The Persistence of Marketing Effects on Sales

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Are marketing efforts able to affect long-term trends in sales or other performance measures? Answering this question is essential for the creation of marketing strategies that deliver a sustainable competitive advantage. This paper introduces persistence modeling to derive long-term marketing effectiveness from time-series observations on sales and marketing expenditures. First, we use unit-root tests to determine whether sales are stable or evolving (trending) over time. If they are evolving, we examine how strong this evolution is (univariate persistence) and to what extent it can be related to marketing activity (multivariate persistence). An empirical example on sales and media spending for a chain of home-improvement stores reveals that some, but not all, advertising has strong trend-setting effects on sales. We argue that traditional modeling approaches would not pick up these effects and, therefore, seriously underestimate the long-term effectiveness of advertising. The paper concludes with an agenda for future empirical research on long-run marketing effectiveness.
(**Econometric Models; Marketing Mix; Advertising and Media Research**)

1. Introduction

In a recent article on declining sports-car sales in the United States, *The Economist* writes "Many dealers worry that the drop in sports-car sales is not just the result of recession, but could herald a permanent decline in their popularity." (August 29, 1992, p. 63). The article goes on to describe pricing and advertising strategies employed by Chevrolet Corvette and its competitors in an attempt to revive product sales. It illustrates an often recurring and important dual question for marketing executives and researchers: are observed upturns or downturns in sales of a temporary or permanent nature, and, in the latter case, how do marketing strategies affect such long-run sales movements?

Answering these questions is essential for the development of marketing strategies that deliver a sustainable competitive advantage. Yet, our understanding of long-run marketing phenomena is far from complete. In a literature that has traditionally devoted more attention to the short-term impact of marketing strategies, four approaches to study long-run marketing phenomena have emerged: the dynamic effects of marketing expenditures have been measured with distributed-lag and/or transfer-function models, the persistence of first-mover advantages has been assessed through cross-sectional designs, Markov transition matrices have been extrapolated to derive equilibrium market shares, and concepts such as brand loyalty, brand equity and brand franchise attempt to develop a distinct metric to capture a brand's long-run performance.

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We argue that the shortcomings of these approaches necessitate a new approach, which we call *persistence modeling*. Marketing is defined to have a *persistent* (or permanent) effect if a portion of its observed short-run impact is carried forward and sets a new trend in performance. On the other hand, marketing has a *temporary* effect if, after a number of periods, the brand returns to its pre-expenditure performance level. To establish persistence, two questions must be answered. First, we must determine if sales are stable or evolving. Stable sales fluctuate temporarily around a fixed mean, while evolving sales have no fixed mean and can deviate permanently from previous levels. We will describe methods for establishing stability or evolution empirically. Second, if we find sales evolution, we examine whether it can be traced to marketing efforts. For that, we perform persistence calculations to determine what fraction of the short-term marketing effects is carried forward and affects the long-run performance.

Persistence modeling introduces a new way of looking at the over-time effectiveness of marketing activities and differs in two important ways from traditional market-response models. First, rather than focusing on the individual carry-over or purchase-reinforcement coefficients, we derive the *total* long-run impact of a marketing action in that an initial outlay receives credit for all subsequent effects that follow from it. We argue that several channels of influence should be taken into account when computing the total long-run impact of a marketing activity: instantaneous, carry-over, purchase-reinforcement and feedback effects, as well as the effects resulting from firm-specific decision rules or competitive reactions. Second, rather than looking at the absolute price or marketing-spending levels, we consider the differential impact of temporary deviations from the brand's expected marketing support.

We use persistence modeling to compare the short- and long-run effectiveness of different advertising media used by a large home-improvement chain. Our results show that temporary advertising increases or shocks can have a permanent effect on a brand's performance, which is empirical evidence for the hysteresis effect described by Little (1979). To the best of our knowledge, our study is the first to quantify hysteresis using ETS (econometric and time-series) models. Our analysis also illustrates that *long-run marketing impact emerges from a complex dynamic interaction of a variety of short-term forces*. Finally, our example demonstrates that different media can have different long-run effects.

The remainder of the paper is organized as follows. In Section 2, we review some approaches that are currently used to study long-run marketing phenomena. Section 3 introduces the persistence concept, and the empirical application is discussed in Section 4. Section 5 summarizes our main findings and indicates some areas for future research.

2. Criteria and Methods for Long-run Modeling

Quantifying long-run marketing effects is essential for developing empirical generalizations and for designing fact-based marketing strategies aimed at the creation of sustainable competitive advantage. Therefore, measures of long-run marketing effectiveness should be able to assess the long-run performance impact of specific marketing actions. Second, some markets are characterized by stable competitor performance over time, while others show up- or downward evolutions. Since marketing effectiveness may well be different in stable as opposed to trending or evolving environments (see, e.g., Gatignon, Weitz and Bansal 1990), models aimed at quantifying the long-run impact of marketing activities should be able to detect these differences, if they exist. Finally, while long-run performance should be the ultimate goal of management, only short-term performance is readily observed. We therefore state as our third criterion that long-run response models should provide a necessary link between the two.

To put our modeling framework in perspective, we assess to what extent currently used approaches to long-run modeling satisfy these criteria.[1]

2.1. The Over-time Distribution of Marketing Effects

Recognizing that marketing expenditures may not have their full impact in the period in which they are incurred, response models have often used distributed-lag structures to quantify the impact of marketing activities. A typical approach is to assume that a fixed fraction (λ, $0 \leq \lambda < 1$) of the effects in one period is retained in the next period. Clarke (1976) used this geometrically decaying pattern to conclude that 90% of the measurable effects of advertising on a mature product's sales are consumed within 3 to 9 months. This conclusion does not address, however, whether marketing can have a persistent effect. The Koyck model implies that sales return to their pre-expenditure level, since $\lim_{n \to \infty} \lambda^n = 0$, for $0 \leq \lambda < 1$. This behavior does not conform with the more complex sales patterns one often observes, such as the presence of prolonged up- or downward trends. Such trends are typically modeled by including deterministic time factors in the response equation, implying that sales will steadily rise or decline, regardless of the level of marketing support. Not only may this result in implausible forecasts, these models also have little appeal in terms of management's control over the long-run evolution of its brands.

2.2. Measuring Long-run Equilibria Using Cross-sectional Designs

Cross-sectional designs have been used to assess long-run equilibria (see, e.g., Urban et al. 1986, Bass, Cattin and Wittink 1978). The long run is then represented by the static solution that arises after all short-term adjustments have taken place. These designs often offer the advantage of large sample sizes. On the other hand, the parameter estimates may be affected by heterogeneity bias and may not reflect the actual long-run relationship for any brand or industry. In terms of our second criterion, interaction terms allow us to capture the different effectiveness of marketing in growing markets (Gatignon et al. 1990). Finally, with cross-sectional designs one cannot consider events that occur over time, nor can one infer how the short-term dynamics translate into long-run equilibria.[2]

2.3. The Extrapolation of Transition Matrices

Markov transition matrices can be extrapolated to derive long-run or equilibrium market shares. Such extrapolation provides a link between a brand's short- and long-run position. The transition matrices, however, are often based on just two purchase occasions, and the long-run inference may be sensitive to the selected snap-shot of the market. Also, the initial Markov models did not include decision variables, making it difficult to derive the contribution of specific marketing actions.

Some authors (see, e.g., Horsky 1976, Givon and Horsky 1990) have linked the transition probabilities to decision variables and have estimated the model parameters on a sequence of market-share observations. By linking the transition probabilities to the brands' marketing support, the notion of a long-run equilibrium share becomes less clearly defined (since it will depend on the future time path of the covariates), and

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[1] Even though many of these issues have been considered most frequently when using aggregate data, they are also relevant with individual-choice data. For example, the lag-structure of price and promotion was considered in Lattin and Bucklin (1989), while Abe (1991) modeled the build-up effect of advertising exposures. Also, the brand-specific intercepts in logit models have been interpreted as a measure of long-term strength, reflecting the use of a distinct metric when making long-run inferences (see Section 2.4).

[2] Kalyanaram and Urban (1992) adopt a pooled cross-sectional/time-series design to overcome this shortcoming of earlier studies on the persistence of first-mover advantages. In their model specification, however, temporary advertising increases do not affect the asymptotic market-share level of any of the contenders, and have only a temporary impact on the speed of convergence towards that long-run market-share level.

attention has shifted towards the interpretation of the individual (short-run) parameters (Givon and Horsky 1990) and their use in the derivation of optimal advertising levels (Horsky 1976). Finally, we are unaware of Markov models that distinguish between stable and evolving environments.

