ at the time of setting his strategy. Given the common knowledge of p, a more internally consistent assumption is that the seller has the same information as the buyers, and also believes his wholesale cost w is distributed Uniform$[0, p]$ when he sets his fee and minimum markup.

This note makes the latter assumption, and reproves all results in ORPM that are affected by this change. On several occasions, the change requires a modification of a proposition or claim. The modifications needed are only in the exact algebraic and numerical expressions, not in the qualitative insights proposed by ORPM.

All headings and section numbers used in this note refer to the original ORPM paper. We start by noting that the analysis of buyer behavior is unaffected. Therefore, §2.2 of the ORPM paper is unaffected. Changes to §§2.3–2.5 as follows.

2.3. Optimal Selling Without Commitment to a Minimum Markup ($m = 0$)

The change to the profit function (Equation 2 in the original paper) is highlighted in **bold red** (only **bold** in printed version) in the equations that follow:

$$\begin{aligned}\Pi(f) &= \Pr(\underline{v} < v)f + E_{w,v>\underline{v}}[\mathbf{1}(b > w)(b - w)] \\ &= [1 - H(\underline{v})]f \\ &\quad + \int_{\underline{v}}^{p}\int_0^p \mathbf{1}\left(\frac{v}{2} > w\right)\left(\frac{v}{2} - w\right)\left(\boldsymbol{\frac{1}{p}}\right) dw\, dh(v) \\ &\quad + [1 - H(p)]\int_0^{p/2}\left(\frac{p}{2} - w\right)\left(\boldsymbol{\frac{1}{p}}\right) dw \\ &= [1 - H(2\sqrt{pf})]f \\ &\quad + \int_{2\sqrt{pf}}^{p}\pi(v)\, dH(v) + [1 - H(p)]\pi(p), \qquad (2')\end{aligned}$$

where

$$\pi(v) \equiv \int_0^{v/2}\left(\frac{v}{2} - w\right)\left(\boldsymbol{\frac{1}{p}}\right) dw = \frac{v^2}{8p}$$

is the profit from a bidder of valuation v.

A modified Proposition 2 implied by Equation (2′) is as follows:

Modified Proposition 2. *When*

$$\frac{3}{4}x - \frac{1-H(x)}{h(x)}$$

is an increasing function of x, the optimal consumer entry threshold of a noncommitment seller satisfies either $(3/4)\underline{v} = (1-H(\underline{v}))/h(\underline{v})$ or $\underline{v} = p$, whichever is lower. When $H(x) = x$ on $[0, p]$, the implied optimal fee given $m = 0$ is

$$\sqrt{f^*(p)} = \min\left(\frac{2}{7\sqrt{p}}, \frac{\sqrt{p}}{2}\right) = \begin{cases} \dfrac{\sqrt{p}}{2} & \text{for } p < \dfrac{4}{7} \approx 0.57, \\[2ex] \dfrac{2}{7\sqrt{p}} & \text{for } p \geq \dfrac{4}{7} \approx 0.57. \end{cases}$$

Proof. Please see the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mksc.2014.0883) for all proofs in this note. □

Interestingly, and unlike in ORPM, all $p \geq 4/7$ screen entering bidders at the same minimum valuation of $\underline{v} = 4/7$.

2.4. Optimal Selling With Commitment to a Minimum Markup ($m \geq 0$)

The main result (dominance of fees over minimum markups in Proposition 3) survives intact, exactly as stated:

Proposition 3. *Suppose $H(x) = x$ on $[0, p]$. Even if the seller can credibly commit to a positive minimum markup, the optimal selling strategy uses zero minimum markup and the positive fee $f^*(p)$ derived in Proposition 2.*

Interestingly, the proof of Proposition 3 is substantially simplified by the more consistent assumption relative to the ORPM proof.

The illustrative example has to be recalculated under the new assumption.

Welfare and Profit Calculation of Illustrative Example in §2.4.

