Intern. J. of Research in Marketing 22 (2005) 159–182

International Journal of Research in Marketing

www.elsevier.com/locate/ijresmar

Assessing potential threats to incumbent brands: New product positioning under price competition in a multisegmented market

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Abstract

We present a model of competitive positioning and pricing of new products in a multisegmented market that is useful not only for new entrants, but also for brand managers of incumbents to assess the potential threats inherent in existing market structures. We do this for a multisegmented market in which the ideal point for each segment is located in a multidimensional discrete-attribute space with fixed demands at a given point in time. Firms launch new products sequentially at positions in this attribute space, incurring fixed and variable costs, and then decide on their product prices. Each firm acts to maximize its profit. We allow free entry, regardless of whether or not an entry location is occupied by an incumbent, and the position and price of a firm's product determine its market share. The number of firms that can make a profit in the market is determined endogenously, and the model determines the number of survivors. Free and endogenous entry removes from the brand manager the need to evaluate millions of potential entry threats from combinations of new products and possible positions. Instead, the methods developed here determine a much smaller set of threats that need to be considered. We adopt from the facility-location literature another equilibrium concept, the stable set, and relate it to the Nash equilibrium. Location decisions are stable, if, and only if, the entrants make a profit (viability) and the non-entrants cannot find any location such that their profit after entry is non-negative (survival). We design a heuristic algorithm based on genetic algorithms to empirically obtain the Nash equilibrium. The illustration involves the prospect of new brands attempting to enter the established liquid detergent market. Using aggregated share data from heavy user and light user segments, we model the segment-level market share as a function of distance from segment-specific ideal points, with segment-specific price sensitivities. We use segment-level shares to locate heavy and light user ideal points in a product-positioning space derived from Consumer Reports ratings of the real brands. The results show that the only open position for successful entry matches the effectiveness of Tide (the market leader) in removing stains, and lowers costs (and price) by sacrificing on the other attribute in the space. The reduced price appeals to the heavy user segment, leading to profitable entry. This position for entry remains profitable even if Tide opportunistically relocates.

Keywords: New product research; Product positioning; Price competition; Nash equilibrium; Defensive marketing strategy

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0167-8116/$ - see front matter
doi:10.1016/j.ijresmar.2004.06.005

1. Introduction

Consumers do not wash more clothes just because a new brand of laundry detergent comes on the market. Successful new-product entry into mature categories typically does not expand market size, nor is it likely to reduce the costs faced by existing brands. Consequently, successful entry can have major implications for the profits of existing brands in the category. Knowing the vulnerable positions where a new brand or brands could attack a mature market is highly useful for possible new entrants, as well as for existing brands seeking to defend their positions. We need to look no further than these obvious reasons to understand why new-product positioning and pricing problems have received great attention from academia and industry.

Concerning product positioning, psychometric models using a multidimensional scaling procedure (MDS) have produced an extensive literature (c.f., Cooper, 1983, Green & Krieger, 1989 and Green & Srinivasan, 1978, 1990 for overviews of these models). However, this research stream does not address the competitive issues in new-product positioning and pricing. The first paper on the competitive product-positioning problem is generally thought to be the early work by Hotelling (1929), who considered product positioning on a line where consumers are distributed uniformly. Many authors use this linear-market assumption (e.g., Eaton & Lipsey, 1975; Economides, 1984; D'Aspremont, Gabszewicz, & Thisse 1979; De Palma, Ginsberg, Papageorgiou, & Thisse 1985, and Shaked & Sutton, 1982).[1]

The literature on product pricing and competitive positioning in a multidimensional space is not very extensive (Carpenter, 1989; Choi, DeSarbo, & Harker, 1990, 1992; Hadjinicola, 1999; Hauser, 1988; Hauser & Shugan, 1983; Lane, 1980). While these game-theoretic models are rigorous and provide a theoretical background for psychometric models, most of them are too restrictive in their model specifications to be used in real decision making. Carpenter (1989) analyzes only duopolistic competition in a single market (single ideal point). Lane (1980) analyzes the sequential-entry problem under oligopolistic competition, but assumes a single market segment and identical cost structures of firms. Hauser and Shugan (1983) and Hauser (1988) address defensive marketing strategy when a firm faces an attack by a new competitive product. In the Defender model, consumers are distributed on a "per-dollar" multiattribute space when few, if any, attributes can be measured on the required ratio scale. Hadjinicola (1999) presents a product-position and pricing model, including economies of scale effects, but the number of ideal points is still limited to one.

The model presented by Choi et al. (1990, 1992, hereafter CDH) is more realistic than the game-theoretic models discussed above. They addressed oligopolistic spatial and price competition in a multi-segmented market, where single brand producers maximize their profits. They describe consumer choice behavior using a multinomial-logit (MNL) model—a realistic extension (compared to linear models of choice) we wish to preserve. However, we are very critical of their choices in the positioning game. In CDH, the analyst or manager predetermines the number of entrants in the market and their positions. We feel entry must be free (rather than a fixed number of entrants) and endogenous (i.e., determined by the competitive model, rather than exogenously specified) to have practical value as explained below. Also in CDH, a firm of interest, entering last, selects a position, while other firms, having entered already, respond only with the price. They consider only variable costs, while we believe both fixed and variable costs should be considered.

To understand the practical necessity of considering free and endogenous entry, rather than predetermined entry, first consider the problem of a manager for an incumbent brand trying to assess the threats to that existing brand's earnings. In the simple example reported later, incumbent brands have positions on a 4-by-4 grid. A single new entrant could take any of 16 positions each of which, under exogenous entry, would have to be evaluated by a manager or analyst for the potential threat. While this would be tedious, even with a well-calibrated market–response model, consider what happens with six potential entrants. The manager would have to evaluate over 16-million potential threats. By understanding and accepting the model's structure and assumptions, the manager obtains an analytical result that greatly reduces the number of defensive scenarios that need to be

[1] For a more complete survey of spatial-competition models in a linear market, see Eiselt and Laporte (1989).

considered. To be practical for brand management, models of competitive entry and positioning must incorporate free and endogenous entry.

In this paper, we present a two-stage model of competitive positioning and pricing of new-products in a multisegmented market. In our model, free entry is assumed so that firms launch their product as long as they make a profit. For this, sequential entry is assumed, since the simultaneous-entry game for a pure Nash equilibrium has never been solved in the product-positioning case. We show, however, that the traditional criticism of the sequential-entry game (i.e., that it is subject to first-mover advantages) does not apply in important cases. Fixed costs (R&D, production, advertisement, and channel-setup cost) play an important role, and the number of firms that can survive in the long run is obtained endogenously. We also solve the problem numerically in the first stage. We implement a genetic algorithm on the discrete attribute space using stable sets (Dobson & Karmarkar, 1987) to identify the set of viable and survivable entrants. The numerical results allow us to check the number of firms in the market, their equilibrium position and price, and first-mover advantage.

In summary, we present the following results in this paper:

- In the first stage, we establish the relationships between Nash equilibrium and another equilibrium concept, the stable set developed by Dobson and Karmarkar (1987) for facility-location problems. They define location decisions as stable, if, and only if, the entrants make a profit (viability) and the non-entrants cannot find any location such that their profit after entry is non-negative (survival). This concept can be directly applied to the product-positioning problem, and we show that the stable set always includes the Nash equilibrium in our problem.
- In the second stage, we show the sufficient conditions for the existence and uniqueness of Nash equilibrium. The sufficient condition for the uniqueness is not addressed by Choi et al. (1990).
- Finally, based on the result of first and second stages, we introduce a genetic algorithm to obtain an equilibrium solution. Then, we illustrate the method, using real data from a liquid detergent market.

This paper is organized as follows: in Section 2, we formulate the two-stage model. We first analyze the pricing problem, and then address the positioning problem. In Section 3, we discuss the computation of equilibria and present an application of our approach to the liquid detergent market. Section 4 concludes.

2. Problem formulation

We formulate a two-stage game-theoretic model for new-product positioning and pricing. In each stage, firms respond to the action of their competitors in order to maximize their profits. In the first stage, firms decide to enter a market by selecting product positions. By assigning an arbitrary position to indicate the “no entry” decision (e.g., location “0”), we include the entry decision in the positioning decision. The attribute space is discrete and represented by a multidimensional grid. Each firm launches a single new product in this discrete attribute space.[2] Variable and fixed costs, which depend on product positions, are incurred in this stage (i.e., the levels of the attributes reflected in the product position determine the fixed and variable costs). These kinds of attribute grids and their associated costs are appropriate in the design phase for new products or the reformulation phase for existing products. In such phases, the consumer preferences are typically mapped on to the physical dimensions of the products to aid understanding of the tradeoffs between features and costs. We assume that each firm introduces its product sequentially and launches its product as long as the fixed cost of launching a product is less than the gross contribution earned from producing the product. Therefore, the number of products (or firms) in the market is determined endogenously in our model.[3]

[2] By assuming each firm launches a single product, we are simplifying the model. In mature markets, some firms have multiple existing products and still could introduce a multiplicity of new products. The extension of the current framework to models that reflect multiple-product firms is a challenging task that is left to future development.

[3] This free-entry approach was also used by Lane, 1980, but with the oversimplifying assumptions of homogeneous costs across products and uniformly distributed customer demand.

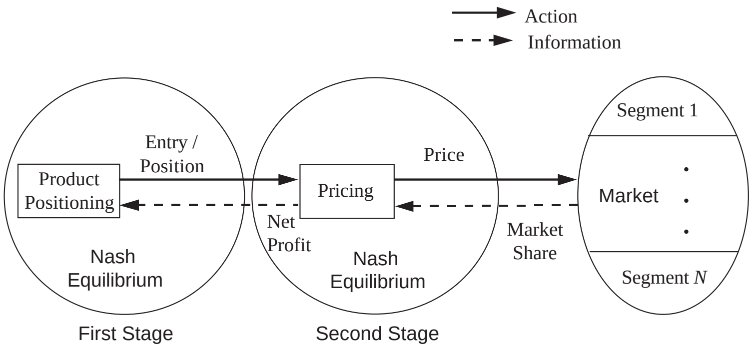


Fig. 1. Conceptual framework of the competitive product-positioning model.

In the second stage, firms decide on the prices for their products. Pricing decisions are made simultaneously. In this stage, the number of products and their positions are given, because pricing is a relatively short-term and flexible decision. This approach has been used in several papers (cf., Lane, 1980; Moorthy, 1988; Prescott & Visscher, 1977) (Fig. 1).

The market share of each product is determined by the positions and prices. In our model, we use a probabilistic market share model. Neoclassical economists have postulated that a consumer's choice process is deterministic. However, according to Anderson, de Palma, and Thisse (1992), fluctuations are inherent in the process of evaluating alternatives and one cannot identify all aspects that affect the choice process even if the choice process is deterministic. In this sense, probabilistic models are more realistic and practical than deterministic models. It is noteworthy that deterministic models are extreme cases of probabilistic market share models.

We utilize the multinomial-logit (MNL) model as a probabilistic market share model. This model has been used extensively in the marketing literature. Anderson et al. (1992) emphasized that the following features make the MNL model useful: First, the MNL model is easy to deal with mathematically and sometimes results in closed-form solutions. Second, the MNL model has a solid theoretical background. It is not only derived from the Luce choice axiom (1959) and random-utility models such as Yellott (1977), but also regarded as a special form of attraction model by Bell, Keeney, and Little (1975). Third, the MNL model has been successfully used to estimate the demand in numerous industries.