2.4. *The Use of a Distinct Long-run Metric*

Recently, new metrics have been designed to measure the long-run impact of marketing investments, such as consumer franchise (Blattberg and Neslin 1989) and brand equity (Simon and Sullivan 1993). In terms of our criteria for long-run measurement, these metrics are rather limited. First, they often cannot be used to measure the long-run impact of a firm's marketing decisions. Simon and Sullivan (1993), for example, perform an event study to assess the impact of major (discrete) marketing events on a financial-market based equity measure. Not only does this limit considerably the scope of activities for which the long-run impact can be quantified, it is also unclear whether the brand's equity will eventually return to its pre-event value. Hence, one cannot use this measure to assess whether an event has a temporary or permanent effect. Finally, it is not clear how these new metrics relate to short-run sales movements. For example, as the annual sales of Porsche in the U.S. have plummeted from 25,000 to 4,000 in the last three years, does this imply that its equity is eroding?

In conclusion, while existing approaches to measuring long-term marketing effectiveness offer promising features, they do not satisfy all criteria we set forward.

3. Persistence Modeling: A New Approach to Long-run Inference

Our framework consists of three major steps. First, we assess the presence of evolution versus stability in sales (or market share) by investigating its behavior over time. If a brand's performance fluctuates around a fixed mean level (i.e., stability), no long-run effects can be inferred from the data, since performance always returns to its pre-expenditure mean. If sales are not mean-reverting, an evolutionary or long-run component is present, and marketing *can* (*but need not*) cause a permanent deviation from previous sales levels. Unit-root tests are introduced in Section 3.1 to empirically distinguish both scenarios. The presence of a unit root implies that a portion of a sudden change in sales (a shock) persists through time and affects its long-run behavior. Univariate persistence measures assess the magnitude of this retained portion and determine how much an estimate of the brand's long-run performance should be updated when its current performance is lower than expected (Section 3.2). Univariate persistence gives a first indication of the relative importance of the long-run performance fluctuations but does not consider the source of the initial shock: in updating the long-run sales forecast, no distinction is made between a 10% sales increase caused by additional advertising, a temporary price reduction or an economic expansion. Multivariate persistence measures make that distinction, as discussed in Section 3.3.

3.1. *Are Sales Performance and Marketing Support Stable or Evolving?*

The important distinction between stability and evolution is formalized through the *unit-root* concept. Consider for simplicity the case where a brand's sales over time[3] (S_t) are described by a first-order autoregressive process:

$$(1 - \phi L)S_t = c + u_t, \tag{1}$$

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[3] We focus on the stable/evolving nature of sales performance, since this determines whether long-run marketing effects are possible. Evolution in marketing spending is neither a necessary nor a sufficient condition for long-run marketing effectiveness but determines what modeling strategy should be used to derive multivariate persistence estimates. Hence, unit-root tests should also be applied to the marketing time series.

where ϕ is an autoregressive parameter, L the lag operator (i.e., $L^k S_t = S_{t-k}$), u_t a series of zero-mean, constant-variance (σ_u^2) and uncorrelated random shocks, and c a constant. Applying successive backward substitutions allows us to write equation (1) as

$$S_t = [c/(1 - \phi)] + u_t + \phi u_{t-1} + \phi^2 u_{t-2} + \cdots, \tag{2}$$

in which the present value of S_t is explained as a weighted sum of random shocks. Depending on the value of ϕ, three scenarios can be distinguished. When $|\phi| < 1$, the impact of past shocks diminishes and eventually becomes negligible. Hence, each shock has only a temporary impact. In this case, the series has a fixed mean $c/(1 - \phi)$ and a finite variance $\sigma_u^2/(1 - \phi^2)$. Such a series is called stable. When $|\phi| = 1$, however, (2) becomes:

$$S_t = (c + c + \cdots) + u_t + u_{t-1} + \cdots, \tag{3}$$

implying that each random shock has a permanent effect on the brand's sales. In this case, no fixed mean is observed, and the variance increases with time. Sales do not revert to a historical level but instead wander freely in one direction or another, i.e., they *evolve*. Distinguishing between both situations involves checking whether the autoregressive polynomial $(1 - \phi L)$ in equation (1) has a root on the unit circle. One could also consider the case where $|\phi| > 1$, i.e., where past shocks become more and more important. However, situations where the past becomes ever more important are unrealistic in marketing, and we will therefore focus our attention on the first two cases.

The previous discussion used the first-order autoregressive model to introduce the concepts of stability, evolution and unit roots. The findings easily can be generalized to the more complex autoregressive moving-average process $\Phi(L)S_t = \Theta(L)u_t$. Indeed, the stable/evolving character of a series is completely determined by whether some of the roots of the autoregressive polynomial $\Phi(L) = (1 - \phi_1 L - \cdots - \phi_p L^p)$ are lying on the unit circle.

Numerous tests have been developed to distinguish stable from evolving patterns. One popular test, due to Dickey and Fuller (1979), is based on the following test equation:

$$(1 - L)S_t = \Delta S_t = a_0 + bS_{t-1} + a_1 \Delta S_{t-1} + \cdots + a_m \Delta S_{t-m} + u_t. \tag{4}$$

The t-statistic of b is compared with the critical values in Fuller (1976), and the unit-root null hypothesis is rejected if the obtained value is smaller than the critical value. Indeed, substituting $b = 0$ in (4) introduces a random-walk component in the model, whereas $-1 < b < 0$ implies a mean-reverting process. The $m\Delta S_{t-j}$ terms reflect temporary sales fluctuations, and are added to make u_t white noise. Because of these additional terms, one often refers to this test as the "augmented" Dickey-Fuller (ADF) test. An important issue in applying the ADF test is the choice of m. Setting m too high results in a less powerful test, while a value that is too small may fail to make the u_t-series white noise and bias the test statistics. Following Perron (1990), we use conventional significance tests on the a_i to determine the cut-off point.

3.2. Univariate Persistence or "How Important are the Long-run Components?"

Unit-root tests were introduced to distinguish stable from evolving markets. The current section focuses on the *quantitative importance* of the long-run components, which will give us a first indication on how effective marketing can be in the long run. Indeed, if the long-run sales fluctuations are very small, most marketing effects still will be temporary.

The presence of a unit root implies that a portion of a shock in sales will persist through time and affect its long-run behavior. The magnitude of this portion determines how much our long-run sales forecast should be changed when the current performance is

lower than expected.[4] In the absence of a unit root, sales return to their pre-shock mean level, and the long-run forecast is not affected by a lower-than-expected current performance. For a pure random walk, the best forecast at any point in time is its current value. Hence, a one-unit sales decrease today translates into a one-unit reduction of the long-run forecast. This is also shown in equation (3), which gave the infinite-shock representation of a random-walk process. It is clear that a unit shock in $(t - k)$ has a unit impact on all future values of S_t. In contrast, for an ARIMA $(0, 1, 1)$ model with $\theta_1 = 0.6$, equation (3) becomes

$$S_t = S_{t-1} + (u_t - 0.6u_{t-1}) = u_t + 0.4u_{t-1} + 0.4u_{t-2} + 0.4u_{t-3} + \cdots. \qquad (5)$$

In this case, only 40% of an initial shock keeps influencing the brand's future sales levels, and an unexpected $100,000 sales decrease in the current period would lead to a $40,000 reduction in the long-run forecast. For still other values of the autoregressive and/or moving-average parameters, the magnitude of the retained portion may be even smaller.