Let $v = 1$ correspond to \$1,500, let $p = 2/3$ (which thus corresponds to \$1,000), and assume consumer valuations are distributed uniformly on [\$0, \$1,500]. The optimal bidding fee to charge is $f^*(2/3) = 6/49 \approx$ \$183, which screens at level $\underline{v} = 4/7 \approx$ \$857. (The same level would hold for all other $p > 4/7$.) Thus, most low consumers do not enter. Those low consumers who do enter bid $v/2$, resulting in bids between \$427 and \$500. In addition, all high consumers enter, and all bid \$500. The expected social welfare W realized through the reverse-pricing seller is the difference $v - w$ when there is a trade, that is, when valuation exceeds $\underline{v}$ and the bid exceeds w

$$\begin{aligned} W\left(f^*\left(\frac{2}{3}\right)\right) &= \left(\frac{1}{p}\right)\int_{4/7}^{2/3}\int_0^{v/2}(v-w)\,dw\,dv \\ &\quad + \left(\frac{1}{p}\right)\int_{2/3}^{1}\int_0^{p/2}(v-w)\,dw\,dv \\ &= \left(\frac{3}{2}\right)\frac{271}{3{,}087} \approx \$197, \end{aligned}$$

with an overall probability of trading of about 14%. The seller's profit is

$$\begin{aligned} \Pi\left(f^*\left(\frac{2}{3}\right)\right) &= \left(1-\frac{4}{7}\right)\frac{6}{49} + \left(\frac{3}{2}\right)\int_{4/7}^{2/3}\frac{v^2}{8}\,dv \\ &\quad + \left(\frac{3}{2}\right)\left(1-\frac{2}{3}\right)\frac{p^2}{8} \\ &= \frac{461}{5{,}292} \approx \$130. \end{aligned}$$

Now consider the optimal minimum markup to charge contingent on bidding fees being zero.

To derive the optimal markup, let $f = 0$ in $\Pi(m, f)$ in the proof of Proposition 3

$$\begin{aligned} \Pi(m|f=0) &= \int_m^p \pi(m \mid v)\,dv + (1-p)\pi(m \mid p) \\ &\Rightarrow FOC_m\colon\ m^* = \frac{3-\sqrt{9-10p+5p^2}}{5} \approx \$260. \end{aligned}$$

This is unchanged from before because the $(1/p)$ term factors out of the profit. However, the welfare and profits are affected

$$\begin{aligned} W\left(m^*\left(\frac{2}{3}\right)\right) &= \left(\frac{1}{p}\right)\left[\int_{m^*}^{p}\int_0^{(v-m^*)/2}(v-w)\,dw\,dv \right. \\ &\quad \left. + \int_p^1\int_0^{(p-m^*)/2}(v-w)\,dw\,dv\right] \\ &= \left(\frac{3}{2}\right)\frac{189+59\sqrt{41}}{6{,}750} \approx \$187. \end{aligned}$$

Plugging the m^* into the profit equation yields

$$\Pi\left(m^*\left(\frac{2}{3}\right)\,\middle|\, f=0\right) = \left(\frac{3}{2}\right)\frac{61+41\sqrt{41}}{8{,}000} \approx \$90.$$

Now consider the optimal bidding fee contingent on setting the minimum markup to $m^* = (9-\sqrt{41})/15 \approx$ \$260. For $p = 2/3$, the proof of Proposition 3 suggests a fee of

$$\begin{aligned} f^*\left(m \,\middle|\, p = \frac{2}{3}\right) &= \frac{6(1-2m)^2}{49} \\ &\Rightarrow f^*\left(\frac{9-\sqrt{41}}{15}\,\middle|\, p=\frac{2}{3}\right) \approx \$78. \end{aligned}$$

Modified Table 1 **Illustrative Example of a Plane Ticket from New York to London that Costs \$1,000 on Expedia and That Consumers Value Uniformly Between \$0 and \$1,500**

	Optimal bidding fee, no minimum markup (\$)	Optimal minimum markup, no bidding fee (\$)	Optimal minimum markup and its associated optimal bidding fee (\$)
Bidding fee (f)	183	0	78
Minimum markup (m)	0	260	260
Screening level ($\underline{v}$)	857	260	820
Social welfare (gains from trade)	197	187	150
Seller profit	130	90	107