We assume that consumers are grouped into several market segments.[4] The assumption of the multi-segmented market is important in decision-making although a single-segment market has been assumed for mathematical tractability in many of the competitive product positioning models (Carpenter, 1989; Lane, 1980). For example, Cooper and Nakanishi (1988) showed the possible inconsistency between market shares and individual choice probabilities when individual choice probabilities and purchase frequencies are both heterogeneous. These authors suggested that, in such a case, one should segment the market and analyze each segment separately in order to escape an aggregation problem (i.e., the parameters estimated for the combined data would fail to reflect the proper sensitivity of either segment to the marketing instruments). Based on this structure, we begin our analysis of the pricing of products.

2.1. Second stage: pricing game

In this subsection, sufficient conditions for the existence and uniqueness of Nash equilibrium are obtained. An algorithm to identify the equilibrium is discussed. We first define the following notation:

N^* The number of firms or products launched
N The number of potential products to be launched, which is large enough that $N \geq N^*$,

[4] General marketing process follows market segmentation, target market selection, and product positioning (Sarvary, 2000). In our approach, we assume that market segmentation is given, and that the target market and position are jointly determined in the model.

Na	The dimension of the product-attribute space,
M	The number of market segments
i	The index for products, $i=1,\ldots, N$
j	The index for market segments, $j=1,\ldots, M$
D_j	The demand of market segment j
$\mathrm{MS}_{x(i)j}$	The market share of product i in market segment j
$x(i)$	The position of the ith product that is launched in the product-attribute space, $x(i)=(x(i)_1,\ldots, x(i)_{\mathrm{Na}})$, $x(i)=(0, \ldots, 0)$ if firm does not launch any product; $X=(x(1), x(N))$
s_j	The ideal position of market segment j in the product-attribute space, $s_j=(s_{j1},\ldots, s_{j\mathrm{Na}})$, $S=(s_1, \ldots, s_M)$
$p_{x(i)}$	The price of product i, $p=(p_{x(1)},\ldots, p_{x(N^*)})$
$c_{x(i)}$	The variable cost of product i, a function of its position $x(i)$
$f_{x(i)}$	The fixed cost of product i, a function of its position $x(i)$
$d_{x(i)j}$	The distance between product i and the ideal point for segment j in the attribute space
π_i	The net profit of the product i; if the product is launched, $\pi_i=[p_{x(i)}-c_{x(i)}]\ \sum_j D_j \mathrm{MS}_{x(i)j}-f_{x(i)}$; otherwise, $\pi_i=0$
$z(k_1,\ldots, k_{\mathrm{Na}})$	The occupancy or number of products at the point $(k_1,\ldots, k_{\mathrm{Na}})$ of attribute space
Z	The occupancy vector of $z(k1,\ldots, kNa)$'s for all $(k1,\ldots, kNa)$. The vector showing the number of products positioned at each point in the attribute space.

Although the MNL model can be derived in several ways, we assume that market share is derived from the axioms of Bell et al. (1975). Then the market share of product i in market segment j is,

$$\mathrm{MS}_{x(i)j}=\frac{A_{x(i)j}}{\sum_{k=1}^{N^*}A_{x(k)j}+A_{0_j}} \quad \text{for all } i,j \tag{1}$$

where $A_{x(i)j}$ is attraction of market segment j toward product i, and A_{0j} is "no purchase" option. Thus, market share is simply the ratio of the attraction of a product over that of all products and no-purchase option. In the MNL model, $A_{x(i)j}$ is assumed to be an exponential function as follows:

$$A_{x(i)j}=\exp\left[-d_{x(i)j}-\gamma_j p_{x(i)}\right]$$

where γ_j (>0) is a price sensitivity. If $A_{0j}=0$, every customer is forced to chose one of the brands regardless of price. In many cases, A_{0j} is assumed to be $A_{0j}\equiv\exp[0]=1$ to assign status-quo utility of zero (see Besanko, Gupta, & Jain, 1998; Choi et al., 1990). The attraction is determined by distance from a segment's ideal point and price, which have forms of ideal-point models and vector models, respectively. These two types are compatible with each other, because the vector model is hierarchically nested within the ideal-point model and an empirical test can assess which form is more appropriate. Any reasonable distance measure can be used for distance $d_{x(i)j}$, although we need to specify a form of the distance measure for the parameter estimation. As shown later, we use a weighted, squared Euclidean distance.

Since firms make positioning decisions in the first stage, the distances $d_{x(i)j}$ are known numbers in the second stage. For known parameter values and distances, firms set the price to maximize their profits, which is expressed as:

$$\pi_i=\left[p_{x(i)}-c_{x(i)}\right]\sum_j D_j\mathrm{MS}_{x(i)j}-f_{x(i)}.$$

A Nash equilibrium in the second stage is defined as a set of prices such that no firm can benefit from a unilateral change in its price decisions. Thus, a Nash equilibrium is obtained by a set of $p_{x(i)}$ $(i=1,\ldots, N^*)$ such that $\partial\pi_i/\partial p_{x(i)}=0$, or

$$p_{x(i)}=c_{x(i)}+\frac{\sum_{j=1}^{M}D_j\mathrm{MS}_{x(i)j}}{\sum_{j=1}^{M}D_j\left(-\partial\mathrm{MS}_{x(i)j}/\partial p_{x(i)}\right)} \tag{2}$$

Since $0\leq\mathrm{MS}_{x(i)j}\leq 1$,

$$\partial\mathrm{MS}_{x(i)j}/\partial p_{x(i)}=-\gamma_j\mathrm{MS}_{x(i)j}\left[1-\mathrm{MS}_{x(i)j}\right]\leq 0. \tag{3}$$

Eq. (3) shows that the price $p_{x(i)}$ is bounded below by the variable cost $c_{x(i)}$. That is, firms set prices no less than their variable costs. Choi et al. (1990) showed that $p_{x(i)}$ is bounded above in a non-segmented market. We show the similar result for the multisegmented market in the following lemma.

Lemma 1. $p_{x(i)}<\infty$. *See Appendix A for the proof.*

This lemma implies that firms have no incentive to increase the price up to infinity and therefore the price is bounded above. Thus, we assume that firms set prices on the closed interval $[c_{x(i)}, rp_{x(i)}]$, where the reservation price $rp_{x(i)}$ is a reasonable upper bound of price that the customers (buyers) have toward product i.

Pure-strategy Nash equilibria exist in the second stage if the following conditions are satisfied: (i) the strategy spaces of the prices $p_{x(i)}$ are nonempty, compact–convex subsets of Euclidean space; and (ii) the payoff functions π_i are continuous and quasi-concave in prices $p_{x(i)}$ (see Fudenberg & Tirole, 1992). Although it is obvious that strategy spaces are non-empty, compact, convex, and the payoff functions are continuous, the profit functions π_i are not generally quasi-concave in a multisegmented market. However, the profit functions are quasi-concave under the condition of the following lemma, and therefore there exist pure-strategy Nash equilibria.

Lemma 2. *If $\gamma_j \leq 2/(rp_{x(i)} - c_{x(i)})$ for all products i and market segments j, then there exist pure-strategy Nash equilibria for the second stage. See Appendix A for the proof.*

A similar condition was proven and discussed by Choi et al. (1990). Although the condition of Lemma 2 guarantees the existence of the Nash equilibria, it does not guarantee the uniqueness. Anderson et al. (1992) proved the existence and uniqueness of the Nash equilibrium only for a single-segment market. We provide a sufficient condition for the unique Nash equilibrium in the following proposition.

Proposition 1. *If $\gamma_j \leq 1/(rp_{x(i)} - c_{x(i)})$ for all i and market segment j, then there exists a unique pure-strategy Nash equilibrium for the second stage. See Appendix A for the proof.*

Sufficient conditions were not provided in Choi et al. (1990). However, uniqueness is essential not only for the first stage, but also for computation of price equilibrium. We identify price equilibrium with the diagonalization algorithm of the variational inequality suggested by Choi et al. (1990). They showed that solving this problem with the diagonalization algorithm is equivalent to solving the following problem:

$$\max_{p^k_{x(i)}} \pi\left(p^k_{x(i)} | p^{k-1}_{-x(i)}\right) \quad \text{for each } i, \tag{4}$$

where p^k_i is the price of product i at kth iteration. The variational inequality problem is to find a vector x^* in a closed convex subset K of R^{m} for a given function F such that

$$(y - x^*)^T F(x^*) \geq 0 \;\text{ for } \forall y \in K$$

To solve this problem, Pang and Chan (1982) developed a diagonalization (or nonlinear Jacobi) method. They also proved the method converges. Applying the algorithm to our problem involves solving the following equation:

$$\sum_{i=1}^{N^*} - \nabla_{p_{x(i)}} \pi\left(p^*_{x(i)}, \bar{p}_{-x(i)}\right)\left(p_{x(i)} - p^*_{x(i)}\right) \geq 0, \forall p_{x(i)} \in \Omega,$$

where Ω is a feasible set for prices and $\bar{p}_{-x(i)}$ is other firm's fixed prices. Choi et al. (1990) showed that solving this equation is equivalent to solving Eq (4). This representation is very simple and fits in well with the definition of Nash equilibrium. The uniqueness condition of Proposition 1 guarantees convergence. The algorithm to obtain the Nash equilibrium for the second stage is described in Appendix A.

2.2. First stage: positioning game

In this subsection, we define Nash equilibria for product positioning. We discuss stable sets introduced by Dobson and Karmarkar (1987), and show the relationship between these two equilibria.

We suppose that the attribute space is discrete and represented by a multidimensional grid. We allow multiple products to be positioned at the same point in the attribute space. In this stage, each firm either enters the market by selecting a product position or stays out of the market. Without loss of generality, we assume that firms launch their product according to the ascending order of an index i. Firms act as a profit maximizer, so that they launch their products as long as they make profits. The profit function of the product i in the first stage is defined as $\pi_i(X) = \pi_i(X, p^*(X))$; that is, the firm producing product i positions the product considering the equilibrium price of the second stage. The profit function π_i can be represented as a function of the position variable only if the equilibrium price is unique. Thus, our developments throughout this paper assume the sufficient condition of Proposition 1 is satisfied.

A pure-strategy Nash equilibrium for the product positioning is defined in the usual manner (i.e., no firm can be better off by unilateral change of its product position). That is, product position X^* is a pure-strategy Nash equilibrium (PNE(P)) in the first stage, if and only if $\pi_i(x(i)^*, -x(i)^*) \geq \pi_i(x(i), -x(i)^*)$ for all $x(i)$ and i, where $-x(i)$ is the strategy profile of the firms except firm i.

We assume that each firm has perfect information about its predecessors' decisions and launches its product with complete information about the followers' strategy set and pricing equilibrium. Thus, the equilibrium obtained is a subgame-perfect Nash equilibrium (see Selten, 1975). The equilibrium is obtained by backward induction and the existence is guaranteed (see Kuhn, 1953).