Campbell and Mankiw (1987) developed a simple procedure to derive a series' univariate persistence as the sum of the moving-average coefficients of the *first-differenced* series. Consider the following univariate ARIMA specification

$$\Phi(L)\Delta S_t = \Theta(L)u_t. \qquad (6)$$

The infinite-shock representation of ΔS_t is given by

$$\Delta S_t = [\Phi(L)]^{-1}\Theta(L)u_t = A(L)u_t = (1 + a_1 L + a_2 L^2 + \cdots)u_t. \qquad (7)$$

The impact of a unit shock in period $t - k$ on the sales *growth* in t is a_k (i.e., the partial derivative of ΔS_t with respect to u_{t-k}). Its impact on the sales *level* in t is $1 + a_1 + \cdots + a_k$, as is easily seen by taking the partial derivative of S_t with respect to u_{t-k} in equation (8)

$$S_t = (1 - L)^{-1}(u_t + a_1 u_{t-1} + \cdots)$$

$$= \sum_{-\infty}^{t} u_i + a_1 \sum_{-\infty}^{t-1} u_i + \cdots + a_k \sum_{-\infty}^{t-k} u_i + \cdots \qquad (8)$$

Thus, the long-run impact on S_t is given by the sum of the moving-average coefficients in (7), and is often denoted as $A(1)$. Since $A(1)$ equals $\Theta(1)/\Phi(1)$, estimates of $A(1)$ can be obtained by fitting ARMA models to ΔS_t, and taking the ratio of the sum of the moving-average coefficients $(1 - \theta_1 - \cdots - \theta_q)$ to the sum of the autoregressive coefficients $(1 - \phi_1 - \cdots - \phi_p)$.

Univariate persistence measures what proportion of *any* (i.e. unspecified) shock will affect sales permanently and can be used to measure the long-run impact of isolated events, such as negative product news or short-lived advertising campaigns. Traditionally, these phenomena have been studied with intervention analyses (Leone 1987). These require a *post*-event history, while persistence models are based on the properties of the *pre*-event history. Persistence models have to assume that the event does not change the process that generates sales, while intervention models investigate (ex post) the need to alter the underlying model. If the necessary data are available, which only happens after a sufficiently long period of time, intervention analysis will be more powerful. However, since persistence estimates do not need post-event data, they may be of more practical use to a decision-maker.

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[4] The error terms (or shocks) in a univariate ARIMA-model can be interpreted as deviations from the series' expected level (Hanssens 1982). The information set used in this expectation formation, however, is restricted to the series *own* past history. Extensions that incorporate information from other variables are discussed in Section 3.3.

3.3. Multivariate Persistence

Multivariate persistence derives the long-run impact of an unexpected change in a control variable. We first discuss its two main distinctive features: the quantification of a marketing decision's *total* impact and the *unexpected* nature of the considered outlay. Next, we discuss the mechanics involved in the derivation of multivariate persistence estimates.

The Total Effect

The marketing literature has identified six channels through which a marketing action can influence a brand's performance: 1) contemporaneous, 2) carry-over, 3) purchase-reinforcement, 4) feedback effects, 5) firm-specific decision rules, and 6) competitive reactions. In quantifying the total long-run impact of a marketing action, all channels of influence should be accounted for. In what follows, we present a brief motivation for each of these effects. For expository purposes, we focus on the advertising-sales relationship.

Contemporaneous effects. Consensus exists in the marketing field that advertising often has a considerable immediate impact. For example, Leone and Schultz (1980) call the positive elasticity of selective advertising one of marketing's first empirical generalizations.

Carry-over effects. Numerous studies have argued that the effect of advertising in one period may be carried over, at least partially, into future periods (see, e.g., Givon and Horsky 1990). Consumers are supposed to remember past advertising messages and create "goodwill" towards the brand that only gradually deteriorates because of forgetting.

Purchase reinforcement. Givon and Horsky (1990) argue that the dynamic impact of advertising on sales can also work indirectly through purchase reinforcements: a given outlay may create a new customer who will not only make an initial purchase but also repurchase in the future. Using a similar logic, Horsky and Simon (1983) assume that advertising gives innovators an incentive to try the product after which an imitation effect takes over, creating a larger customer base and higher future sales. According to Bass and Clarke (1972) and Hanssens et al. (1990), current advertising should receive credit for these subsequent sales.

Feedback effects. Bass (1969) warned that advertising spending may be influenced by current and past sales, and should not be treated as exogenous. This is certainly the case when percentage-of-sales budgeting rules are applied. To illustrate the importance of feedback effects in the derivation of an expenditure's total impact, consider the following chain reaction initiated by a one-period advertising increase: increased advertising in period $t \Rightarrow$ increased sales in $t \Rightarrow$ increased advertising in $t + 1 \Rightarrow$ increased sales in $t + 1 \Rightarrow \cdots$. Credit should be given to the initial advertising increase for the subsequent sales increases since without it, none of these effects would have occurred. Obviously, when assessing the profit implications, both the additional revenues and expenses should be taken into account.

Firm-specific decision rules. Traditional single-equation models treat advertising as exogenous and do not model the dependence of current on previous expenditure levels. Empirical evidence contradicts this "independence" assumption: published time-series models often find significant autoregressive components in a firm's spending pattern (Hanssens 1980). Here again, a chain reaction may occur that affects the total long-run impact.

Competitive reactions. Competitive activities may change advertising's effectiveness drastically. For example, even though the instantaneous sales response may be positive, its long-run effect could be zero because of competitive reactions. We refer to Leeflang and Wittink (1992) and Metwally (1978) for a detailed discussion on this self-canceling effect.

Modeling Unexpected or Shock Movements

The second distinctive feature of persistence calculations is their focus on tracing the over-time impact of *unexpected* movements (shocks), as opposed to more traditional market-response models that consider *absolute* spending or price levels.

A first, important step in any policy analysis is the formulation of a baseline forecast against which policy changes can be evaluated. A logical choice is *a no-change scenario* in which all historically observed spending and reaction patterns are assumed to persist in the future (Litterman 1984). Within this framework, one-step-ahead sales and advertising forecasts (i.e., $\hat{S}_{t+1}$ and $\hat{A}_{t+1}$) can be interpreted as the performance and expenditure levels that are expected on the basis of the available information up to period t (Darnell and Evans 1990; Hanssens 1982). Deviations from the one-step ahead forecasts reflect unexpected shocks, whose *differential* impact can be traced over time.

Support for looking at aggregate marketing shocks is found in Helmer (1976), Hanssens (1982), Kleinbaum (1988) and Simon (1982). Helmer and Hanssens use univariate ARIMA models to derive expected support levels, and include their residuals instead of the actual levels in a market-response model, while Kleinbaum uses the residuals of VAR models to characterize competitive reaction patterns in the American truck market. Finally, Simon (1982) uses adaptation-level theory to support the notion that the same advertising level can generate different responses depending on the magnitude of the "anchor value" against which current spending is compared.

Multivariate Persistence Estimates

We use a vector-autoregressive (VAR) model to derive multivariate persistence estimates, because it easily captures multiple channels of influence and does not require the imposition of a priori structural restrictions. For ease of exposition, and without loss of generality, we consider a bivariate model between advertising and sales. If both series are stable, the VAR model can be written as

$$\begin{bmatrix} S_t \\ \text{ADV}_t \end{bmatrix} = \begin{bmatrix} \pi_{11}^1 & \pi_{12}^1 \\ \pi_{21}^1 & \pi_{22}^1 \end{bmatrix} \begin{bmatrix} S_{t-1} \\ \text{ADV}_{t-1} \end{bmatrix} + \cdots + \begin{bmatrix} \pi_{11}^I & \pi_{12}^I \\ \pi_{21}^I & \pi_{22}^I \end{bmatrix} \begin{bmatrix} S_{t-I} \\ \text{ADV}_{t-I} \end{bmatrix} + \begin{bmatrix} u_{S,t} \\ u_{\text{ADV},t} \end{bmatrix}, \quad (9)$$

where I is the order of the model, which may be determined using Akaike's Information Criterion, and where $\vec{u}_t = [u_{S,t}, u_{\text{ADV},t}]'$ is a white-noise vector. All elements in $\vec{X}_t = [S_t, \text{ADV}_t]'$ are related to all elements in $\vec{X}_{t-i}$ ($i = 1, \ldots, I$), making VAR models very useful for describing the lagged structure in the data. Contemporaneous effects cannot be captured directly, but information on such effects is contained in the covariance matrix of the residuals (Σ). This matrix can detect significant effects but cannot establish their direction.