Since the combination (m^*, f^*) is now on the locus of optimal fees given markups, we can simply plug the m^* into the profit equation in the proof of Proposition 3 to find that the profit is $\approx$ \$107. As expected, this profit is more than $\Pi(m^*(2/3) \mid f = 0)$, but less than $\Pi(m = 0, f^*(2/3))$. Interestingly, the resulting screening level is similar to that with the optimal fee and no markup, namely, $\approx$ \$820. The welfare is thus decidedly lower because of bid shading

$$W(m^*, f^*(m^*)) = \left(\frac{1}{p}\right)\int_{3/5}^{2/3}\int_0^{(v-m^*)/2}(v-w)\,dw\,dv + \left(\frac{1}{p}\right)\int_{2/3}^{1}\int_0^{(p-m^*)/2}(v-w)\,dw\,dv = \left(\frac{3}{2}\right)\frac{18+277\sqrt{41}}{27{,}000} \approx \$150.$$

Modified Table 1 displays the results for (1) the optimal bidding fee given the minimum markup set to zero, (2) the case of an optimal minimum markup given no bidding fee, and (3) for the case of the optimal fee to use given the minimum markup suggested in (2).

2.5. Should the Seller Who Charges a Bidding Fee Facilitate or Hinder Consumer Learning About the Current Bid-Acceptance Threshold?

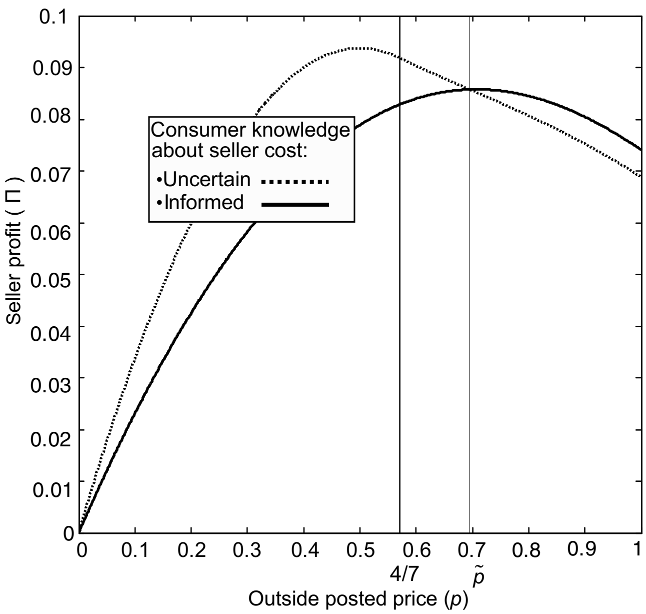
Claim 1. The claim that when potential consumers learn the marginal cost before their entry decision, the optimal bidding fee is a solution to $2f_I = E[\min(p, v) \mid v > f_I]$ holds exactly because the $(1/p)$ correction factors out of the profits (see the proof in the online appendix for details).

Whereas the *FOC* is unaffected, the profit of the seller facing informed consumers is $(1/p)$-times higher than in the original paper. The profit of the seller facing uncertain consumers is different as well (as discussed previously), so the numerical details of Proposition 4 change.

Modified Proposition 4. *A unique outside price $4/7 < \tilde{p} < 1$ exists such that facilitating consumer learning about the seller's current bid-acceptance threshold is profitable for the seller when $p > \tilde{p}$, and vice versa.*

The modified Figure 4 illustrates modified Proposition 4.

Modified Figure 4 **Seller Profit Under Two Consumer-Information Scenarios**



Notes. This figure illustrates modified Proposition 4. The two curves represent seller profits as a function of the price on the outside posted-price market. The solid curve involves consumers informed about seller cost before making their entry decision. The dashed curve involves consumers uncertain about seller cost at the time of their entry decision.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2014.0883.

Reference

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