While the Nash equilibrium is the most frequently used solution concept for competition models, other solution concepts have been used depending on the context of the other problems (see Ghosh & Harche, 1993). One of the other solution concepts is stable sets—introduced by Dobson and Karmarkar (1987) for facility-location problems. Location decisions are defined to be stable, if, and only if, the entrants make a profit (viability) and the non-entrants cannot find any location such that their profit after entry is non-negative (survival). Dobson and Karmarkar (1987) define several variants of stable sets according to the context of the problem. These variants include independently or jointly viable, strong or weak survival, and restricted or unrestricted entry. Entry is restricted if a competitor can open only one location. Strong survival implies the case that the firm of concern makes more profit than the competitors, while weak survival means non-negative profit. Also independence implies a location makes profit independently with respect to the set of open locations. The stable set used in this current effort, according to their definition, has the properties of weak survival, restricted entry, and independent viability (so-called WRI). This solution concept is general enough to be applied to our problem. Under this solution concept, the entry sequence of the firms does not matter—firms are indistinguishable in this aspect. Thus, the decision variable under this solution concept is the number of products at each point of the attribute space (represented by occupancy vector Z), rather than the position of each product. For the formal definition of stable sets, we categorize firms into two sets: entrants (E) and non-entrants (NE). Then the definition of stable sets is as follows:

Definition 1 (*Stable sets (SS) in the first stage*). Occupancy vector Z is an element of a stable set, if and only if:

i) $\pi_i \geq 0$ for all $i \in E$ (*viability*), and
ii) $\pi_i < 0$ for all $i \in \mathrm{NE}$, if they launch their product after the entry of firms of set E (survival).

Before we investigate the relationship between Nash equilibria and stable sets, we define a "pre-assigned game" (P_{01}). In this game, firms have positions pre-assigned at what would be the best position for their products, if they were to decide to enter. But all firms must simultaneously decide whether or not to enter. In this game, the decision of each firm can be represented by a binary value: 0 if a firm decides not to enter; 1 if a firm decides to enter. Then we can show that the set of PNE(P) is a subset of stable set and that the set of PNE(P_{01}) is equivalent to the stable set. These two conditions are formalized in Proposition 2.

Proposition 2. *(i) SS $\supset$ PNE(P); (ii) SS = PNE(P_{01}). The proof is provided in Appendix A.*

Proposition 2 provides us with a different way to understand Nash equilibria in product-positioning games. The proposition indicates that the Nash equilibria must satisfy viability and survival conditions; otherwise, firms have incentives to change their actions. For example, if the profit of an entrant is negative, which is a violation of viability, then the entrant will move out of the industry. So, in equilibrium all entrants are profitable (i.e., viable). If a non-entrant could make a profit by launching its product, counter to the survival condition, then the firm would enter the industry. Thus, in equilibrium, no non-entrant can find a position that is profitable (i.e., the survival condition holds). These results explain why the later entrants are blockaded, even though multiple products at each position are not prohibited.

Our prime motivation for proving Proposition 2 is that it gives us an efficient way to produce a pure-strategy Nash equilibrium for the sequential-entry game (PNE(P)) by narrowing the search of game tree. First, it provides the maximum number of firms in an industry for the free-entry model so that the depth of the game tree is determined a priori. Once the depth of the game tree is restricted, one needs only to look at the branches generated by the stable set rather than the whole game tree. An example is given later in Fig. 3. This restricted search implies that players may foresee the ends, but only the limited ends. The algorithm may be sketched as follows:

Algorithm 1 (A1). *Design of the algorithm to identify PNE(P):*

Step1. Obtain elements of the stable set.

Step2. Find PNE(P) among the elements of the stable set.

Implementation of the algorithm is introduced in detail in Appendix B. In the next example, we explain briefly the idea of the algorithm (A1). Also we show that first-mover advantage may not be guaranteed.

Example 1. In a real situation, technical constraints or high fixed costs may make certain combinations of attributes impossible or impractical to formulate. To illustrate the entry and positioning game in such cases, we suppose that three potential product positions and ideal points are located in a circular form in a two-dimensional attribute space as shown in Fig. 2. Attraction $A_{x(i)j}$ is assumed to be an exponential function as follows:

$$A_{x(i)j} = \exp\left[-\sum_{h=1}^{2}\beta_{jh}\left(x(i)_h - m_{jh}\right)^2 - \gamma_j P_{x(i)}\right]$$

where, h: the index for attributes, $h=1,2$, m_{jh}: the ideal point of market segment j in hth attribute dimension, β_{jh}: the distance sensitivity of market segment j in hth attribute dimension, γ_j: the price sensitivity of market segment j, $\gamma_j>0$.

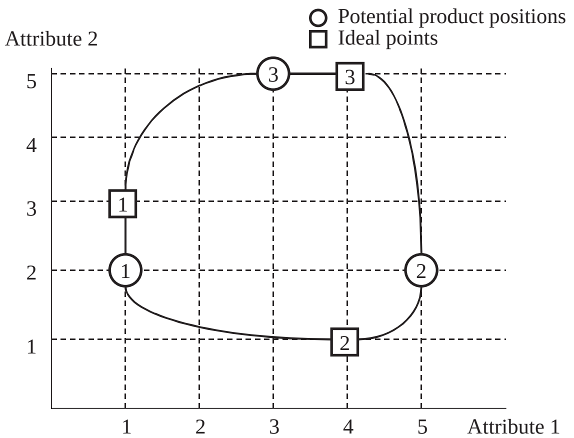


Fig. 2. A case with three market segments and potential positions.

Table 1
Cost and demand parameters

β_{jh}	γ_j	D_j	$c_{x(i)}$	$f_{x(i)}$	$rp_{x(i)}$
0.5	0.5	50	0.1	190	3.91

The distance metric is assumed to be a weighted, squared Euclidean distance (c.f., Carroll, 1980). For simplicity, we suppose $A_{0j}=0$. In this preliminary example, costs and demand parameters are assumed to be the same for each product and given in Table 1. Thus, differences in demand are generated only from products taking different positions in the attribute space.

In this example, occupancy vectors are generated by enumeration.[5] Table 2 shows a stable set and corresponding payoffs, identified from the occupancy vectors. The stable set limits the number of products to two, so that we can construct a game tree to identify PNE(P), as in Fig. 3. The branches are strategy profiles of the entrants in the first stage. The numbers at the end of the leaves are payoffs of entrants launching their products. PNE(P) is identified by *restricted* backward induction. For instance, the second entrant selects the best strategy by comparing the payoffs. Then, the game is reduced to a profit-maximization problem of the first entrant. Since selecting position 3 is the best strategy for the first entrant, [3,1] is identified as PNE(P). In this procedure, the relationship established in Proposition 3 enables us to limit depth of the game tree (two entrants) and to restrict the search only to the solid branches. The size of the tree to be searched is reduced by more than 25% in this example.

Note in Fig. 3 that the profit of the first entrant is less than the second entrant. Ghosh and Buchanan

[5] The algorithm can determine the maximum number of entrants that can take any particular position in the attribute space, given that no other positions are occupied. Since an occupant at any other position would have non-negative demand, we can show that this maximum cannot be exceeded if other positions are occupied. Give an upper bound for the number of entrants at any position, enumeration entails listing all possible combinations of entrants and the positions they could take, up to the maximum. This is a large, but finite number of candidates for evaluation as potential stable sets for the Nash Equilibrium process.

Table 2
Occupancy–payoff table on the attribute space in Fig. 2

Occupancy vector (Z (1,2), Z (5,2), Z (3,5))	$\pi_{Z\ (1,2)}$	$\pi_{Z\ (5,2)}$	$\pi_{Z\ (3,5)}$
(1,1,0)	7.29	184.21	0
(1,0,1)	179.87	0	11.63
(0,1,1)	0	4.58	186.92
(2,0,0)	95.75	0	0
(0,2,0)	0	95.75	0
(0,0,2)	0	0	95.75

(1988) called this phenomenon (i.e., that the first mover is not better off than the follower) the "first-entry paradox." They discuss the first-entry paradox with a duopolistic-location model in a linear market, and address the relationship between the first-entry paradox and the non-existence of a Nash equilibrium for the simultaneous-entry game. Rhim, Ho, and Karmarkar (2003) generalized this result by dealing with an oligopolistic model on a network. The same result can be constructed for the product-positioning game. Before we present the result, we define a "simultaneous-entry game" (P_S). This game is different from the sequential-entry game P only in that firms decide on entry (and positioning) simultaneously in the first stage. Pricing games in the second stage of P_S and P are identical. Then, the result on the first-mover advantage in the product-positioning game is as follows (the proof appears in Appendix A).

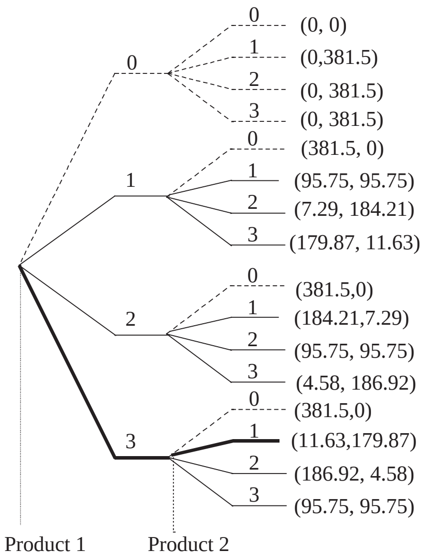


Fig. 3. Game tree for restricted backward induction.

Proposition 3. *In the sequential entry game P, first-mover advantage is ensured if PNE of P_S exists and PNE(P) $\subset$ PNE(P_S).*

In the product-positioning game, we deal only with the sequential-entry game partly because the simultaneous-game solution has never been shown to exist. Our sequential solution could be considered more robust if we could demonstrate that the first mover is not always in the advantaged position. Proposition 3 is demonstrated as follows: Nash equilibria of the simultaneous-positioning game for the problem set in Example 1 may be identified by a two-dimensional payoff matrix as shown in Fig. 4. However, this figure shows that a pure-strategy Nash equilibrium does not exist, and thus the first-mover advantage in the sequential-positioning game may not be guaranteed. Non-existence of a Nash equilibrium can happen when potential product positions and ideal points have a special pattern, giving entrants incentive to deviate from the current decision. In the example, if player 1 selects position 1, player 2 chooses position 2. Then player 1 will move to position 3, which will make player 2 move to position 1. Thus, the two players will hop around the positions endlessly, precluding an equilibrium for the simultaneous game. This phenomenon was called "dancing" by Teitz (1968). Labbé and Hakimi (1991) also observed this in the context of facility location. In our example, when dancing happens, being the first mover is less advantageous than being the follower.

If the simultaneous game, given this grid and these entrants, produces "dancing," then in the sequential-entry game with the same grid and entrants, the second mover has the advantage. This finding makes our analysis of the sequential-entry game more robust since the equilibrium for the sequential-entry game is still solvable (while the equilibrium for simultaneous-

		Player II 1	Player II 2	Player II 3
Player I	1	(95.75, 95.75)	(7.29, 184.21)	(179.87, 11.63)
	2	(184.21, 7.29)	(95.75, 95.75)	(4.58, 186.92)
	3	(11.63, 179.87)	(186.92, 4.58)	(95.75, 95.75)

Fig. 4. Two-dimensional payoff matrix of simultaneous-positioning game.

entry game is not), and first-mover advantages are not automatic.