To analyze the impact of marketing shocks over time, it is useful to introduce the mathematically equivalent (infinite-order) vector-moving-average (VMA) representation:

$$\begin{bmatrix} S_t \\ \text{ADV}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_{S,t} \\ u_{\text{ADV},t} \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix} \begin{bmatrix} u_{S,t-1} \\ u_{\text{ADV},t-1} \end{bmatrix}$$
$$+ \begin{bmatrix} a_{11}^2 & a_{12}^2 \\ a_{21}^2 & a_{22}^2 \end{bmatrix} \begin{bmatrix} u_{S,t-2} \\ u_{\text{ADV},t-2} \end{bmatrix} + \cdots \quad (10)$$

a_{12}^k gives the impact on S_t of a one-unit advertising shock that happened k periods ago. When dealing with stable sales series, these effects eventually wear out, and the brand's performance returns to its preshock mean level. A sequence of successive a_{ij} is called an impulse-response function and can be derived by direct estimation of a finite-order VMA model or by simulating the impact of a shock in an easier-to-estimate VAR model. The latter procedure is illustrated in Appendix A for a first-order VAR model. Impulse-

response functions reflect the complex interactions of all included channels of influence, and provide a complete description of the system's dynamic structure. When depicted graphically, they communicate more effectively than the common listings of individual parameter estimates.

When dealing with evolving variables, $\vec{X}$ is replaced by $\Delta\vec{X} = [\Delta S, \Delta ADV]'$ in equations (9–10), in which case a_{ij}^k gives the impact of a unit shock on the sales *growth* k periods later. A simulation similar to the one illustrated in Appendix A can be used to trace the over-time impact of shocks on both ΔS_t and S_t. The first difference of an evolving variable is stationary, and the corresponding impulse-response functions converge to zero. However, the response functions tracing the impact on an evolving variable can converge to a non-zero level, and this level corresponds to the multivariate extension of Campbell and Mankiw's $A(1)$ measure (see, e.g., Evans 1989). For example, when dealing with evolving variables, equation (10) becomes

$$\begin{bmatrix} \Delta S_t \\ \Delta ADV_t \end{bmatrix} = \begin{bmatrix} a_{11}^0 & a_{12}^0 \\ a_{21}^0 & a_{22}^0 \end{bmatrix}\begin{bmatrix} u_{S,t} \\ u_{ADV,t} \end{bmatrix} + \begin{bmatrix} a_{11}^1 & a_{12}^1 \\ a_{21}^1 & a_{22}^1 \end{bmatrix}\begin{bmatrix} u_{S,t-1} \\ u_{ADV,t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} a_{11}^2 & a_{12}^2 \\ a_{21}^2 & a_{22}^2 \end{bmatrix}\begin{bmatrix} u_{S,t-2} \\ u_{ADV,t-2} \end{bmatrix} + \cdots, \quad (11)$$

where $a_{11}^0 = a_{22}^0 = 1$ and $a_{21}^0 = a_{12}^0 = 0$. A straightforward generalization of $A(1)$ suggests $(0 + a_{12}^1 + a_{12}^2 + \cdots)$ as a measure of the long-run sales impact of advertising shocks. Indeed, following a similar logic as in the univariate case, a_{12}^k measures the impact on the sales *growth* k periods later, while $\Sigma_k(a_{12}^k)$ gives the long-run impact on the sales *level*. However, advertising shocks can have an influence directly through the a_{12}^k, and indirectly through their correlation with $u_{S,t}$. The proposed measure captures all lagged effects, but omits the instantaneous effects since $a_{12}^0 = 0$. Hence, it does not reflect the total impact of an incremental advertising outlay. Put differently, when advertising has an instantaneous effect on sales (as reflected in the correlation between $u_{S,t}$ and $u_{ADV,t}$), one should not consider the long-run impact of a change in $u_{ADV,t}$ alone.

For this reason, persistence calculations are often performed within a transformed VAR model where the error terms have a diagonal covariance matrix. Using a Cholesky decomposition, Σ can be written as $\Sigma = T^{-1}D(T^{-1})'$, where D is a diagonal matrix, and T an upper triangular matrix with unit diagonal elements. When dealing with evolving variables, we replace $\vec{X}$ by $\Delta\vec{X}$ in equation (9), which, after premuitiplication by T becomes

$$\Delta S_t = \gamma\Delta ADV_t + \alpha(L)\Delta S_{t-1} + \beta(L)\Delta ADV_{t-1} + e_{S,t}$$

$$\Delta ADV_t = \delta(L)\Delta S_{t-1} + \eta(L)\Delta ADV_{t-1} + e_{ADV,t}. \quad (12)$$

In this equation, $(e_{S,t}, e_{ADV,t})' = T(u_{S,t}, u_{ADV,t})'$. Hence, $e_{ADV,t} = u_{ADV,t}$ and $\text{cov}(e_{S,t}, e_{ADV,t}) = 0$. The first property implies that a shock in $e_{ADV,t}$ corresponds to a shock in the original formulation, which avoids the interpretational problems that would arise when the new advertising shock is a linear combination of $u_{S,t}$ and $u_{ADV,t}$. The second property eliminates the problem of working with correlated errors, and ensures the efficiency of OLS.

Equation (12) corresponds to a Wold-recursive form in which ADV_t has been assigned causal priority. By ordering advertising first, any contemporaneous correlation between sales and advertising is attributed to the advertising shocks. In other words, we assume that a contemporaneous effect of sales on advertising can be precluded on logical grounds. When advertising is ordered logically prior, $e_{S,t}$ reflects that portion of an unexpected sales increase that is not correlated with, or cannot be attributed to, advertising fluctuations.

For a two-variable case, the long-run sales impact of a given shock can easily be derived analytically, as illustrated in Appendix B. When more than two variables are involved, an analytical expression becomes cumbersome to derive. One can, however, derive the entire impulse-response function and see at what level it stabilizes. Indeed, $g(1)$ and $h(1)$ in Appendix B correspond to $S_{t+\infty}$ in (12), obtained by simulating the over-time impact of, respectively, $(e_{S,t}, e_{\text{ADV},t})$ equal to $(0, 1)$ and $(e_{S,t}, e_{\text{ADV},t})$ equal to $(1, 0)$.

Nevertheless, in some situations we may lack prior knowledge about the nature of the contemporaneous relationships, and as the level of aggregation becomes more coarse, bidirectional relationships may become more likely. In those instances, a number of different strategies could be adopted. First, one could use different causal orderings and assess the sensitivity of the resulting persistence estimates. Second, one could use a persistence operationalization that does not require a prior ordering. When the disturbances are joint normally distributed, the expected vector of shock values, given a shock of known magnitude to one of the variables (e.g., $u_{i,t} = k$), becomes a linear function of that initial shock: $E(u_{jt} | u_{it} = k) = k\sigma_{ij}/\sigma_{ii}$ (Evans and Wells 1983). After deriving the expected values for all shock variables, the over-time impact of the entire vector can be simulated. Pesaran, Pierse and Lee (1993), on the other hand, use a spectral-density based estimate that immediately incorporates contemporaneous effects.[5] Their approach, however, does not offer a formal link between short- and long-run dynamics.

In conclusion, multivariate persistence is a comprehensive, yet empirically tractable approach to assessing long-term marketing effects. In the following section, we present a simple yet managerially relevant case study to illustrate the proposed modeling framework.

4. A Case Study: The Persistence of Media-mix Effects

4.1. Description of the Data

Seventy-six monthly observations are available on a large home-improvement chain's sales figures, gross margins, advertising budget, and expenditures on print and TV/radio advertising. All data are expressed in constant dollars. The number of outlets remained stable in the time period under study (1980–1986), so we cannot assess response effects of distribution in the model. A graph of sales revenue and total advertising is given in Figure 1. Print advertising refers to fliers inserted in newspapers or to newspaper ads, which were used to announce temporary price reductions. We, therefore, can expect that the duration of its effect will be comparable to that obtained when analyzing the over-time impact of price promotions directly. Previous research suggests that promotions typically have an effect in the same period and in a few subsequent periods through purchase reinforcement (Blattberg and Neslin 1989). Radio and TV advertising, on the other hand, was used to improve the chain's image by broadcasting a "customer value" theme and can be expected to have longer-lasting effects.

4.2. Are Sales and Advertising Stable or Evolving?

The data plots in Figure 1 suggest the presence of an upward trend or evolution in both sales and total advertising spending. To substantiate this visual impression, we used the formal unit-root test described in equation (4). This test was applied not only to

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[5] Central to their approach is the fact that any unit-root series can be written as the sum of a random walk and a stationary process, in which the former (latter) carries the permanent (temporary) part of a shock to the series. The (normalized) variance-covariance matrix of the series' random-walk parts measures the long-run association between the series of interest, and provides another persistence operationalization (see also Van de Gucht, Kwok and Dekimpe 1993).
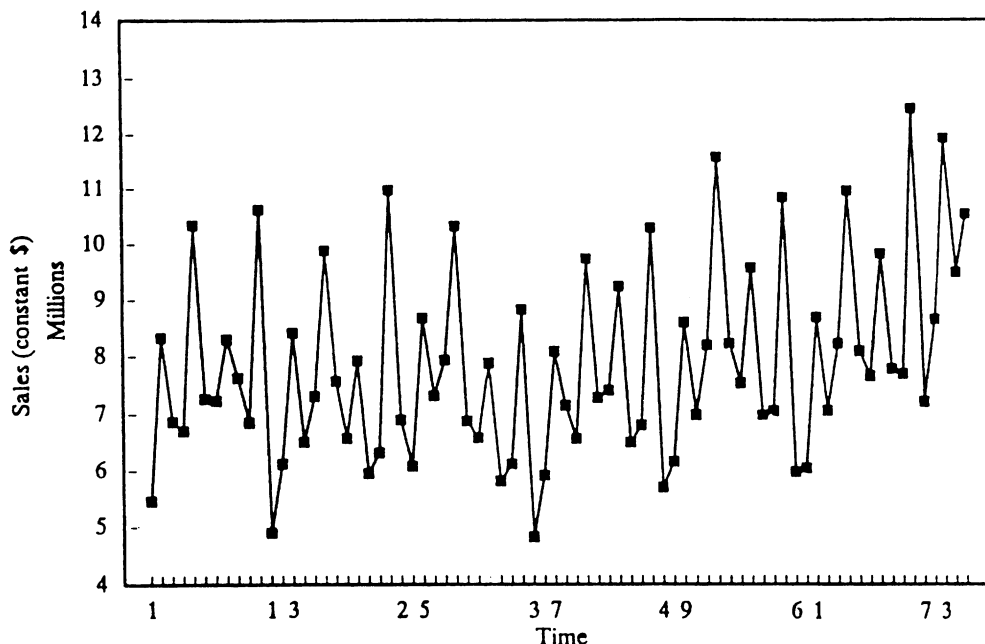
FIGURE 1A. Sales Series.

performance and total advertising but also to TV/radio advertising, print advertising and gross margins. In the test equations, we included seasonal dummy variables to allow for deterministic seasonal effects,[6] and standard F-tests were used to determine how many lagged-difference terms should be included. In the second column of Table 1, we indicate the smallest number of lagged differences that resulted in insignificant F-tests for one, two and three extra terms. For the sales variable, for example, we initially estimated a model without any lagged difference terms (i.e., $m = 0$), but the corresponding F-statistics indicated the need to increase the value of m, i.e., to add ΔS_{t-1} to the test equation. The F-tests reported in columns three to five indicate that there was no need to further augment the test equation with ΔS_{t-i}, $i = 2, 3, 4$. Columns six and seven show the coefficient of S_{t-1} (b) and its associated t-statistic. To determine whether b is significantly different from zero (i.e., no unit root), we compare the t-statistic with the critical value (-2.89) listed in Fuller (1976). If the computed t-statistic is smaller than -2.89, the unit-root null hypothesis is rejected.

Our test results, which are summarized in the last column of Table 1, show that sales and total advertising spending have a long-run or evolving component. Therefore, unexpected changes in the chain's performance can have a continuing impact, and VAR models on the differences should be used to determine whether advertising shocks indeed have a long-run impact. Similar conclusions were obtained for print and TV/radio advertising. On the other hand, the conclusion for gross margins is borderline between stability and evolution. Indeed, the results are sensitive to the significance level adopted for $F(1, \cdot)$. As we will indicate in Section 4.4., this ambiguous result does not affect our conclusions about the long-run profitability of advertising.

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[6] Ghysels, Lee and Noh (1991) show that a failure to account for deterministic seasonal effects distorts the test results when these effects are present in the data-generating process: a bias is introduced in the size of the test, while there is also a considerable power reduction. Including redundant seasonal dummy variables also tends to reduce the power of the test, but to a much lesser extent than their erroneous omission.
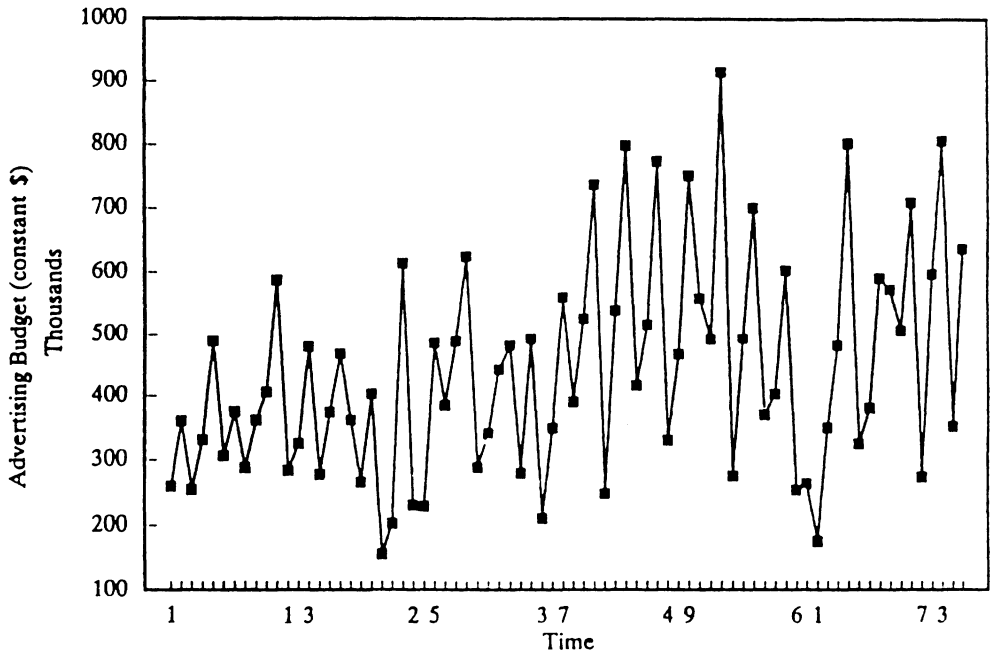
FIGURE 1B.   Total Advertising Budget.

4.3. *How Important Are the Long-run Sales Fluctuations?*

Having revealed the *existence* of a long-run component in the company's sales figures, we now quantify its *relative importance*. Put differently, we already know that the long-run performance forecast should be updated after an unexpected change in current sales, but by how much remains to be determined. Table 2 gives Campbell and Mankiw's $A(1)$ measure for several low-order ARMA models. Rather than trying to determine an "optimal" ARMA model for ΔS_t using likelihood-ratio tests, we estimated a sequence of low-order ARMA models, and computed the ratio of the sum of the MA-coefficients to the sum of the AR-coefficients for each of these models. As such, we could assess the robustness of the univariate persistence estimates. To account for the potentially con-

TABLE 1

Testing for a Long-run Component: ADF-Test

Series	m	$F(1, \cdot)$	$F(2, \cdot)$	$F(3, \cdot)$	b-coeff	t-stat	Unit Root?
Sales	1	0.69	0.54	0.60	−0.06	−0.64	yes
Total Adv.	2	0.75	1.16	0.84	−0.28	−2.04	yes
Print	2	1.07	1.27	1.10	−0.37	−2.45	yes
TV/radio	2	0.56	1.01	0.80	−0.35	−2.15	yes
Margins	0	3.10	1.76	1.17	−0.34	−3.51	?
	1	0.47	0.27	0.31	−0.26	−2.50	

[1] m gives the number of lagged differences in the augmented test equation, and the F-statistics test the significance of one to three additional lagged differences. b is the coefficient of S_{t-1} in equation (4), and t-stat is the corresponding t-statistic which is compared against the 5% critical value of -2.89. We also tested for a second unit root, but no such evidence was found. The results for sales and all three advertising media were validated using Johansen's multivariate full-information maximum-likelihood procedure for cointegration testing (Johansen 1988).