3. Computation with example

In this section, we present an overview of a heuristic algorithm based on using genetic algorithms to converge on a stable set, and using an approach adapted from Selten (1975) for estimating the subgame-perfect Nash equilibrium. The details of some specific steps in the genetic algorithm are presented in Appendix B. Using this combined heuristic algorithm, we solve an example constructed from a real data set on a liquid detergent market.

The genetic-algorithm aids us by converging on a stable set. The stable set tells us which positions in the attribute space are occupied and how many products occupy each position. But it does not tell us which products fill these slots. Which products go into the occupied slots and the prices for those products result from the computation of subgame-perfect Nash equilibria. Obtaining subgame-perfect Nash equilibria with real data is very time consuming and sometimes intractable. [6] Enumeration is one possibility for identifying which products go where, if the size of the problem is small. However, if the attributes are measured by a fine scale, or if the dimensionality of attribute space is greater than two, enumeration may not be practical. In this case, we need to use search algorithms such as genetic algorithms. This approach agrees with the real decision-making process where firms figure out the industry structure with only bounded rationality and select the best-possible position based on it.

Rhim (1997) presents the implementation of the genetic algorithms for a competitive facility-location problem on a network addressed by Rhim et al. (2003). Since the discrete attribute space can be transformed to a network, the first step of the product-positioning problem is analogous to that of the facility-location problem. Thus, the algorithm developed for the facility-location problem is directly applicable to our problem. The general procedure of genetic algorithms is provided in Fig. 5.

[6] Identifying subgame-perfect Nash equilibria was proven to be a hard problem (NP-complete) by Gilboa and Zemel (1989).

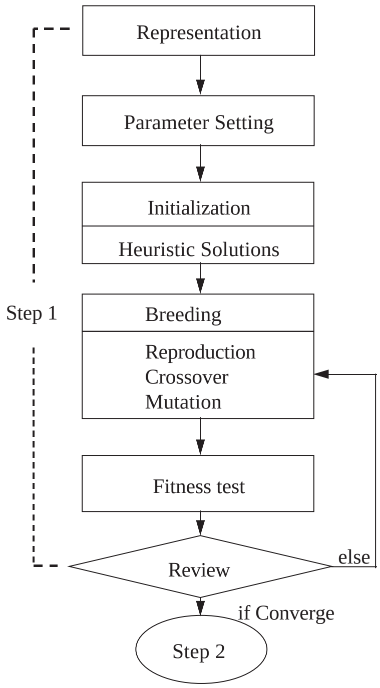


Fig. 5. Flow of genetic algorithms for Step 1 of (A1).

3.1. Representation

Deciding the encoding scheme of decision variables is critical to the performance of genetic algorithms. Since the goal of Step 1 is to obtain a stable set, the decision variable is the number of products at each point of attribute space (summarized by the occupancy vector Z), which may take on non-negative integer values. Thus, we follow a non-binary representation scheme, which we believe to be more intuitive and realistic than a binary occupancy vector. So the individuals being bred by this genetic algorithm are occupancy vectors. The chromosomes controlling the breeding specify how many products are located at each position in the attribute space. These chromosomes are evaluated for their fitness, reproduced, crossed, and mutated in accord with parameters and fitness functions discussed below.

3.2. Parameter setting

A genetic algorithm uses a set of parameters that determine the size of the population and the probability that each breeding operations is exercised.

Population size is regarded to be the most critical parameter, since improper population size leads to either premature convergence and/or ineffective search. We implemented a genetic algorithm with varying population size, as recommended by Arabas, Michalewicz, and Mulawka (1994). This method introduces the concept of "age" and "lifetime" for a chromosome. "Age" is defined as the number of generations since the birth of a chromosome, and "lifetime" is defined as maximum number of generations that a chromosome can survive. A chromosome with higher fitness can survive more generations than the one with lower fitness. Thus, the population size is more controlled by a natural-selection mechanism than if it is fixed. Most of the other parameters (e.g., reproduction, crossover, and mutation) for genetic algorithms are set by trial and error. We used the values of parameters Rhim (1997) tested for a competitive facility-location problem that was very similar to the present context.

3.3. Initialization

The initial population is generated by a probabilistic add/drop heuristic (see details in Appendix B).

3.4. Breeding

The initial population evolves through reproduction, crossover, and mutation operations. Reproduction involves selecting a set of occupancy vectors to mate based on the current population. Since the varying-population method is used, selections are influenced by an aging process. Lifetime in the aging process is determined by the fitness test described below. The crossover and mutation operations generate new occupancy vectors in the next generation of the reproduced population. Crossover is a binary operator combining two occupancy vectors, while mutation is a unitary operator providing diversity to populations. New crossover operators (geographical crossover and projection crossover) are implemented to utilize the spatial structure. Details are provided in Appendix B.

3.5. Fitness test

Evolution is directed by a fitness test. Genetic algorithms seek a balance between population diversity and selective pressure. While breeding operators generate diversity among the population, fitness guides the breeding process through the aging process. Occupancy vectors having higher fitness are likely to have longer lifetime. Unlike optimization problems, identifying stable sets is a yes-or-no type question rather than a more-or-less type one. Thus, we define fitness of an occupancy vector to stable sets applying the membership-function (or grade of membership) concept of fuzzy-set theory.[7]

3.6. Review

The convergence of the evolution process is tested by checking the size of the stable set. For example, if the size of the stable set does not increase for a certain number of cycles, the first step is stopped and the restricted backward induction explained in Example 1 is started in Step 2.

The program was developed using object-oriented-programming concepts (i.e., Visual C+). Compared with the second stage, computation of the first stage remains a difficult one. Rhim (1997) tested performance of the algorithm on the competitive facility-location problem.[8] Cases with five to seven demand–supply co-location nodes in a linear form are tested (Fig. 6). For each number of nodes, samples of size 10 are randomly generated. Each problem set is solved using both enumeration method and genetic algorithms and results are compared in Table 3. Percentages of stable sets identified by the GA and percentages of proper Pure Nash equilibria compared with complete sets found by enumeration are recorded. Stable-set hit-rate decreases as the number of nodes increases. The PNE hit-rate increases in this range, but we conjecture this rate will also decrease on average as the number of nodes increases.

[7] If X is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pairs: $A=\{(x, \mu_A(x))|x \in X\}$. $\mu_A(x)$ is called the membership function or grade of membership of x in A which maps X to the membership space M. When M contains only the two points 0 and 1, A is non-fuzzy and $\mu_A(x)$ is identical to the characteristic function of a non-fuzzy set. (Zimmermann, 1991).

[8] Results on the competitive facility-location problem can be directly used to understand the performance of our algorithm for the product-positioning problem, since the second-stage algorithms for facility-location and product-positioning models generate exact solutions.

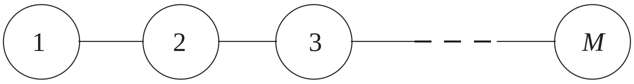


Fig. 6. Linear market case.

In the following example, we present the application of the model to a real problem using a set of data from a liquid detergent market.

Example 2. For the proposed model, we require a set of data containing market shares for each segment, prices, product-attribute data, and costs. Market share and price data are drawn from A. C. Nielsen Company scanner-panel data in Sioux Falls, South Dakota for the period of 1986–1987 (64 weeks). During this period, 12 major brands explain 83.8% of total demand. In these data, a single company owns several brands. For example, Procter & Gamble own Cheer, Tide, Era, Solo, and Bold. However, we assume that these brands compete with each other in the eyes of the consumers. Market share for each brand is generated week by week. Prices are averaged over stores every week. Product-attribute data for liquid detergents sold in 1986 are obtained from Consumer Reports (1987). Total demand is obtained from Wilkinson (1990) and Ainsworth (1995). Wilkinson (1990) estimated the 1989 detergent market at $3 billion and expected that the split between powder and liquids would remain 60% to 40%. Ainsworth (1995) reported that the U.S market for household cleaning products was basically flat with growing rate less than 1% per year. Thus, we estimate the liquid detergent market in 1988 as worth $1.2 billion (40% of $3 billion).

We segment the market according to individual-household purchase volume for the whole period. Cooper and Nakanishi (1988) show that market shares may be replaced by individual choice probabilities in case of homogeneous purchase frequencies and homogeneous choice probabilities. By differentiating heavy users and light users, aggregation problems can be minimized. Therefore, based on the purchase volume during the period, we divide the market into two equally populated segments: heavy and light users.[9] In our data, heavy users explain 86.7% of the total demand. Thus, the heavy half should get more attention than the 80/20-rule suggests. Considering 12 major brands' shares (83.8%), heavy users' share (86.7%), average retail price over products ($3.57), and the share of non-purchase option (19.3%), total demand of heavy and light users are 244.2 and 37.5 million units, respectively.

Table 3
Exactness of genetic algorithm

No. of nodes	SS identified	PNE obtained
5	85.58%	60.00%
6	80.82%	70.00%
7	75.29%	80.00%

In order to estimate demand parameters, we need to specify a distance metric. We postulate a weighted, squared Euclidean distance as in Example 1 (Cooper & Nakanishi, 1983). To introduce price elasticity into the model, we suppose $A_{0j} \equiv \exp[0] = 1$. Then, the market share is expressed as follows:

$$\mathrm{MS}^t_{x(i)j} = \frac{A^t_{x(i)j}}{1 + \sum_{k=1}^{N*} A^t_{x(k)j}}$$

$$A^t_{x(i)j} = \exp\left[-\sum_{h=1}^{\mathrm{Na}} \beta_{jh}\left(x(i)_h - m_{jh}\right)^2 - \gamma_j p^t_{x(i)}\right], \qquad (5)$$

where t is an index for the week, $t = 1, \ldots, T$

Extending the method by Cooper and Nakanishi (1983), market share can be transformed into the following linear equations:

$$\begin{aligned}\log\frac{\mathrm{MS}^t_{x(i_1)j}}{\mathrm{MS}^t_{x(i_2)j}} &= \sum_{h=1}^{\mathrm{Na}} \beta_{jh}\left[\left(x(i_2)_h^2 - x(i_1)_h^2\right) - 2m_{jh}\left(x(i_2)_h - x(i_1)_h\right)\right] + \gamma_j\left[p^t_{x(i_2)} - p^t_{x(i_1)}\right] \\ &= \sum_{h=1}^{\mathrm{Na}}\left[\beta_{jh1}\left(x(i_2)_h^2 - x(i_1)_h^2\right) + \beta_{jh2}\left(x(i_2)_h - x(i_1)_h\right)\right] + \gamma_j\left[p^t_{x(i_2)} - p^t_{x(i_1)}\right] \qquad (6)\end{aligned}$$

where $i_1 < i_2$, $\beta_{jh1} = \beta_{jh}$ and $\beta_{jh2} = -2\beta_{jh}\, m_{jh}$. Since the values of $\mathrm{MS}^t_{x(i)j}$, $x(i)_h$, and $p^t_{x(i)}$ are given, we can

[9] Although there are many theoretical approaches for market segmentation, volume segmentation is popular in practice (Cooper, 1993; Haley, 1995). The methods developed here can be used with any segmentation scheme.

estimate parameters β_{jh}, γ_j, and ideal points m_{jh} in two stages, using the following equation:

$$Y^t_{i_1 i_2 j} = \alpha_j + \sum_{h=1}^{\mathrm{Na}} \left[\beta_{jh1}\mathrm{ATSQ}_{i_1 i_2 h} + \beta_{jh2}\mathrm{ATTR}_{i_1 i_2 h}\right] + \gamma_j \mathrm{PR}^t_{i_1 i_2 j} + \varepsilon^t_{i_1 i_2 j}, \tag{7}$$

where

$$Y^t_{i_1 i_2 j} = \log \frac{\mathrm{MS}^t_{x(i_1)j}}{\mathrm{MS}^t_{x(i_2)j}},$$

$$\mathrm{ATSQ}_{i_1 i_2 h} = x(i_2)^2_h - x(i_1)^2_h,$$

$$\mathrm{ATTR}_{i_1 i_2 h} = x(i_2)_h - x(i_1)_h,$$

$$\mathrm{PR}^t_{i_1 i_2 j} = p^t_{x(i_2)} - p^t_{x(i_1)},$$

α_j: an intercept, ε^t_{i1i2j}: the error term.