TABLE 2

Univariate Persistence Estimates for the Sales Series

(p, q)	$\hat{A}(1)$ (S.e.[$\hat{A}(1)$])	(p, q)	$\hat{A}(1)$ (S.e.[$\hat{A}(1)$])
(0, 1)	0.570 (0.106)	(1, 3)	0.555 (0.146)
(0, 2)	0.570 (0.126)	(1, 4)	0.696 (0.214)
(0, 3)	0.557 (0.148)	(1, 5)	0.717 (0.187)
(0, 4)	0.646 (0.166)	(3, 1)	0.563 (0.128)
(0, 5)	0.764 (0.187)	(4, 1)	0.545 (0.098)
(1, 1)	0.570 (0.126)	(5, 1)	0.649 (0.224)

Mean = 0.589; median = 0.569.

founding effects of deterministic seasonal factors, these models were estimated on the residuals of a prior regression of ΔS_t on seasonal dummies rather than on ΔS_t itself.[7]

Each low-order ARMA model provides a somewhat different approximation to the infinite-shock representation of ΔS_t, but the associated persistence estimates are quite robust: they all fall between 0.545 and 0.764. The mean and median of the $A(1)$ estimates in Table 2 are, respectively, 0.589 and 0.569. These figures suggest that approximately 60% of an initial sales increase persists in the long run, and that about $100 - 60 = 40\%$ is temporary. Hence, management should update its long-run sales forecast by approximately \$60,000 after an unexpected \$100,000 increase in current performance.

4.4. Quantifying the (Long-run) Impact of Advertising Shocks

We now investigate whether variations in advertising spending have any trend-setting effects. First, we consider a bivariate model between sales and total advertising. This analysis does not yet take coordinated decision making across the media into account but illustrates the method in a simple setting. Next, we compare the short- and long-run effectiveness of different media in a trivariate framework.

Do Advertising Pulses Have a Persistent Effect?

Based on Akaike's Information Criterion, a VAR model of order 2 was selected. Again, we allowed for deterministic seasonal effects. Moreover, Wald- and likelihood-ratio tests were performed to check whether ΔS_t and ΔA_t are influenced by ΔS_{t-12} and ΔA_{t-12}, but no such evidence was found. Since we knew a priori that sales could not have a contemporaneous feedback effect, we could uniquely identify the Cholesky decomposition that transforms the VAR model into a system with uncorrelated error terms. Figure 2 traces the *differential* impact on sales and advertising of a one-unit advertising shock. Figure 2 is derived from a restricted VAR model in which all coefficients with a *t*-statistic less than one in absolute value have been restricted to zero (Pesaran et al. 1993). The impulse-response functions derived from the unrestricted model are similar in shape but result in persistence estimates with larger standard errors.

Figure 2 reveals, first, that the impact of advertising shocks extends well beyond the three periods that are explicitly included in the VAR model. For example, the incremental

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[7] A similar implementation can be found in Pesaran and Samiei (1991). To be able to detect overdifferencing, the order of the moving-average part was at least one and a maximum-likelihood procedure was used which did not preclude the occurrence of a unit root in the MA-polynomial. We estimated all models with $0 \leq p \leq 5$ and $1 \leq q \leq 5$, but report only those models for which 1) convergence was obtained within 50 iterations, and 2) the bounds of stationarity/invertibility were not exceeded in any iteration. Overall, little evidence of overdifferencing was found, and the low-order ARMA models provide further support for the evolving nature of S_t.
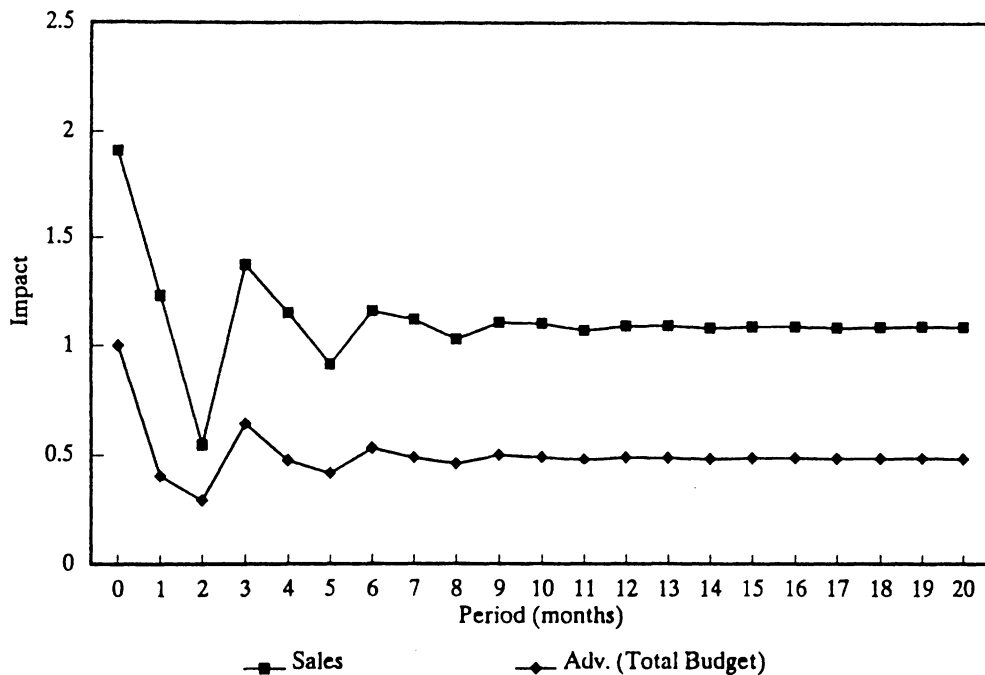
FIGURE 2.   Impact of a Unit Advertising Shock.

sales resulting from advertising carry-over can affect future sales in a number of ways: repeat purchases by customers who bought the product in the first period because of the additional advertising support, and first purchases by customers affected by additional word-of-mouth communication. Moreover, these higher sales can feed back to higher advertising spending in subsequent periods, etc. Because of such chain reactions, a cyclical sales response emerges whose fluctuations gradually decrease. Second, the incremental sales and advertising expenditures stabilize at a *nonzero* level, providing evidence *and* a quantification of Little's hysteresis effect. Specifically, an extra advertising dollar in the current period updates the long-run sales forecast by $1.086, and the long-run advertising forecast by $0.486. These figures were derived in two ways: through a simulation of the response function and by a substitution of the parameter estimates into equation (B.6). Asymptotic standard errors were derived using the delta method, and the resulting *t*-statistics are 3.27, 10.13 and 1.94 for the long-run impact on sales, on advertising and for the net effect, respectively. Linear response models were used to derive these persistence estimates. Diminishing-returns-to-scale effects are implicitly accounted for by using advertising shocks: the same $10,000 advertising spending would be all shock if historical spending was zero, but would only have a $4,000 shock value if historical spending resulted in an expected advertising level of $6,000. Our persistence estimates show that as more is spent on advertising in the current period, higher spending levels in future periods become more likely and a net long-run impact on sales results of $0.600 (1.086 − 0.486). This net effect shows that *current advertising investments result in a positive dollar inflow in the long run*.

    What about the long-run profitability of advertising spending? Given a sales persistence of 1.086, and a long-run advertising impact of 0.486, the required margin to break even is 0.448 (i.e., 1.086 × 0.448 = 0.486). Gross margins are borderline stable or evolving, as indicated before. If they are stable, we can interpret the sample mean (0.348) as the best long-run forecast of the chain's margin. This sample mean is well below the required

break-even margin. If they are evolving, we should look at all gross margins over time: during the six and a half years under study, the break-even margin 0.448 was never attained. Therefore, we can safely conclude that *even though advertising has a positive net sales effect, it does not have a positive long-run profit impact*. In our break-even calculations, we assessed the profit implications *per period* after the market response has stabilized, i.e., after both impulse-response functions have converged. We did not incorporate the initial one-dollar outlay in our calculations since it is a sunk cost that does not affect the profitability in periods 21, 22, ... This one-dollar outlay, however, should be taken into account when determining from what point onwards the initial expenditure is fully recovered.

Are the Persistence Measures Robust?

To validate our findings, three approaches were adopted. First, we investigated if a long-run effect was attributed to advertising because other trend-setting factors were omitted from the model. For that purpose, we derived persistence estimates from two extended models. In a first model, we added a deterministic trend to the sales equation as a proxy for other potentially important factors, such as population growth or a gradual improvement in the chain's positioning or segmentation scheme. In the second model, we included an economic indicator to account for changes in the economic climate. The selected indicator was the unemployment percentage in the state where all outlets were located.[8] In both cases, the persistence estimates were very similar to the ones reported. For the model with a linear trend in the sales equation, the long-run impact of an advertising shock on the long-run sales and advertising level was 0.951 and 0.482, respectively. For the model with the economic indicator, the corresponding values are 1.123 and 0.503.