Eq. (7) can be transformed to the following equations:

$$Y^t_{i_1 i_2 j} - \overline{Y}_{i_1 i_2 j} = \gamma_j\left(\mathrm{PR}^t_{i_1 i_2 j} - \overline{\mathrm{PR}}_{i_1 i_2 j}\right) + \hat{\varepsilon}^t_{i_1 i_2 j} \tag{8}$$

$$\frac{1}{T}\sum_{t=1}^{T}\left(Y^t_{i_1 i_2 j} - \gamma_j \mathrm{PR}^t_{i_1 i_2 j}\right) = \alpha_j + \sum_{h=1}^{\mathrm{Na}}\left[\beta_{jh1}\mathrm{ATSQ}_{i_1 i_2 h} + \beta_{jh2}\mathrm{ATTR}_{i_1 i_2 h}\right] + \varepsilon_{i_1 i_2 j} \tag{9}$$

For each segment, ordinary-least-squares (OLS) methods are applied.[10] At first, price sensitivity γ_js are estimated using Eq. (8), and then distance sensitivity and ideal-point parameters β_{jh1}, β_{jh2}s are obtained, using Eq. (9). In order to escape from possible collinearity problems associated with using both squared and linear terms $\mathrm{ATSQ}_{i_1 i_2 h}$ and $\mathrm{ATTR}_{i_1 i_2 h}$, we use deviation scores in estimating βs and restore the original attribute values when computing ideal points.

Consumer reports presented nine attributes concerning liquid detergents, excluding cost. Anti-redeposition is a property of detergents such that, once removed, dirt and stain do not resettle over the entire wash-load. Inoue (1996) showed Anti-redeposition to be highly correlated with another attribute, Whitening, so we only needed one of these two. The other measures reflected the detergents' ability to remove stains caused by dirt, makeup, spaghetti sauce, grape juice, grass, tea, ink, and motor oil. We formed a composite index, Effectiveness, reflecting the sum of the stain-removing capabilities. Factor analysis supported using two dimensions to capture the nine attributes (cf. Inoue, 1996). Data on these two attributes and summary of estimation results are provided in the following tables (Tables 4 and 5).

Heavy users appear more sensitive to price and Effectiveness than light users, and the light users are more sensitive to Anti-redeposition. The value of R^2 ranges from 0.09 to 0.11 for Eq. (8) and around 0.29 for Eq. (9). This implies that market share is explained more by product attributes than price. Since the values of β_{jh1}s are significant, we have ideal-point models rather than vector models. Since the values of β_{jh1}s are negative, the ideal positions of segments are actually anti-ideal points for both attributes. Attractiveness increases as products move away from the anti-ideal points on this discrete grid. The anti-ideal points of segments, obtained from Eq. (6), are (1.87, 3.00) for light users and (1.60, 3.00) for heavy users. These points should be located on the axis of Anti-redeposition, because deviation score for the Effectiveness is anchored around 3 and values of β_{jh2}s are zero. Positions of existing brands and anti-ideal points estimated are presented in Fig. 7.

Since costs are not generally open to academic researchers, we need to estimate them—a challenging exercise. Horsky and Nelson (1992) estimated variable costs using the equation derived from joint

Table 4
Two attributes of liquid detergents

No.	Brand name	Anti-redeposition	Effectiveness
1	All	3	4
2	Arm and Hammer	3	3
3	Bold	1	3
4	Cheer	4	4
5	Dynamo	3	4
6	Era	3	5
7	Fab	1	4
8	Purex	2	5
9	Solo	3	4
10	Tide	3	6
11	Wisk	3	3
12	Yes	3	5

[10] Since the tracking data in this illustration eliminate sampling errors, OLS estimates should be very similar to GLS estimate (cf., Cooper & Nakanishi, 1988, pp. 125–128).

Table 5
Summary of estimation results with deviation scores

Variable	Parameters	Light users (t-value)	Heavy users (t-value)
Price	γ_j	0.58 (9.13)[a]	0.72 (19.34)[a]
R^2		0.09	0.11
Intercept	α	–	−0.23 (−1.69)[b]
Anti-redeposition	β_{jh1}	−0.23 (−2.08)[a]	−0.21 (−1.84)[b]
	β_{jh2}	−0.38 (−2.85)[a]	−0.45 (−3.50)[a]
Effectiveness	β_{jh1}	−0.07 (−2.78)[a]	−0.12 (−3.40)[a]
	β_{jh2}	–	–
R^2		0.29	0.29

[a] 5% significance level.
[b] 10% significance level.

maximization of the player's profit functions for equilibrium. They assumed variable costs are expressed as a function of product attributes. Berry, Levinsohn, and Pakes (1995) present various empirical models and methods to obtain estimates of demand and cost parameters in oligopolistic markets. In their base model, variable cost is a log-linear function of product attributes and estimated by OLS.[11] Based on these models, we suppose that variable cost is a log-linear function of attributes and product-specific dummy variables as follows:

$$\ln c_{x(i)} = \lambda_0 + \sum_{h=1}^{2} \lambda_h x(i)_h + \sum_{j \in \{3,7,11\}} \delta_j \mathrm{DUM}_j + \varepsilon_i \tag{10}$$

where λ_hs and δ_j are the parameters to be estimated and ε_i is an error term. DUM_js are dummy variables for product-specific costs. Dummy variables are added only for the economy brand of each multi-product firm, which are Bold, Fab, Wisk. Thus, $\mathrm{DUM}_j = 1$ if $j = i$; otherwise 0. $c_{x(i)}$ is produced from Eq. (2), assuming the average price is in equilibrium. Average price is adjusted by subtracting average retail margin (27.5%), [12] (cf., Saporito, 1988) from the price. Again, parameters are estimated by OLS. Results are summarized in Table 6 and estimated variable costs are provided in Fig. 7.

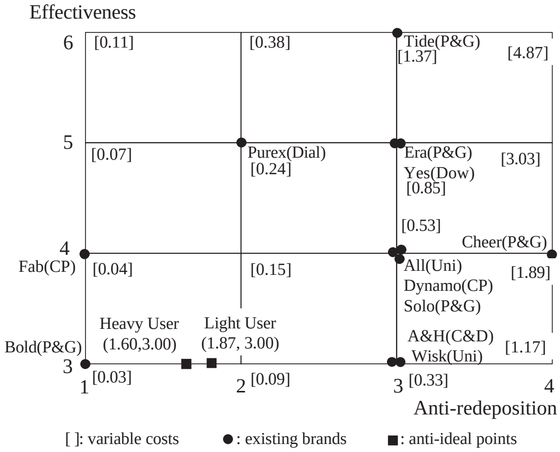


Fig. 7. Positions of products and markets.

Fixed costs at non-empty positions are estimated based on net-profit-to-sales ratios obtained from 10K report (1987). Since 10K reports do not provide brand-level data, the estimation based on firm-level data generates only rough results. Thus, we assume uniform fixed costs over product positions. Horsky and Nelson (1992) also estimated a uniform fixed cost from other reports on automobile industry. In our case, weighted average of fixed costs by brand's market share produces \$16.68 million for the fixed costs at each of the product positions.

A reasonable upper bound of price is obtained by subtracting retail margin from the transaction prices. The upper bound is set to \$3.91, covering 99% of the transaction prices. Changing this upper bound to the maximum observed prices (\$6.18) does not alter the equilibrium result. The empirical data contain 12 brands that already exist in the market and we suppose that they react only in the second stage. Then we compute the number of new entrants and their positions. The results appear in Table 7.

Under the given cost structure, our model indicates that 19 new entrants can enter the market and survive.[13] They take the extreme position on Effectiveness, but save costs by taking the position on Anti-redisposition of Bold and Fab. Pursuing both attributes at the same time is too expensive, considering the increase in marginal cost. Because of the moderate

[11] More recent paper by Besanko, Dube, and Gupta (2002) assumes that variable cost is a function of raw materials and jointly estimates the function with demand. However, we follow the approach used in the first two papers for simplicity.

[12] This value was produced by averaging margins of hypermarkets, wholesale clubs, discount stores, and supermarkets.

[13] The number of survivors at this location is a function of fixed costs as explained at the end of this section.

Table 6
Regression result for cost equation

Variable	Parameters	Estimates (t-value)
Intercept	λ_0	− 6.34 (− 3.96)[a]
Anti-redeposition	λ_1	1.27 (3.54)[a]
Effectiveness	λ_2	0.47 (2.32)[b]
Dummy for Bold	δ_3	3.60 (3.63)[a]
Dummy for Fab	δ_7	3.50 (3.85)[a]
Dummy for Wisk	δ_{11}	1.63 (2.76)[a]
R^2		0.77

[a] 5% significance level.
[b] 10% significance level.

marginal cost, they set reasonable price ($1.60), and beat the existing brands with better performance.

Therefore, the best strategy for introducing a new brand in this market is to take a position of focused functional quality and reasonable price. But we have to ask, “How realistic is this result?” This result indicates that a market that is estimated currently to be profitable for all existing brands (see Table 8 for current shares and estimated profits) turns it into one that is profitable only for the new entrants. Each new entrant grabs a 4.11 share—around twice the share of incumbent brands after entry. As shown in Table 9, even if Tide gives up Anti-redeposition and opportunistically repositions itself to match the benefits of the new entrants, Tide and the new entrants survive and are profitable, while all other brands are not profitable.

Our analysis and simulations describe a market that Tide leads by providing a substantial tangible benefit, supported, of course, by the advertising and distribution clout of Procter & Gamble. To succeed, new entrants in this market must match Tide on Effectiveness—no easy task since no current brand achieves this. If this parity can be achieved, the new entrants become profitable by offering less Anti-redeposition (the attribute less desired by the heavy users), but at a lower price, which appeals to the more price-sensitive heavy users. This represents a severe threat to the established leader in the category, and indeed to all incumbent brands. As indicated above, Tide could profitably reposition and drop price, but this does not seem like a desirable alternative for the long-time category leader.