Second, to assess the stability of our findings, we calculated the persistence estimates from VAR models estimated on, respectively, the first and last 50 observations. The long-run impact on sales and advertising of an advertising shock was 1.291 and 0.475 for the first 50 observations, versus 0.921 and 0.468 for the last 50 observations, indicating a considerable degree of stability.

Finally, to assess the validity of the VAR model from which we derived the persistence calculations, we compared its forecasting performance with a number of competing specifications. Specifically, we derived one-step ahead forecasts from a model estimated on 60, 61, . . . , 75 observations, and compared the mean squared (MSE) and mean absolute (MAE) error with the forecasting performance of four popular alternative specifications: the Koyck model and the partial-adjustment model, each with and without a deterministic trend. Overall, the VAR model performed significantly better than these competing models, reducing their MAE by 31 to 69%, and their MSE by 52 to 87%.[9]

Do Print and TV/Radio Pulses Have a Different Short- and Long-run Effectiveness?

Based on the AIC criterion, a second-order model was selected to assess the impact of print and TV/radio expenditures on sales. For the unrestricted VAR(2) model, the residual correlation between print and TV/radio was only 0.010 and not significant ($t = 0.174$). Consequently, the imposed causal ordering should not have a major im-

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[8] At the U.S. level, unemployment has been shown to both Granger cause and to be Granger caused by GNP (Evans 1989). We treated unemployment as an exogenous variable, and, based on the ADF test, used the first difference of the variable.

[9] A detailed description of these results is available from the authors upon request. It should be noted that because of the imposed causal ordering, and since we only considered the one-step ahead sales forecasts, we did have "a fair comparison" between our two-equation VAR model and the competing single-equation models.
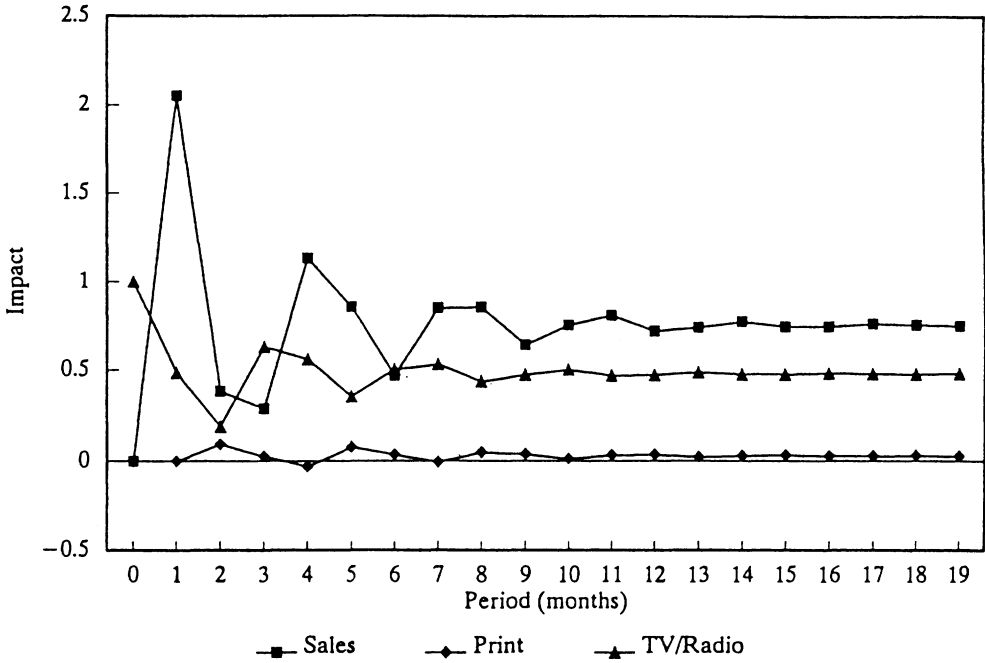
FIGURE 3A.   Impact of a Unit Shock to TV/Radio Advertising.

pact. In what follows, we assume that the medium that is "shocked" is ordered first and assess the sensitivity of our findings to this assumption. Sales are always ordered last, since they cannot realistically feed back into media spending in the same period. The
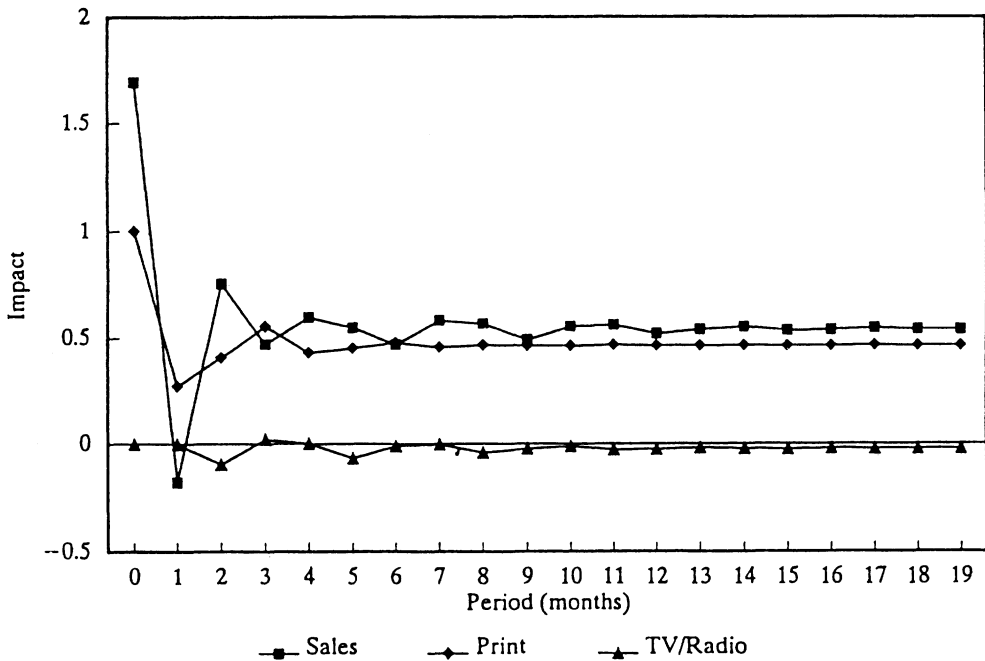

FIGURE 3B.   Impact of a Unit Shock to Print Advertising.

TABLE 3

Persistence Calculations in a Trivariate Model

A. Impact of a unit shock to print advertising	
Persistent effect on sales	0.541
Persistent effect on print adv.	0.465
Persistent effect on TV/radio adv.	−0.021
Net long-run effect	0.097

B. Impact of a unit shock to TV/radio advertising	
Persistent effect on sales	0.752
Persistent effect on print adv.	0.029
Persistent effect on TV/radio adv.	0.481
Net long-run effect	0.242

C. Impact of a unit shock to sales	
Persistent effect on sales	0.761
Persistent effect on print adv.	0.029
Persistent effect on TV/radio adv.	−0.001

[1] All figures are based on a restricted VAR(2) model. In panel A, print advertising is ordered first, and in panel B TV/radio advertising is ordered first. The figures in panel C are obtained irrespective of the ordering between print and TV/radio advertising. The sales series is always ordered last. Overall, the causal ordering of both advertising media did not have a significant impact.

impulse-response functions are given in Figure 3, and Table 3 summarizes the persistence estimates.[10]

When comparing the short- and long-run effectiveness of both media, the following observations can be made. First, even though print advertising has a significant instantaneous impact, no meaningful *net* long-run impact is observed. After an unanticipated $100,000 increase in the chain's current spending on print advertising, the long-run sales forecast is updated by $54,100, but long-run print-advertising spending is updated by $46,500. So, increased print-advertising spending leads to about as much additional spending in the long run as it does to additional sales revenue. On the other hand, the image-oriented TV/radio messages do not have a significant instantaneous impact, but result in a much larger long-run effect. Indeed, even though the TV/radio forecast is updated by an amount ($48,100) similar to the print-advertising forecast ($46,500), the long-run sales forecast is now adjusted by $75,200. However, the company's profit margins are not sufficient to pay for the added spending, so once again we cannot find a positive *profit* impact in the long run. Other findings from the VAR model are that long-run forecasts of the spending level in one medium are only marginally affected by an unexpected increase in the other medium. Finally, the long-run sales impact of a unit shock to $e_{S,t}$ (i.e., non-advertising shocks) is 0.761, which is comparable to the value obtained in the bivariate model (0.760).

[10] To assess the validity of the trivariate model, we again estimated two extended models. When the economic indicator was added to the model, the persistence estimates were not affected. When a deterministic trend was added, the persistence estimates were again very similar. For example, the long-run sales impact of a TV/radio shock was 0.774.

An executive summary on media effectiveness is as follows: for achieving short-term promotional goals such as reducing store inventories, print advertising is more effective. For developing long-run sales, the image-oriented TV/radio spending is the most effective, with an absolute (net) persistence level of about 75 (24) cents per extra dollar spent in that medium. However, building long-run sales through advertising is costly, as the company's profit margins are too low to absorb the additional expenditures. This example shows how persistence modeling can be used to assess the tradeoffs between strategies for developing long-run sales and strategies for improving profitability.

As a final comment on the empirics, it is important to realize that various shocks and their persistence may all affect sales at the same time. So, when we argue that a fraction of the advertising shock effects is permanent, we do not imply that this persistence on sales will actually occur, causing the series to grow infinitely after a given shock. Other shocks will affect sales in the future, some of which with an opposite long-run effect. Our analyses, however, have *disentangled* the various sources of long-run movements in sales in a way that allows us to make managerially important inferences.

5. Conclusion—Areas for Future Research

We have introduced a new method to measure the long-run effectiveness of marketing, called persistence modeling. It differs from previous approaches in that it computes the *total* long-run impact of *unexpected* shocks in any marketing variable. Persistence modeling satisfies our three criteria for long-run effectiveness measures: (1) it quantifies the long-run impact of specific marketing actions, (2) it distinguishes long-run effectiveness in stable versus evolving environments, and (3) it provides a formal link between marketing's short- and long-run effects.

We discussed the different steps of the proposed framework and used them to compare the short- and long-run effectiveness of the advertising media used by a home-improvement chain. Several managerially relevant conclusions emerged from our analyses. First, advertising effects did not dissipate within one year but had a persistent effect on the chain's sales evolution. This finding differs from Clarke's (1976) conclusion that 90% of the measurable effects of advertising on sales are consumed within a few months. Our findings suggest that Clarke's conjecture may be valid in stable environments but should not be generalized to evolving markets. Hence, if the distinct nature of evolving environments is not taken into account, *one may seriously underestimate the long-run effectiveness of advertising*. Second, our multivariate persistence estimates provide empirical support for Little's hysteresis effect. To the best of our knowledge, this study is the first to quantify hysteresis using ETS models. Third, a substantial net long-run sales effect was observed only for image-oriented TV/radio pulses. This provided empirical support for the notion that a growing emphasis on sales promotions may not be helpful to a brand's long-run performance.

Possible areas for future research remain wide open. First, more work is needed on the dynamic optimization of shock-based strategies. Several studies have compared the short-term impact of pulsed and even-spending advertising policies. However, these studies considered only the short-term implications of the different spending patterns and did not model the shock effect of the pulse. Indeed, repeated shocks may affect the baseline forecast down the line and may influence what is considered unexpected in the future (see, e.g., Winer 1986). Second, the study of temporal aggregation bias may receive a new impetus: the presence or absence of unit roots is *not* affected by the level of aggregation, and disaggregate univariate persistence estimates can be derived from their aggregate counterparts (Dekimpe 1992). More research is needed to see whether this also holds for their multivariate extensions. Third, different persistence levels can be hypothesized and measured for positive versus negative deviations from a brand's expected support

level. Fourth, we could examine the stability of market-response and spending-feedback behavior. Specifically, we could assess whether certain *major* events can still be treated as regular shocks that do not change the model parameters, or whether they should be modeled as structural breaks (see, e.g., Perron 1990). Finally, other empirical studies on marketing persistence should be conducted, especially on databases at the household level. They will contribute to the development of empirical generalizations on the long-run effectiveness of promotions, image-oriented advertising and other marketing efforts.

We conclude by revisiting our opening example of sports-car sales. The first question facing dealers, is the downturn in sales of a permanent or temporary nature, can be answered by testing for unit roots in the time series of sports-car sales. If there are no unit roots, we have little to say about long-run sales movements, and we can only make short-term inferences about marketing effects. However, if evolution is established, we can test to what extent long-run sales of sports cars are affected by marketing strategies such as advertising spending and price levels, by calculating the appropriate persistence measures. This new knowledge can be used in the design of marketing strategies that create a sustainable competitive advantage.[11]

Appendix A

To isolate the *differential* effect of a one-unit advertising shock in a bivariate sales-advertising (S-ADV) model, one can estimate a VAR model, assume both variables equal to zero prior to t, and set $(u_{S,t}, u_{ADV,t})$ equal to $(0, 1)$. Next, one solves recursively for ADV_{t+k} and S_{t+k} ($k = 0, 1, \ldots$) under the assumption that no further shocks occur to the system, i.e., assuming $(u_{S,t+k}, u_{ADV,t+k}) = (0, 0)$ for $k = 1, 2, \ldots$. This procedure is illustrated for a first-order model:

$$\begin{bmatrix} S_t \\ ADV_t \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} * \begin{bmatrix} S_{t-1} \\ ADV_{t-1} \end{bmatrix} + \begin{bmatrix} u_{s,t} \\ u_{ADV,t} \end{bmatrix}. \tag{A.1}$$

With the starting conditions given before, the sales and advertising levels in period t become:

$$\begin{bmatrix} S_t \\ ADV_t \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{A.2}$$

Equations (A.3)–(A.4) give the corresponding values for period $t + 1$ and $t + 2$:

$$\begin{bmatrix} S_{t+1} \\ A_{t+1} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} * \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pi_{12} \\ \pi_{22} \end{bmatrix}, \tag{A.3}$$

$$\begin{bmatrix} S_{t+2} \\ A_{t+2} \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} * \begin{bmatrix} \pi_{12} \\ \pi_{22} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \pi_{11}\pi_{12} + \pi_{12}\pi_{22} \\ \pi_{21}\pi_{12} + \pi_{22}\pi_{22} \end{bmatrix}. \tag{A.4}$$

A similar procedure can be applied to derive the sales and advertising levels in $t + 3, t + 4, \ldots$.

Appendix B

For a two-variable case, the long-run impact of a given shock easily can be derived analytically. Starting from equation (12), one can combine terms to get

$$[1 - \alpha(L)L]\Delta S_t = [\gamma + \beta(L)L]\Delta ADV_t + e_{S,t}, \tag{B.1}$$

$$[1 - \eta(L)L]\Delta ADV_t = [\delta(L)L]\Delta S_t + e_{ADV,t}. \tag{B.2}$$

(B.2) can be rewritten as

$$\Delta ADV_t = \frac{\delta(L)L}{1 - \eta(L)L} \Delta S_t + \frac{1}{1 - \eta(L)L} e_{ADV,t}. \tag{B.3}$$

Substituting (B.3) into (B.1), we get

$$[1 - \alpha(L)L]\Delta S_t = \frac{[\gamma + \beta(L)L][\delta(L)L]}{1 - \eta(L)L}\,\Delta S_t + \frac{\gamma + \beta(L)L}{1 - \eta(L)L}\,e_{\mathrm{ADV},t} + e_{S,t}. \tag{B.4}$$

(B.4) can be rewritten as

$$\Delta S_t = h(L)e_{S,t} + g(L)e_{\mathrm{ADV},t}, \tag{B.5}$$

which is the moving-average representation of ΔS_t. Using a similar logic as in the univariate case, $g(1)$ measures the long-run effect of a unit advertising shock, and $h(1)$ gives the long-run impact of a unit increase in $e_{S,t}$. By combining terms in (B.5), it is easy to show that

$$h(L) = \frac{1 - \eta(L)L}{[1 - \alpha(L)L][1 - \eta(L)L] - [\gamma + \beta(L)L][\delta(L)L]}, \tag{B.6a}$$

$$g(L) = \frac{\gamma + \beta(L)L}{[1 - \alpha(L)L][1 - \eta(L)L] - [\gamma + \beta(L)L][\delta(L)L]}. \tag{B.6b}$$

$g(1)$ and $h(1)$ are obtained after substituting 1 for L.

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