Given this analysis, what should the incumbent brands do? First, the market leader needs to determine if patents or trade secrets protect its position on Effectiveness. Can a new entrant actually achieve a “6” on this scale? If not, they are secure. But if Tide is imitable on Effectiveness, the next step would be to compare our cost estimates to the ones the manufacturers hold privately. If the costs associated with the (1,6) position are higher than we estimate, the incumbents may not face as severe a threat as we have identified. If, on the other hand, the costs are justified, more extreme measures may be required. Perhaps launching flanker brands backed by the

Table 7
Results of new entrants under passive reaction from existing brands

Position (anti-red., effect.)	No. of existing brands	No. of new entrants	Profit per brand (mil. $)	Price ($)	Market share per brand (%)
(1,3)	1	0	− 10	1.50	1.60
(1,4)	1	0	− 9.3	1.51	2.78
(1,6)	0	19	0.63	1.60	4.11
(2,5)	1	0	− 8.22	1.70	2.05
(3,3)	2	0	− 9.38	1.80	1.77
(3,4)	3	0	− 9.57	1.99	1.72
(3,5)	2	0	− 8.70	2.31	1.93
(3,6)	1	0	− 6.89	2.83	2.37
(4,4)	1	0	− 10.57	3.36	1.48

Table 8
Market share, profit, and price of existing brands before entry of new brands

Position (anti-red., effect.)	No. of existing brands	Profit per brand (mil. $)	Price ($)	Market share per brand (%)
(1,3)	1	15.77	1.59	7.40
(1,4)	1	19.33	1.61	8.16
(2,5)	1	24.69	1.82	9.31
(3,3)	2	19.00	1.89	8.11
(3,4)	3	18.10	2.09	7.92
(3,5)	2	22.37	2.42	8.83
(3,6)	1	37.31	2.97	10.64
(4,4)	1	13.13	3.43	6.84

Table 9
When the position of tide is moved to (1,6)

Position (anti-red., effect.)	No. of existing brands	No. of new entrants	Profit per brand (mil. $)	Price ($)	Market share per brand (%)
(1,3)	1	0	− 10.19	1.50	1.56
(1,4)	1	0	− 9.48	1.51	1.74
(1,6)	1	19	0.17	1.60	4.01
(2,5)	1	0	− 8.44	1.70	2.00
(3,3)	2	0	− 9.57	1.79	1.72
(3,4)	3	0	− 9.76	1.99	1.68
(3,5)	2	0	− 8.91	2.31	1.88
(4,4)	1	0	− 7.15	2.83	2.31

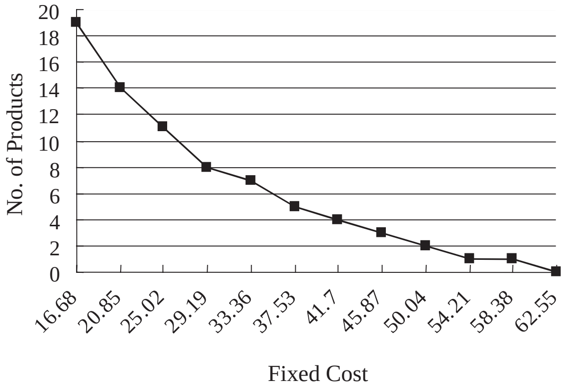


Fig. 8. Sensitivity analysis on the number of products in the market.

distribution and advertising clout of the market leader could discourage further entry. Here we can see the usefulness of extending the modeling framework to include product line decisions (i.e., multiple products from a single firm that have coordinated strategies). Even without such model extensions, this analysis has identified a threat that needs to be considered in strategic brand planning.

Finally, we report a sensitivity analysis using our model. How many new products can be introduced into the market? Number of products in the market is determined endogenously in our free-entry model. With other parameters fixed, increasing fixed costs works as an entry barrier. Fig. 8 shows that the number of new products decreases as fixed costs increase from the original value ($16.68 million). For each fixed-cost level, the best position for the new product does not change. This is an important result that is quite different from Example 1 where dancing and first-mover disadvantage occur. The stability that this finding implies makes our finding more practically important. We conjecture that the stability of the best position comes from monotone increasing variable costs, uniform fixed costs, and absence of limited positioning.

4. Conclusion

In this paper, we presented a model for the new-product-positioning problem with pricing decisions. Competition is addressed by means of a game-theoretic approach. Game-theoretic models are usually based on strong assumptions to obtain analytic solutions. However, we formulate the problem with an emphasis on application by combining the essential requirements for practical applicability (i.e., multiple segments, multiple positioning attributes, and free and endogenous entry) and by making more reasonable assumptions where needed.

In spite of our practical perspective, more research is needed in this field. If the data cannot satisfy the sufficient conditions for the existence and uniqueness, the model fails to obtain equilibrium solutions. In this case, different modeling approaches such as deterministic models or other market share functions may be appropriate. Another major assumption is that each firm can launch only one product. This assumption is critical, and may not hold in some markets. In such cases, other approaches such as nested logit models (Anderson & De Palma, 1992) need to be considered. In the empirical part, we have added a number of rough assumptions, especially concerning the estimated cost structure. Horsky and Nelson (1992) recommend obtaining cost data from design engineers. This is a sound approach, not available to us. We also know that advertising, co-op advertising, distribution, merchandising, and other marketing actions impact the attractiveness of brands. Note these are not design attributes that would increase the dimensionality of the attribute space. These are policy variables that would enter our analysis just as price does. While not including such instruments limits somewhat the applicability of our methods, attempting to expand the methods to include other marketing actions increases the complexity of the analytical model enormously, and is beyond the scope of the present effort. Therefore, we leave all these generalizations to future research and improvement.

Another limitation concerns the simultaneous setting of prices in the second stage of the game. The structure of the game can be thought of as a traditional two-stage model. All brands position their brands (sequentially), and then all simultaneously choose prices. The problem proposed, however, is the launch of a new brand into an existing market. Presumably, in such a case, some brands will have already selected positions and prices in a previous round of the game. Posing the two-stage game in the traditional manner limits our findings somewhat. We hope that future efforts can remove that limitation. A related limitation concerns our treatment of brands as independent competitors (i.e., a firm launches only one product), despite the fact that four brands are part of Procter & Gamble and two brands belong to Unilever. In the empirical literature, brand choice models typically do not recognize the possible dependencies that this corporate brand ownership might imply (e.g., by choosing a nested-logit framework over a conditional-logit model), but this should be recognized as a limitation of the current work as well as a limitation of much of the empirical literature.

We could use this framework to analyze brands entering new markets (i.e., markets with no incumbents). To do so, however, would require a very different approach to demand estimation than the real market number we employ in the current application. While conjoint analysis could possibly provide estimates of demand for new products in new markets, this too is beyond the scope of the present effort.

We have shown how to extend previous work in new-product entry and optimal pricing in an existing multisegmented market-providing a normative, analytical framework for free and endogenous entry, as well as empirical methods to apply this framework to real markets. We hope these efforts facilitate further work that bridges between normative models and empirical markets.

Acknowledgements

The authors wish to thank Andrew Ainslie, Bart Bronnenberg, and Ely Dahan for their helpful suggestions and comments on this article. We also thank the editor for his suggestions and insights, and three referees for their useful perspectives.

Appendix A

Proof of Lemma 1. Showing that price is bounded above.

$$\lim_{p_{x(i)}\to\infty}\pi_i = \lim_{p_i\to\infty}\left[\sum_j D_j \frac{\left(p_{x(i)}-c_{x(i)}\right)}{\mathrm{MS}^{-1}_{x(i)j}} - f_{x(i)}\right]$$

$$= \sum_j D_j \lim_{p_{x(i)}\to\infty}\frac{\left(p_{x(i)}-c_{x(i)}\right)}{\mathrm{MS}^{-1}_{x(i)j}} - f_{x(i)}$$

$(p_{x(i)}-c_{x(i)})\to\infty$ and $\mathrm{MS}_{x(i)j}{}^{-1}\to\infty$ as $p_{x(i)}\to\infty$. By L'Hospital's rule,

$$\lim_{p_{x(i)}\to\infty}\frac{\left(p_{x(i)}-c_{x(i)}\right)}{\mathrm{MS}^{-1}_{x(i)j}} = \lim_{p_{x(i)}\to\infty}\frac{1}{\frac{\gamma_j \mathrm{MS}_{x(i)j}\left(1-\mathrm{MS}_{x(i)j}\right)}{\mathrm{MS}^2_{x(i)j}}}$$

$$= \lim_{p_{x(i)}\to\infty}\frac{\mathrm{MS}_{x(i)j}}{\gamma_j\left(1-\mathrm{MS}_{x(i)j}\right)} = 0$$

and

$$\lim_{p_{x(i)}\to\infty}\pi_i = -f_{x(i)} < 0$$

Therefore, the firm has no incentive to increase price infinitely, which proves the lemma. □

Proof of Lemma 2. Showing the existence of pure-strategy Nash equilibria.

Since $p_{x(i)}\in[c_{x(i)}, rp_{x(i)}]$, the π_is are continuous, and quasi-concave. We need to show that π_is are, in fact, concave, which will be the case if their second derivative is positive.

$$\frac{\partial^2\pi_i}{\partial p^2_{x(i)}} = \sum_j D_j\left[2\frac{\partial \mathrm{MS}_{x(i)j}}{\partial p_{x(i)}} + \left(p_{x(i)}-c_i\right)\frac{\partial^2 \mathrm{MS}_{x(i)j}}{\partial p^2_{x(i)}}\right]$$

$$= \sum_j D_j\gamma_j \mathrm{MS}_{x(i)j}\left(1-\mathrm{MS}_{x(i)j}\right)\left[-2+\left(p_{x(i)} - c_{x(i)}\right)\gamma_j\left(1-2\mathrm{MS}_{x(i)j}\right)\right]$$

Since $D_j\ \gamma_j\ \mathrm{MS}_{x(i)j}(1-\mathrm{MS}_{x(i)j})>0$, we need to show that $-2+(p_{x(i)}-c_{x(i)})\ \gamma_j\ (1-2\mathrm{MS}_{x(i)j})<0$ for all j.

$$-2+\left(p_{x(i)}-c_{x(i)}\right)\gamma_j\left(1-2\mathrm{MS}_{x(i)j}\right) < -2+\left(p_{x(i)} - c_{x(i)}\right)\gamma_j \leqslant -2+\left(p_{x(i)}-c_{x(i)}\right)\left(\frac{2}{rp_{x(i)}-c_{x(i)}}\right)\leq 0$$

Therefore, π_is are concave and there exist pure-strategy Nash equilibria. □

Proof of Proposition 1. Showing the uniqueness of pure-strategy Nash equilibria.

Since the profit function π_i is concave, if $\gamma_j\leq 2/(rp_{x(i)}-c_{x(i)})$, there exists a unique best-reply price $p^{br}_{x(i)}(p_{-x(i)})$, where $p_{-x(i)}=p_{x(1)}\ \ldots\ p_{x(i-1)},\ p_{x(i+1)}\ \ldots\ p_{x(N)})$. A sufficient condition for a unique equilibrium is that the best-reply function is a contraction (Friedman, 1986): that is,

$$\sum_{k\neq i}\left|\partial p^{br}_{x(i)}/\partial p_{x(k)}\right| < 1 \text{ for all } i. \tag{11}$$

From $\partial\pi_i/\partial p_{x(i)}=0$ and the implicit-function theorem,

$$\frac{\partial^2\pi_i}{\partial p_{x(i)}\partial p_{x(k)}}+\frac{\partial^2\pi_i}{\partial p_{x(i)}^2}\cdot\frac{\partial p_{x(i)}}{\partial p_{x(k)}}=0,\text{or}$$

$$\frac{\partial p^{br}_{x(i)}}{\partial p_{x(k)}}=-\left(\frac{\partial^2\pi_i/\partial p_{x(i)}\partial p_{x(k)}}{\partial^2\pi_i/\partial^2_{x(i)}}\right)$$

If $\gamma_j\leq 1/(rp_{x(i)}-c_{x(i)})$, then $\partial^2\pi_i/\partial p_{x(i)}^2<0$, and

$$\begin{aligned}\frac{\partial^2\pi_i}{\partial p_{x(i)}\partial p_{x(k)}}&=\sum_j D_j\gamma_j\mathrm{MS}_{x(i)j}\mathrm{MS}_{x(k)j}\big[1+\big(p_{x(i)}\\&\quad-c_{x(i)}\big)\gamma_j\big(2\mathrm{MS}_{x(i)j}-1\big)\big]>\sum_j D_j\gamma_j\\&\quad\times\mathrm{MS}_{x(i)j}\mathrm{MS}_{x(k)j}\big[1-\big(p_{x(i)}-c_{x(i)}\big)\gamma_j\big]\\&\geq\sum_j D_j\gamma_j\mathrm{MS}_{x(i)j}\mathrm{MS}_{x(k)j}\Bigg[1-\big(rp_{x(i)}\\&\quad-c_{x(i)}\big)\cdot\frac{1}{\big(rp_{x(i)}-c_{x(i)}\big)}\Bigg]=0\end{aligned}$$

Thus, Eq. (11) is equivalent to

$$\sum_{k\neq i}\frac{\big(\partial^2\pi_i/\partial p_{x(i)}\partial p_{x(k)}\big)}{\big(-\partial^2\pi_i/\partial p^2_{x(i)}\big)}<1\ \text{ for all } i,\text{or}$$

$$\sum_{k\neq 1}\frac{\partial^2\pi_i}{\partial p_{x(i)}\partial p_{x(k)}}<-\frac{\partial^2\pi_i}{\partial p^2_{x(i)}}\ \text{ for all } i \qquad (12)$$

Since

$$\begin{aligned}\sum_{k\neq 1}\frac{\partial^2\pi_i}{\partial p_{x(i)}\partial p_{x(k)}}&=\sum_j D_j\gamma_j\mathrm{MS}_{x(i)j}\big[1+\big(p_{x(i)}-c_{x(i)}\big)\\&\quad\times\gamma_j\big(2MS_{x(i)j}-1\big)\big]\Bigg[\sum_{k\neq i}\mathrm{MS}_{x(k)j}\Bigg]\end{aligned}$$

and

$$\begin{aligned}-\frac{\partial^2\pi_i}{\partial p^2_{x(i)}}&=\sum_j D_j\gamma_j\mathrm{MS}_{ij}\big[2+\big(p_{x(i)}-c_{x(i)}\big)\gamma_j\\&\quad\times\big(2\mathrm{MS}_{x(i)j}-1\big)\big]\big(1-\mathrm{MS}_{x(i)j}\big),\end{aligned}$$

Eq. (12) holds, which proves the uniqueness. □

Proof of Proposition 2. Showing (i) a proof that the pure-strategy Nash equilibrium is a subset of the stable set, and (ii) that the stable set equals the pure-strategy Nash equilibrium for the "pre-assigned" game.

(i) SS $\supset$ PNE(P).

Suppose that there exists X* such that $X^*\notin$ SS. Then, it induces two cases such that X* is not viable, or viable but not stable in the survival.

Case 1: X* is not viable. It implies that there exists a product i such that $x^*_{x(i)}\neq 0$, and

$$\pi_i\left(x^*_{x(i)},x^*_{-x(i)}\right)<0.$$

Since

$$\pi_i\left(0,x^*_{-x(i)}\right)=0>\pi_i\left(x^*_{x(i)},x^*_{-x(i)}\right),$$

$X^*=(x^*_{x(i)},\ x^*_{-x(i)})$ is not PNE(P).

Case 2: X* is viable, but not stable in the survival. It implies that there exists a firm i such that $x^*_{x(i)}=0$, and

$$\pi_i\left(x_{x(i)},x^*_{-x(i)}\right)\geq\pi_i\left(x^*_{x(i)},x^*_{-x(i)}\right)$$

Thus, $X^*=(x^*_{x(i)},\ x^*_{-x(i)})$ is not PNE(P).

Therefore, in both cases, X* cannot be PNE(P).

(ii) SS=PNE(P_{01}).

(a) SS $\subset$ PNE(P_{01}): For the proof, we define new variables y_r such that $y_i=1$ if the firm enters the market at a pre-assigned position or $y_i=0$. Let $Y=(y_1,\ldots,\ y_N)$. Suppose that there exists Y* such that $Y^*\notin$ PNE(P_{01}). Then there exists a firm i such that

$$\pi_r\left(y_i,y^*_{-i}\right)>\pi_r\left(y^*_i,y^*_{-i}\right) \qquad (13)$$

Since y^*_i can have only two values, we examine the following two cases.

Case 1: $y^*_i=0$, $y_i=1$. Then Eq. (13) implies that $Y^*=(y^*_i,\ y^*_{-i})$ is not stable in the sense of survival.

Case 2: $y^*_i=1$, $y_i=0$. From Eq. (13), $\pi_r(0,\ y^*_{-i})=0>\pi_r(1,\ y^*_{-i})$. Thus, $Y^*=(y^*_i,\ y^*_{-i})$ is not viable.

Therefore, in both case, $Y^*=(y_i^*,\ y^*_{-i})$ does not belong to SS.

(b) SS $\supset$ PNE(P_{01}): The proof is similar to that of (i). □

Proof of Proposition 3. Showing sufficient conditions for the first-mover advantage.

Let $X=(x(1),\ldots x(N^*))$ be a PNE of P. Suppose that there exists a firm r such that $\pi_r<\pi_{r+1}$. For a given $x(1)-x(r-1)$, the game tree for backward induction is reduced to a two-person game (firm r and $r+1$), since they can foresee equilibria of the subgames starting from firm $(r+1)$'s decision node. Suppose that firm r can relocate its facility after firm $(r+1)$'s location decision, but does not want to. Then firm r should have selected firm $(r+1)$'s site in its initial decision. Therefore, the firm must relocate its facility, which is contradictory to $\mathrm{PNE}(P)\subset \mathrm{PNE}(P_S)$. □

Algorithm for the second stage. Determining the price equilibrium for products after entrance.

Step 0: $p^0_{x(i)}\leftarrow c_{x(i)}$ for all i.

Step 1: $k\leftarrow k+1$; $\max_{p^k_{x(i)}} \pi\left(p^k_{x(i)}|p^{k-1}_{x(i)}\right)$ for all i, using Eq. (2).

Step 2: If $|p^k_{x(i)}-p^{k-1}_{x(i)}|<\varepsilon$ for all i, stop; otherwise, return to Step 1.

Appendix B

This appendix provides further detail on the heuristic algorithm used in the endogenous determination of number of new entrants, location of new entrants, and the empirical estimation of the pure Nash equilibrium. A genetic algorithm is used in the first step of the algorithm to generate and ultimately converge on a stable set. The second step finds the subgame-perfect Nash equilibrium on the generated stable set (see Fig. 5).

B.1. Representation

The individuals being bred by this genetic algorithm are occupancy vectors. The chromosomes controlling the breeding specify how many products are located at each position in the attribute space. These chromosomes are evaluated for their fitness, reproduced, crossed, and mutated in accord with parameters and fitness functions discussed previously in Section 3, except for the concept of lifetime discussed below.

B.2. Parameter setting

The lifetime of a chromosome (introduced by Arabas et al., 1994) should increase when it is more fit compared to other chromosomes:

$$\mathrm{lt}(i) = \min\left(\mathrm{MinLT} + \frac{(\mathrm{MaxLT}-\mathrm{MinLT})}{2} \bullet \frac{\mathrm{fit}(i)}{\mathrm{avgfit}}, \mathrm{MaxLT}\right) \quad (14)$$

where lt(i): lifetime of chromosome i, $i=1,\ldots,$ Nc; MinLT: minimum lifetime of chromosomes; MaxLT: maximum lifetime of chromosomes; fit(i): fitness of chromosome i, $i=1,\ldots,$ Nc; avgfit: average fitness of chromosomes.

B.3. Initialization

The initial population of chromosomes specifying occupancy vectors is generated by a probabilistic add/drop heuristic. This heuristic is a variant of conventional add/drop heuristics for facility-location problems. In conventional add/drop heuristics, the site that a facility is to be added to or dropped from is selected by a deterministic criterion (c.f., Francis, McGinnis, & White, 1992). In our heuristic, the position where the product is to be added is selected from a probability distribution generated from some deterministic criteria. The details are as follows:

B.3.1. Add/drop heuristic for initialization

Step 0: $Z=(0,\ldots 0)$.

Step 1: (1) For each position j, obtain profit π_j when only one product exists at position j. (2) Produce probability $p_j=\frac{\pi_j^c}{\sum_k \pi_k^c}$, where c is a weight coefficient.

Step 2: If $p_j=0$ for all j, stop; otherwise go to Step 3.

Step 3: (1) Select a position s according to a probability distribution $(p_1,\ldots,p_L)$. (2) If $\pi(z_s+1)<0$, $p_s=0$ and go to Step 2; else $z_s\leftarrow z_s+1$.

Step 4: If some positions are not viable, drop the products one by one from less profitable positions until all positions are viable:

Go to Step 2 of the add/drop heuristic.

To maintain diversity within the initial population, we produce several selection probabilities using various coefficients: $c=0$ for random selection; $c=1$ for greedy selection; $c=1/2$ for mixed selection.

B.4. Breeding

In this stage, new populations of occupancy vectors are generated from a parent population. For breeding, the following operators are used:

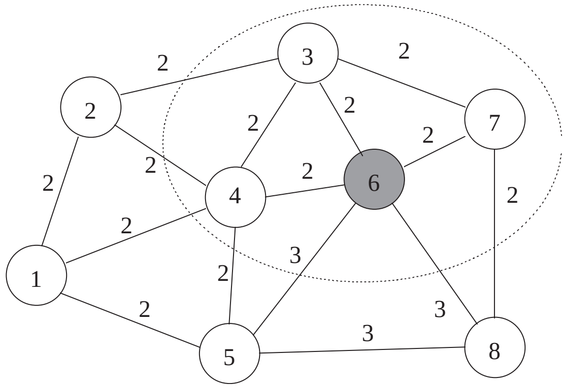
(1) Reproduction:

In reproduction, we randomly select a set of chromosomes (i.e., an occupancy vector) from the current population using a reproduction-rate parameter (See Table 11). In standard genetic algorithms, reproduction is used as a selection mechanism for the next generation such that the chromosomes with high fitness have a better chance to be selected for the next generation than those with low fitness. In our algorithm, selections are made naturally through an aging process that incorporates fitness, described in Eq. (14). Thus, reproduction is performed only to prepare a set for other operations such as crossover and mutation by random selection.

(2) Crossovers:

Crossover is a binary operator combining two chromosomes. The basic idea of crossover is that the best solutions can be constructed from the best partial solutions of previous trials (Goldberg, 1989). This operation is performed on the reproduced set of chromosomes with a certain probability (or crossover rate). The values for the crossover probabilities are listed in Table 11.

Three kinds of crossovers are considered: generic, geographic, and projection. A generic crossover is an operator that is independent of problem specifications. As a generic crossover, we use a two-point crossover. Suppose that $L=5$, and Z^1, Z^2 are selected chromosomes for generic crossover as in Fig. 9. The separators are placed at random. Then new chromosomes produced by the generic crossovers are $Z^{1'}$, $Z^{2'}$.



BEFORE CROSSOVER AFTER CROSSOVER

$Z^1 = (1,2,\mathbf{1},\mathbf{2},1,\mathbf{2},\mathbf{1},2) \quad \Rightarrow \quad Z^{1'} = (1,2,\mathbf{2},\mathbf{1},1,\mathbf{1},\mathbf{2},2)$

$Z^2 = (2,1,\mathbf{2},\mathbf{1},2,\mathbf{1},\mathbf{2},1) \qquad\quad Z^{2'} = (2,1,\mathbf{1},\mathbf{2},2,\mathbf{2},\mathbf{1},1)$

Fig. 10. Geographic crossover.

The geographical crossover is a problem-specific operator, introduced by Karmarkar and Saxena (1993) for the facility-location problem on a network. Geographical crossover has attempted to preserve facilities around a site; that is, it randomly selects a site and maintains the facilities within a randomly determined distance. For example, suppose that in Fig. 10, node 6 is selected as the site to perform the geographical crossover and the distance is determined to be 2. Then, nodes 3, 4, and 7 including node 6 are preserved in the chromosomes Z^1, Z^2, and therefore the resulting chromosomes are like $Z^{1'}$ and $Z^{2'}$. This network-based crossover can be applied to an attribute space. Each product position in an attribute space is regarded as a node in a network.

Finally, we present a new crossover, the projection crossover. This crossover is similar to the generic crossover except that the mating chromosome is a zero vector. Thus, this operation mimics a drop heuristic. An example is provided in Fig. 11.

BEFORE CROSSOVER AFTER CROSSOVER

$Z^1 = (1,2|\mathbf{1},\mathbf{2}|1) \quad \Rightarrow \quad Z^{1'} = (1,2,\mathbf{2},\mathbf{1},1)$

$Z^2 = (2,1|\mathbf{2},\mathbf{1}|2) \qquad\quad Z^{2'} = (2,1,\mathbf{1},\mathbf{2},2)$

Fig. 9. Generic crossover.

BEFORE CROSSOVER		AFTER CROSSOVER
$Z^1 = (1,2\|\mathbf{1},\mathbf{2}\|1)$	$\Rightarrow$	$Z^{1'} = (1,2,\mathbf{0},\mathbf{0},1)$
$Z^2 = (2,1\|\mathbf{2},\mathbf{1}\|2)$		$Z^{2'} = (0,0,\mathbf{2},\mathbf{1},0)$

Fig. 11. Projection crossover.

The crossover selected from the three is determined by certain probabilities. The probabilities are found by trial and error, as are the other parameters.

(3) Mutation:

Mutation is a unary operator that provides diversity to populations. We design two mutation operators following the add/drop heuristic as follows (these operators are selected at random).

Mutation 1:

Step 1: Select a chromosome and a bit within the chromosome according to a mutation rate.

Step 2: Increase the number of products at the selected position by one.

Step 3: Drop products in other positions if they are not viable.

Step 4: Stop if addition of one more products to the selected bit returns negative profit to the product of the bit; otherwise, go to step 2 of Mutation 1.

Mutation 2:

Step 1: Select a chromosome and a bit within the chromosome according to the mutation rate.

Step 2: Increase the number of products up to the maximum assuming that other positions are empty.

Step 3: Drop non-viable facilities one by one if they exist.

B.5. Fitness test

Since elements of a set can be described by several characteristics, membership needs to be represented by several objectives. Thus, we utilize a multi-objective function for membership. One objective is survival and the other objective is the number of firms:

(1) Survival

Stability is defined by two conditions: viability and survival. Firms first require viability when they enter the industry. The survival condition is satisfied after the last entry of firms. Thus, we always maintain viability condition as a constraint during the evolution process and use the survival condition as one objective of the membership function. The first objective function of chromosome i, $f_1(i)$ is defined as follows:

$$f_1(i) = \frac{\sum_{i=1}^{L} \frac{z_j}{z_j^{\max}\left(z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_L\right)}}{L} \tag{15}$$

where L: number of potential positions; z_j: number of firms at position j; $z_j^{\max}$ (.): maximum number of firms that position j can accommodate when $z_1, \ldots, z_{j-1}, z_{j+1}, \ldots, z_L$ are given.

In Eq. (15), the objective function is defined as the average closeness of a chromosome to the maximum number of firms that sites can accommodate. The survival objective is to maximize this function. However, calculating $z_j^{\max}$ (.) for every occupancy vector is time consuming. In order to save the computing time, we use z_j^{cur} as a proxy for $z_j^{\max}$, where $z_j^{\mathrm{cur}} = \max \{z_j(i) |$ occupancy vector i belongs to the stable set$\}$ for all positions j.

(2) Number of Firms

The survival objective has a tendency to generate chromosomes that contain a large number of firms by giving high scores to these chromosomes. However, there may exist chromosomes that contain small numbers of firms. Thus, the second objective is maximizing the inverse of the number of firms divided by the average number of firms. The objective function of chromosome i, $f_2(i)$ is defined as follows:

$$f_2(i) = \frac{\sum_{k=1}^{Nc} N(k)/\mathrm{Nc}}{N(i)} \tag{16}$$

where $N(i)$ is the number of firms of chromosome i.

(3) Overall fitness

From the two objectives, we define the overall fitness of a chromosome i, fit(i) as follows:

$$\mathrm{fit}(i) = \frac{f(i)}{\sum_{k=1}^{\mathrm{Nc}} f(k)} \tag{17}$$

and

$$f(i) = \alpha f_1(i) + (1-\alpha) f_2(i) \tag{18}$$

where $0 \le \alpha \le 1$.

B.6. Review

In our overall heuristic algorithm, three cycles exist: Cycle I is a single generation; Cycle II consists

Table 10
A list of parameter values tested

Parameters	Values
Initial population size	20, 30, 40, 50
Crossover rate	0.1, 0.2,. . ., 0.9
Mutation rate	0.2,. . ., 0.9
Reproduction rate	0.1, 0.2,. . ., 0.5
Removal rate of a member of the stable set	0, 0.1, 0.2,. . ., 0.5
Probability for three crossovers (generic, geographic, projection)	(1,0,0), (0,1,0), (0,0,1), (1/3,1/3,1/3), (2/5,1/5,1/5), (1/5,2/5,1/5), (1/5,1/5,2/5)
Weight for the first objective of fitness function α	0, 0.3, 0.5, 0.7, 1, (0.1,0.4), (0.3,0.7), (0.6,0.9)

of generating populations, transferring identified stable chromosomes from the population to a pool for the stable set, and removing part of stable chromosomes in the population;[14] Cycle III consists of several Cycle IIs and the second step in which we obtain an equilibrium from the identified stable set. The stable set is sorted in lexicographical order in order to prevent the existence of multiple copies of the same chromosome. We remove part of chromosomes at the end of Cycle II with certain probability (removal rate) to increase the diversity of population and prevent premature convergence of the evolution process.

We review the heuristic system to test the convergence of the evolution process. Reviews can be made at the end of Cycle II or Cycle III. If the system is reviewed at the end of Cycle II, the convergence is tested by checking the size of the stable set—the normal convergence test for ending the genetic algorithm. For example, if the size of the stable set does not increase for a certain number of cycles, we stop the first step and proceed to obtaining the equilibrium in the second step. On the other hand, if the system is reviewed at the end of Cycle III, we observe the obtained equilibrium, and stop if the obtained equilibrium does not change for a certain number of cycles. In this application, we reviewed at the end of Cycle II because of the substantial computing time at the second step.

[14] The genetic algorithm is involved in Cycles I and II.

B.7. Second step

In this step, we find a Nash equilibrium using the stable set identified by the genetic algorithm. Suppose we identified a stable set (SS) in the first step. The stable set (list of occupancy vectors satisfying viability and survival conditions) limits the search on the game tree. (See Fig. 3.) The game tree is searched by backtracking (depth-first) approach. Searching is implemented recursively (i.e., the subroutine calls itself). Suppose SS_A is a subset of SS that is maintained to trace the possibility of branching. Initially, $SS=SS_A$. For instance, for kth product, position j can be branched only when SS_A has at least a single occupancy vector such that $z_j>0$. In this case, after branching into position j, z_j's are reduced by 1, and now decision is for $k+1th$ product. If z_j has a negative value, the corresponding occupancy vector is removed from SS_A until search returns to kth product. Let $N_{\max}$ be the maximum number of products obtained from SS. Using SS_A and $N_{\max}$, we describe the sketch of the algorithm as follows:

B.7.1. Algorithm for restricted backward induction

Subroutine TreeSearch (suppose we are at the branch of position j of the kth product)

If (size of $SS_A=1$), then assign positions to $k+1$ to $N_{\max}$th products in the decreasing order of profits.
Else if (size of $SS_A>1$) and ($k=N_{\max}$), then compare the profit of leaf j with those of other branched leaves of kth product.
Else if (size of $SS_A>1$) and ($k<N_{\max}$), then (for all possible position l's of SS_A)
{$z_l \leftarrow z_l - 1$ for all occupancy vectors of SS_A;
If ($z_l<0$) then remove the occupancy vector from SS_A;
Call TreeSearch(node l, $k+1$th product);

Table 11
Selected parameters

Initial population size	50
Crossover rate	0.7
Mutation rate	0.5
Reproduction rate	0.4
Removal rate	0.1
Crossover Distribution	(1/5,1,5,2/5)
Weight for the first objective of fitness function α	(0.3,0.7)

Update optimal choice of kth product;
Recover z_l and SS_A;}

B.8. Parameter set

The parameters are selected by trial and error. The selection criteria we use are the average size of the stable set over a time period AS(T), and the size of stable set at the end of the run, $S(T)$. Both measures are normalized by the size of the true stable set as follows:

$$AS(T) = \frac{\sum_{t=1}^{T} (\mathrm{Size}_t/\mathrm{Size}^*)}{T} \tag{19}$$

$$S(T) = \frac{\mathrm{Size}_T}{\mathrm{Size}^*} \tag{20}$$

where Size_t: the size of the stable set obtained at time t, $t=1,\ldots T$; Size*: the size of true stable set.

Cycle I (i.e., a single generation) is used as a time unit for this measure. Size* is obtained by enumeration. In order to consider both average and final performance, we select the first and second best parameters in AS(T) as candidates, and then select the better parameter in $S(T)$ between those candidates.

As an experiment, we use the homogeneous-cost, linear-market model presented in Fig. 6 (facility-location problem). Total run time T is set to 150. A list of parameter values tested is provided in Table 10.

For some parameters, we include extreme probabilities such as 0 or 1. The zero probability for the removal rate is to test whether this operator is essential or not. The extreme distribution for the three crossovers is to test which operator works best. The extreme values for the weights in the fitness function are to determine whether the multiobjective approach is necessary in our problem. We also include the strategy that the weights of fitness function vary according to the uniform distribution during the run. For example, (0.3, 0.7) represents the case that the weight for the first objective α follows the uniform distribution U[0.3, 0.7]. The selected parameters are summarized in Table 11.

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